Characteristics of the Dynamic of Mobile Networks

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Outline

- MOSAR Project
  - Project overview

- Dynamic Network Characterization
  - Motivation
  - Statistical analysis of snapshots of graphs
  - Towards a global analysis of the dynamics
  - Modeling of the dynamics

- Conclusion
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Conclusion
Deployment of a large-scale dynamic networks

Control of antimicrobial resistance of bacteria responsible for major and emerging nosocomial infections.

MOSAR Experiment
- Medical / staff / Patients (500 people)
- Individual antibiotic use;
- Characterization of the isolates bacteria and their epidemicity;
- 7/24 during 6 month long period

Document contact frequencies
- Associate 1 sensor with each actor
- monitor the dynamic (inter & intra contact)
Patient room

Patient room
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E. Fleury
Multi modal / multi time scale
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Objectives

MOSAR project

- Better understand the intrinsic characteristics / **properties** of dynamic networks
- Model / analyze interaction between node/users
- Describe accurately the dynamics

Two central questions:

- Obtaining random models that reproduce “these” properties
- How do their functionalities constrain the structures of real network?
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Overview

MOSAR Project

Dynamic Network Characterization

Conclusion

Preliminary data


“Toy” traces are now available

- 41 nodes, 3 days (254 151 sec), every 120sec
- 820 possible links,
- “[...] inter contact time distribution can be compared to the one of power law [...]”

“Power law...”

- What do power law really signify?
- Is it the ultimate argument?

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Methodology

Descriptive: Standard graph properties

1. as a function of time to provide an empirical statistical characterization of the dynamics.
2. temporal evolution of the snapshots
3. statistical signal processing

Analysis: global indicators

- connected components, triangles, and communities

Model

- We propose models to perform random dynamic networks simulations.
Standard graph properties

Snapshots $G_t = (V^0, E_t)$

- Active links: $E(t) = |E_t|$
- Connected vertices: $V(t) = |\{u \in V^0, d_{G_t}(u) > 0\}|$
- Average degree of connected vertices is $D(t) = \sum_{u \in V^0} d_{G_t}(u) / V(t)$
- Number of connected components (maximal subgraph such as every node of the subgraph is connected to each another node): $N_c(t) = |C_{G_t}|$
- Number of triangles: $T(t) = |T_{G_t}|$

<table>
<thead>
<tr>
<th>Property</th>
<th>IMOTE</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Corr. Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#Active links</td>
<td>$E(t)$</td>
<td>21.9</td>
<td>12.4</td>
<td>5200</td>
</tr>
<tr>
<td>#Connected vertices</td>
<td>$V(t)$</td>
<td>19.9</td>
<td>4.7</td>
<td>7400</td>
</tr>
<tr>
<td>Avg degree</td>
<td>$D(t)$</td>
<td>2.1</td>
<td>0.8</td>
<td>3600</td>
</tr>
<tr>
<td>#CC</td>
<td>$N_c(t)$</td>
<td>4.8</td>
<td>2.1</td>
<td>5600</td>
</tr>
<tr>
<td>#Triangles</td>
<td>$T(t)$</td>
<td>6.9</td>
<td>8.30</td>
<td>4700</td>
</tr>
</tbody>
</table>
Standard graph properties (cont)

**Probability distribution**

- time bin of 1s <<< period.
- PDF obtained are not heavy tailed
- variability is not very large (stdv is a good measurement of the variability)
Network is sparse

- less than 10% of active links among the 820 possible links
- at no time the network is a single connected component.
- many nodes remain isolated during long times (around 50% on average for daytime and more than 90% for nighttime).
Standard graph properties (cont)
Standard graph properties (cont)

**differential sequence:** \( DS[k] = D[k + 1] - D[k] \)

- log-log representation of the covariance in the wavelet domain
- \( S_j \) is roughly the average of the wavelet coef. at scale \( j \)
- Hurts exponent is close to the special value 0.5.
- no long range \( \rightarrow \) Independent Identically Distributed (IID)

\(^a\) P. Abry and D. Veitch, Wavelet analysis of long-range dependent traffic, TIT, 1998
Standard graph properties (cont)

**Large number of triangles**

- \( \mathbb{E}(T(G(p, n))) = \binom{N}{3} 3! p^3 \) & \( \mathbb{E}(E(G(p, n))) = p \frac{N(N-1)}{2} \)

- When there is \( k \) links, \( \mathbb{E}(T(G(n, k))) \sim \frac{8k^3(N-2)}{N^2(N-1)^2} \)

- 70 links (max) \( \rightarrow 40(60) \)

- 22 links (avg) \( \rightarrow 1(7) \)
# Dynamical characteristics

## Correlation times

- **temporal evolution** \((X(t): \text{univariate time-series})\)
- **The autocorrelation function of** \(X(t):\)**

\[
C_X(\tau) = \langle X(t + \tau)X(t) \rangle_t - \langle X(t) \rangle_t^2
\]

