Diffusion in Dynamic Networks: Tentative for Measuring an Exposure

Jérémie Dumas
Christophe Crespelle
Éric Fleury

D-NET/INRIA, ENS de Lyon, Université de Lyon 1

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Introduction

In situ experiments

- I-Bird (Hospital) : 800 pers., 6 months.
- **Tubexpo** (Hospital) : 88 pers., 98 days.
- Infocom (Conference) : 40 pers., 3 days.
- Reality Mining (MIT) : 100 pers., 9 months.

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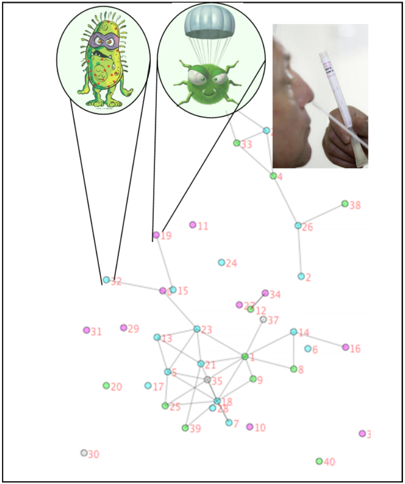

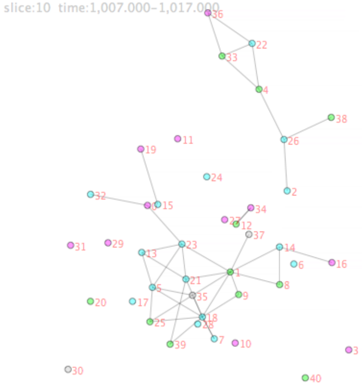

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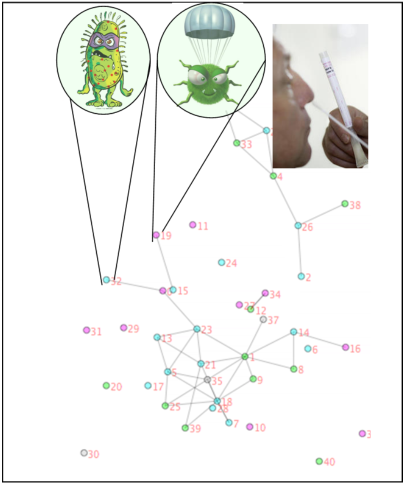

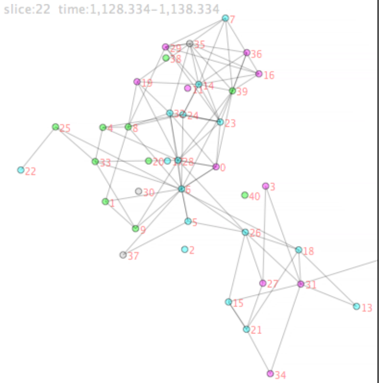

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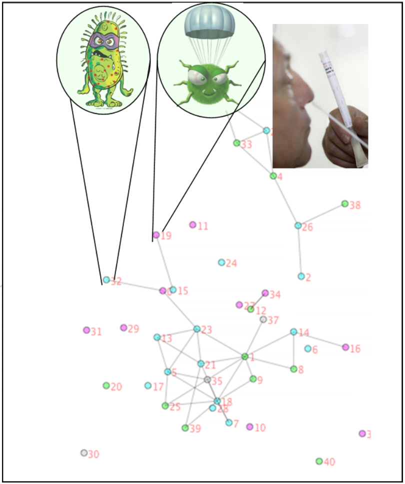

The main goal

- Analyze a spreading process in a dynamic network (contact graph)
- Characterize "*one*" dynamic for the dynamic graph
- Estimate the exposure of nodes to spreading process in a dynamic network
- Several definitions / scores

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Context

Concepts handled

- Dynamic graphs, *i.e.*, time evolving graph
- Nodes are 'fixed' $\longrightarrow V$
- Edges are dynamic $\longrightarrow E_t$
- Spreading from a set of seeds $S_0 \subseteq V$