- **correlation time:** first time where the function \(C_X(\tau)\) goes to zero

## Notes

- correlation times of \(E, V\) and \(N_c\) are rather large: \(\sim 1h15.\)
- \(D\) and \(T\) have comparable correlation times.
- **This suggests that these properties evolve under a common cause.**
Dynamical characteristics (cont)

Mean : 140; $\alpha = 1.66$  
Mean : 3680; $\alpha = 0.60$

Contact and inter-contact durations

- $P[X > x] \sim c x^{-\alpha}$.  
- $\alpha > 2$: finite mean/variance; $\alpha < 2$, infinite variance (heavy tailed).  
- $\alpha < 1$, infinite mean/variance.
Dynamics of links creation and deletion

\[ E_\oplus(t) = |\{ e ∈ E_t, e \notin E_{t-1}\}|, \text{ the number of links added at time } t \]
Dynamics of links creation and deletion (cont)

\[ E_\ominus(t) = \left| \{ e \in E_{t-1}, e \notin E_t \} \right|, \] the number of links removed at time \( t \)

<table>
<thead>
<tr>
<th>Property</th>
<th>( E_\ominus(t) )</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Corr. Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge creation</td>
<td>( E_\oplus(t) )</td>
<td>0.15</td>
<td>0.55</td>
<td>680 ( \sim ) 12min</td>
</tr>
<tr>
<td>Edge deletion</td>
<td>( E_\ominus(t) )</td>
<td>0.15</td>
<td>0.55</td>
<td>680 ( \sim ) 12min</td>
</tr>
</tbody>
</table>
Multivariate statistics of graph properties

Cross-correlations

- Strong influence $E(t)$ over $V(t)$;
- $N_c(t)$ related to $E(t)$
- Less related: $N_c(t)$ and $V(t)$
- $E_\oplus(t)$ and $E_\ominus(t)$: mostly uncorrelated

<table>
<thead>
<tr>
<th></th>
<th>$E(t)$</th>
<th>$V(t)$</th>
<th>$N_c(t)$</th>
<th>$D(t)$</th>
<th>$T(t)$</th>
<th>$E_\oplus(t)$</th>
<th>$E_\ominus(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(t)$</td>
<td>1</td>
<td>0.85</td>
<td>-0.56</td>
<td>0.95</td>
<td>0.90</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td>$V(t)$</td>
<td>0.85</td>
<td>1</td>
<td>-0.20</td>
<td>0.70</td>
<td>0.66</td>
<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>$N_c(t)$</td>
<td>-0.56</td>
<td>-0.20</td>
<td>1</td>
<td>-0.70</td>
<td>-0.41</td>
<td>-0.16</td>
<td>-0.15</td>
</tr>
<tr>
<td>$D(t)$</td>
<td>0.95</td>
<td>0.69</td>
<td>-0.69</td>
<td>1</td>
<td>0.86</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td>$T(t)$</td>
<td>0.90</td>
<td>0.66</td>
<td>-0.41</td>
<td>0.86</td>
<td>1</td>
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<td>$E_\oplus(t)$</td>
<td>0.19</td>
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<td>0.15</td>
<td>1</td>
<td>0.03</td>
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<td>$E_\ominus(t)$</td>
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<td>0.11</td>
<td>-0.15</td>
<td>0.16</td>
<td>0.10</td>
<td>0.03</td>
<td>1</td>
</tr>
</tbody>
</table>
Multivariate statistics of graph properties

Joint distributions

- $P_{XY}(x, y) = P[X = x \text{ and } Y = y] = P[X = x / Y = y]P[X = x]$
- variation of the # links is not constant over the # vertices
Multivariate statistics of graph properties

**Link correlations**

- Most pairs of links have a very low correlation coefficient.

**Markovian evolution**

1. Correlation time link creation/deletion is small
2. Independent from the evolution of other graph properties
3. Links are independents
Multivariate statistics of graph properties

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Towards a global analysis of the dynamics

- not directly interpretable in the sequence of static graphs
- stability of connected components
- communities embedded in the network
- proportion of creation of triangles
Towards a global analysis of the dynamics

- not directly interpretable in the sequence of static graphs
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- proportion of creation of triangles
Triangles in the graphs

<table>
<thead>
<tr>
<th></th>
<th>$P_{+/tri+}$</th>
<th>$P_{+/tri-}$</th>
<th>$f_{+/tri+}$</th>
<th>$f_{+/tri-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMOTE</td>
<td>44 %</td>
<td>56 %</td>
<td>6 %</td>
<td>94 %</td>
</tr>
<tr>
<td>RANDOM</td>
<td>10 %</td>
<td>90 %</td>
<td>5 %</td>
<td>95 %</td>
</tr>
</tbody>
</table>

links / triangles

- $P_{+/tri+}$: link creation $\rightarrow$ triangle
- $f_{+/tri+}$: inactive link $\rightarrow$ triangle
- 40% of link creations increase the number of triangles
- Proportion of inactive links that would create a triangle is very low
- More potential links does not imply higher $P_{+/tri+}$
Modeling of the dynamics

Simulation algorithm

- transition model with Markovian property
- links $e$ are independent
- state of the network
- links $e$ changes with $P_{tr}(e, G_t)$
- duration $\tau(e)$ since the link $e$ has last changed its status

Ingredients

- contact / inter contact duration distribution
- elaborated graph properties ($E(t)$, $V(t)$, $N_C(t)$, $D(t)$)
- dynamical information (triangles)
Modeling of the dynamics

**Input**: Simulation time

**Output**: Random Dynamic Graph

```plaintext
foreach Simulation Time Step t do
    foreach link e do
        \( P_{tr}(e, G_t) = \text{TransitionProbability}(e) \) given the state \( G_t \);
        \( p_r = \text{Uniform}(0,1) \);
        if \( p_r \leq P_{tr}(e) \) then
            ChangeState(e);
    end
end
```
Ingredients I

Contact distribution

- heavy-tailed distributions for contact $P_{ON}$ and inter-contact $P_{OFF}$ durations

- $P_{+}(\tau)$: probability that one link that was OFF since $\tau$ ($\tau \geq 1$) is activated

- $P_{ON}(\tau) = P_{-}(\tau) \times \prod_{i=1}^{\tau-1} (1 - P_{-}(i))$

\[
P_{-}(\tau) = \frac{P_{ON}(\tau)}{\prod_{i=1}^{\tau-1} (1 - P_{-}(i))}, \quad \tau \geq 2, \quad P_{-}(1) = P_{ON}(1) \tag{1}
\]

\[
P_{+}(\tau) = \frac{P_{OFF}(t)}{\prod_{i=1}^{\tau-1} (1 - P_{+}(i))}, \quad \tau \geq 2, \quad P_{+}(1) = P_{OFF}(1) \tag{2}
\]
Rejection Sampling based on a Metropolis-Hastings algorithm

new state $G'_t = \{ G_t + S_e(t)\}^{\text{changed}}$, is accepted with probability

$$P_{RS}(G_t, G'_t) = \min \left(1, \frac{F(x(G'_t))}{F(x(G_t))} \right)$$

$F$ is the target PDF for the graph

The total probability of transition of link $e$ is then:

$$P_{tr}(e, G_t) = P_{-/+}(\tau(e)) \cdot P_{RS}(G_t, G'_t).$$
Ingredients III

**Imposed dynamics of triangles**
- reproduce the correct dynamical transition process concerning triangles
- do not want to change the mean probabilities of transition
- The weighted probabilities are then:

\[
P_{tr}(e, G_t) = \begin{cases} 
  P_+(\tau(e)) \frac{P_{+/tri-}}{f_{+/tri-}} & \text{for link creation without new triangle}, \\
  P_+(\tau(e)) \frac{P_{+/tri+}}{f_{+/tri+}} & \text{for link creation with a new triangle}.
\end{cases}
\]
Investigated models

- **A**: imposed empirical contact and inter-contact duration distribution only.
- **B**: imposed distributions of contact / inter-contact durations, and of number of connected components.
- **C**: distributions imposed contact / inter-contact durations and of number of connected vertices.

---

— Imote / o Model A / * Model B / + Model C

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Simulation results (cont)

--- Imote / o Model $\mathcal{A}$ / * Model $\mathcal{B}$ / + Model $\mathcal{C}$

$\mathcal{A}$: sole contact and inter-contact duration fails

- the number of connected vertices is strongly over-estimated
- the number of connected components is under-estimated
The density of the connected components (the groups) is underestimated

Links are spread uniformly in the graph

A, B and C fail!
Weighted models

- does not have an impact on the contact and inter-contact duration distributions
- the density of connected components is comparable to the experimental data
Simulate results (cont)

Density of frequent connected components

- $(\tau = 7 \text{ and } \sigma = 6)$
- classical models fail to create dense frequent connected components
- the number of frequent connected subgraphs is larger in the simulated data than in the original
MOSAR Project

Project overview

Dynamic Network Characterization

Motivation

Statistical analysis of snapshots of graphs

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Modeling of the dynamics

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contributions

- rigorous / coherent set of properties (basic / advanced)
- probability distribution of contacts and inter contacts is only one parameter
- global analyses to characterize the dynamics of the graph as a whole:
  - correlation between links
  - stability of the connected components
  - number of triangles
  - evolution of communities inside the interaction networks.
- simple / accurate models that generate random interaction graphs with satisfactory temporal properties.

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Conclusion

Futur / On going works

- Introduce non-stationarity (piecewise stationary model)
- Dynamic community computation
- Overlapping community detection
- Trajectories of individuals as a signature
- Large in situ test beds to be deployed...
Some references

Dynamic networks