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Dynamic graph

Definition

- Dynamic graph : sequence $(G_t = (V_t, E_t))_{0 \leqslant t < \delta}$
- Neighborhood of u in G_t : $\mathcal{N}_t(u)$
- Dynamic path (journey) : séquence $c_{t_0} = u_0, \ldots, u_p$ s. t. $\forall i \in [\![1, p]\!], u_{i-1}u_i \in E_{i-1+t_0} \vee u_{i-1} = u_i$
  - Only one hop / step
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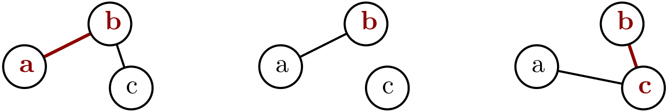

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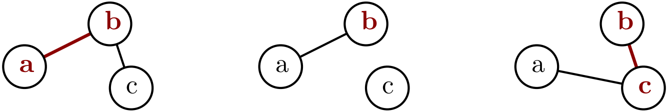

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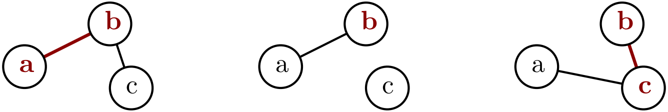
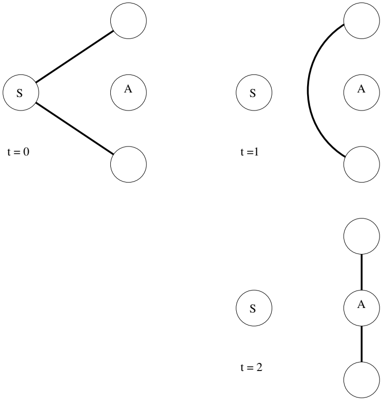


Example



Spreading time	$\mathcal{T}_{\{s\}}(a)(2) = 2$
# of paths	$\mathcal{C}_{\{s\}}(a)(2) = 2$

Spreading time
and number of spreading paths

Definition

- $\mathcal{T}_{S_0}(v)(t) = \min \left\{ d(c), \begin{array}{l} c \text{ path from } S_0 \text{ to } v \\ \text{with an arrival date } t_a(c) = t \end{array} \right\}$
- $\mathcal{C}_{S_0}(v)(t) = |\{c, d(c) = \mathcal{T}_{S0}(v)(t) \wedge t_a(c) = t\}|$, number of such paths

Complexity

Compute $\mathcal{T}_{S_0}$ and $\mathcal{C}_{S_0}$ for all v, t : $\mathcal{O}((n + m_{max})\delta)$ where δ is the period length.

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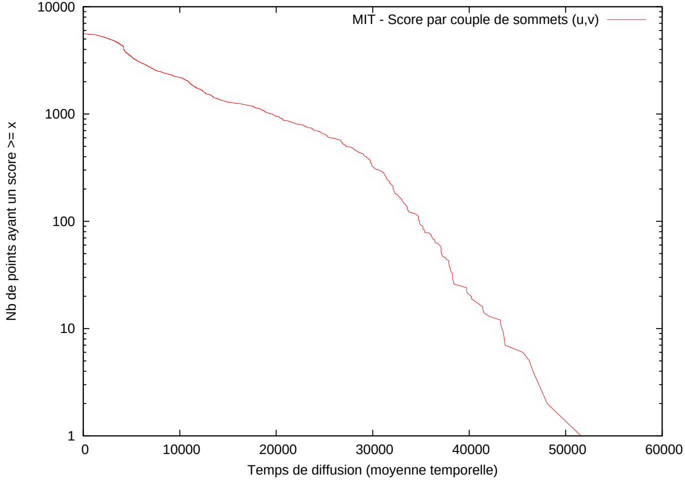
Inverse cumulative distribution



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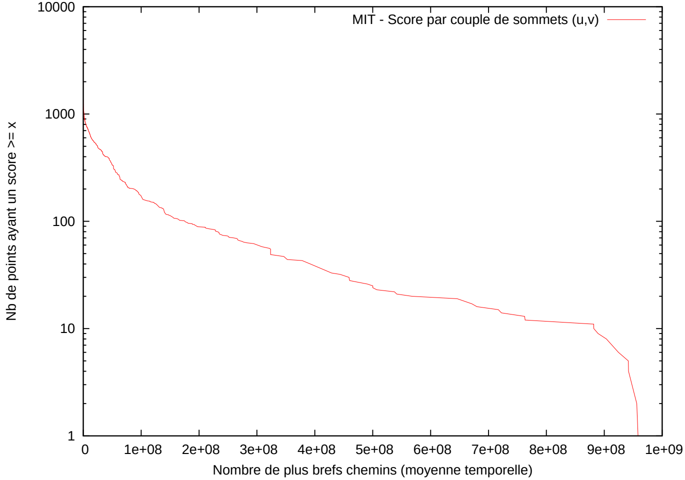

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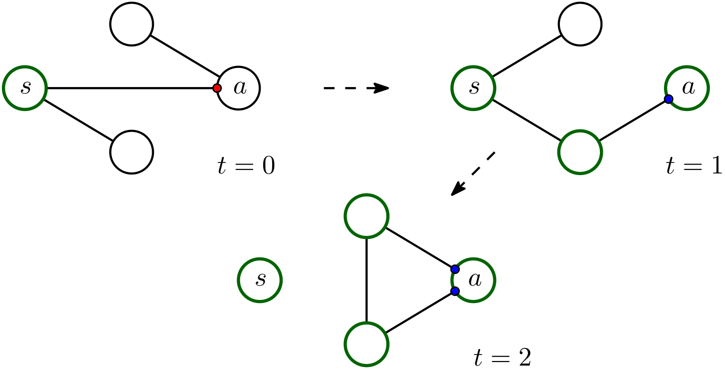

Simple contact	$\mathrm{I}_{\{s\},\gamma}(a)(2) = 1$
Extended contacts	$\mathrm{I}^{*}_{\{s\},\gamma}(a)(2) = 4$

Number of contacts

Simple contacts

Underlying idea

Compute the number of contacts between S_0 and v on the last γ snapshot/graphs.

Definition (Contact index)

$$\mathrm{I}_{S_{0},\gamma}(v)(t) = \sum_{i=t-\gamma+1}^{t} |S_0 \cap \mathcal{N}_i(v)|$$

Complexity

Compute $\mathrm{I}_{S0,\gamma}$ for all v, t : $\mathcal{O}((n + m_{max})\delta \times \gamma)$

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Number of contacts
Extended contacts

Spreading

Now, S_0 evolves, starting at t_d : $t \rightarrow S_t = f_c(S^0, t_d, t)$.

- Neighbors of S_t in G_t are aded to S_t.
- Same time complexity that for $\mathrm{I}_{S^0,\gamma}(v)(t)$

Definition (Extended contact index)

$$\mathrm{I}^*_{S_0,\gamma}(v)(t) = \sum_{i=t-\gamma+1}^{t} |f_c(S_0, t-\gamma+1, i) \cap \mathcal{N}_i(v)|$$

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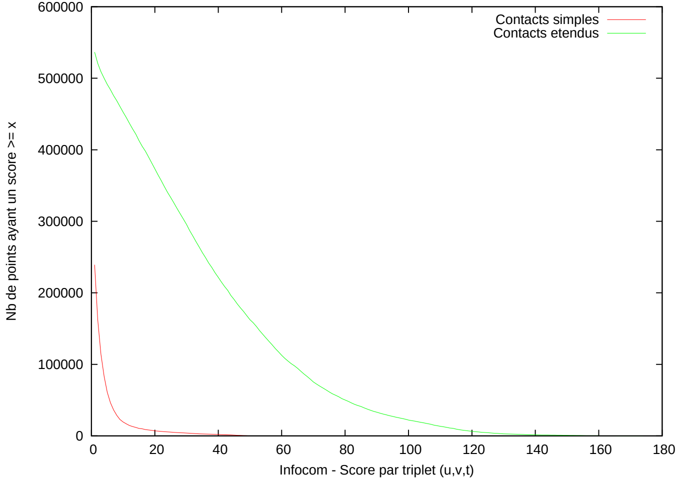

Dynamic Flow

Definition

Let $(G_t)_t$ a weighted dynamic graph + capacity on edges $(c_t)_t$.
Let (s, t_0) the source and (a, t_1) the sink.
We define $(\varphi_t)_{t_0 \leqslant t \leqslant t_1}$ a dynamic flow if it verifies the constraints :

1. **Positivity** : $\forall t, \forall x, y \in V, \varphi_t(x,y) \geqslant 0$
2. **Capacity** : $\forall t, \forall x, y \in V_t, \varphi_t(x,y) \leqslant c_t(x,y)$
3. **Conservation** :
   $\forall t_0 \leqslant t < t_1, \forall x \in V, \sum_y \varphi_t(y,x) = \sum_y \varphi_{t+1}(x,y)$
4. **Condition limits** :
   4.1 $\forall x, y \in V, x \neq s \Rightarrow \varphi_{t_0}(x,y) = 0$
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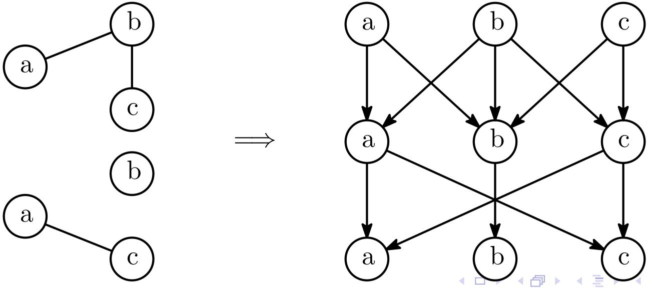
Transition graph

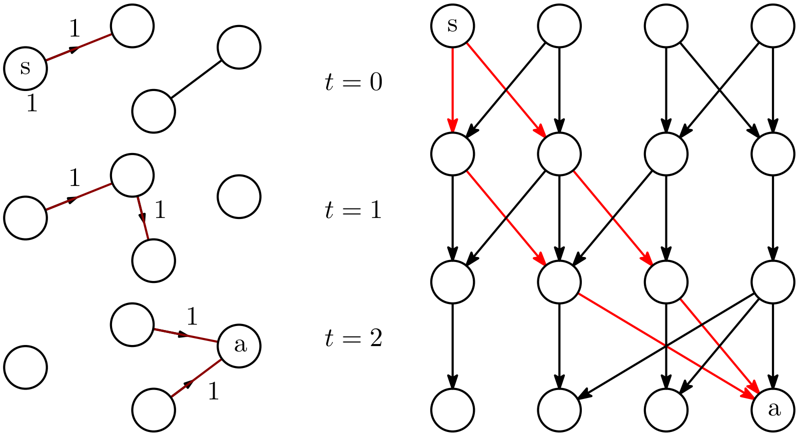
Definition

It's a unique graph that represents the sequence G_t.
Let $\mathcal{G} = (V^\mathsf{T}, E^\mathsf{T})$ the oriented graph such that :

- $V^\mathsf{T} = \{(x,t), x \in V, 0 \leqslant t \leqslant \delta\}$
- $E^\mathsf{T} = \{(x,t)(y,t+1), x = y \vee (x,y) \in E_t, 0 \leqslant t < \delta\}$



Correspondence with an ordinary flow



Dynamic flow	$\mathcal{Q}_{s,\gamma}(a)(2) = 2$

Compiting the dynamic flow

Remarks

- Bounded by $I \leqslant \mathcal{Q} \leqslant I^*$
- Load the transition graph in memory : expensive
- Run the Edmonds-Karp's algorithm on $\mathcal{G}$: not optimized
- but doable on actual data...

Complexity

Computing $\mathcal{Q}_{s,\gamma}$ for all v, t : $\mathcal{O}(n\gamma \times ((m_{max} + n)\gamma)^2)$

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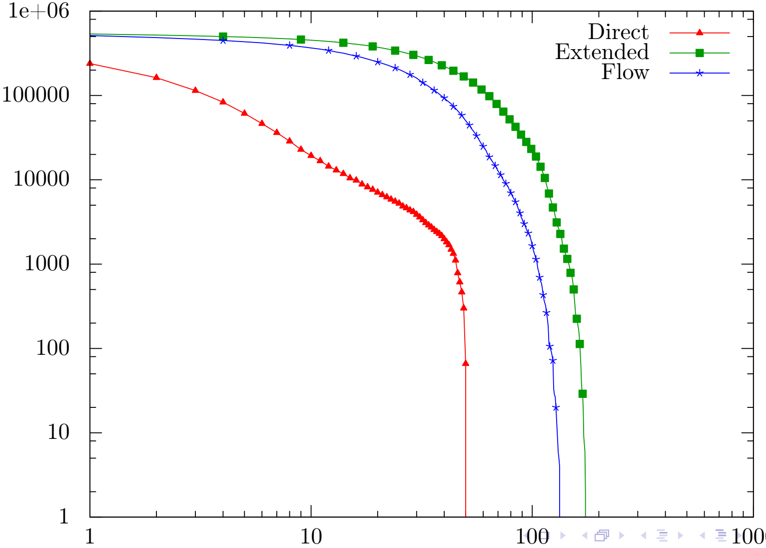
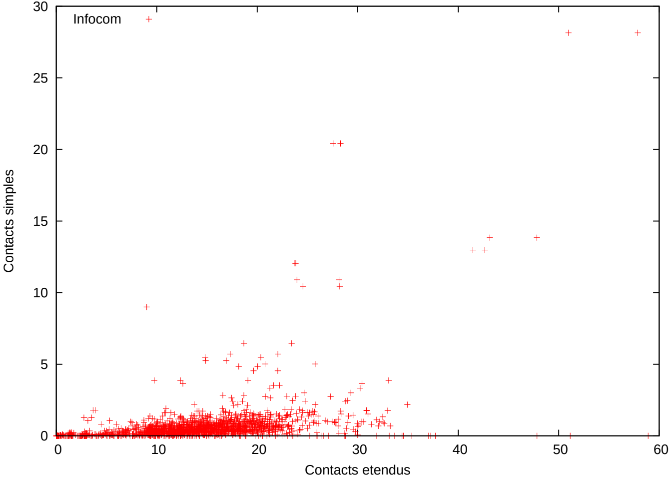
Inverse cumulative distribution



Correlations



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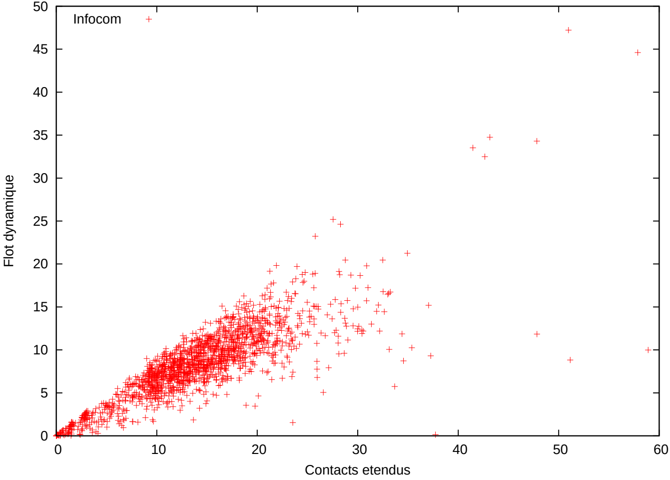

Conclusion

- Number of paths : combinatory explosion (to be refined).
- Dynamic flow : capture the essence of the dynamic.
  - Still expensive to compute on a brute force approach.
  - Need more optimisation / refinements.
- Extended contacts : a usefull rough boundmaun/compromise.
- Some *merits* for measuring the exposure

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- A Special Issue of Computer Networks on Complex Dynamic Networks : Tools and Methods
  - Paper submission : 18-02-2011
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