On the joint dynamics of network diameter and spectral gap under node removal

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Presentation Outline

- Background
- Goals
- Analysis
- Conclusion
- Future work

Known facts

- There are many complex network models
  - In particular
    - Erdős–Rényi (ER) – random graphs
    - Barabási-Albert (BA) – scale-free graphs
  - Well known general behavior under failures and attacks [Albert et al., 2000]
- Normalized Laplacian
  - Information about the network structure

Background

- Laplacian Matrix: $L = D - A$
- Normalized Laplacian: $\mathcal{L} = D^{-1/2} L D^{-1/2}$
  - Normalized Laplacian is less sensitive to graph's size
  - $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \leq 2$
  - λ_2 = Spectral Gap
  - λ_2 bounds conductance

Goals

What are our goals?

- Study λ_2 dynamic behavior
- Dynamic relationship to network diameter

And what aren't...

- Network general behavior under attack or failure
- Network static properties
- λ_2 static properties

Analyzed Graphs

- Barabási-Albert (BA) Scale-free graphs
  - 1000 nodes, 2 connections per new node
  - Initial volume 3992
  - Initial diameter ~ 7

- Erdös-Rényi (ER) Random graphs
  - 1000 nodes, p = 0.00045
  - Initial volume ~ 4400
  - Initial diameter ~ 10

Experiment Process

- Three node removal methods
  - Strategic – highest degree first (targeted attack?)
  - Weighted – random with degree bias
  - Random (random failure?)
- If network breaks, continue with largest component
- Experiment ends when graph reaches maximum diameter
- Experiment parameters are persisted
- Experiment can be repeated
- Network topology can be retrieved at any point
- Log generated for each node removal

Result Log

BA Strategic

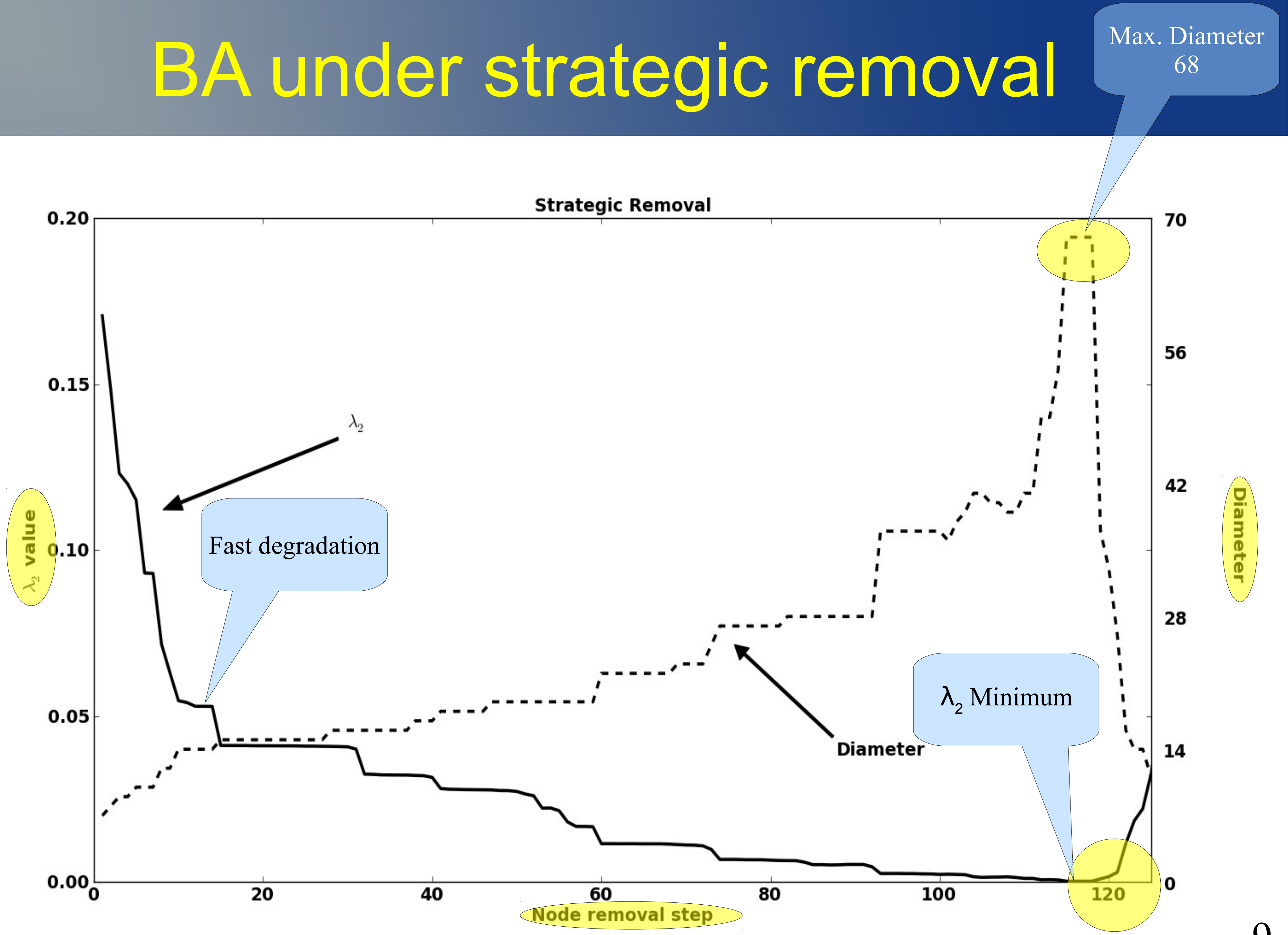
count	Diameter	Degree	DeletedNode	Order	Volume	Components	λ2
1	7			1000	3992	[1000]	0.170608851563
2	8	104	4	999	3784	[999]	0.148342636032
3	9	53	7	996	3678	[996, 1, 1]	0.123159939517
4	9	50	15	994	3578	[994, 1]	0.11994386321
5	10	47	5	992	3484	[992, 1]	0.115077865602
6	10	47	8	990	3390	[990, 1]	0.0930911333146
7	10	33	27	988	3324	[988, 1]	0.0929774907175
113	49	5	396	458	928	[458, 8, 3, 1, 1]	0.000722066327663
114	54	5	413	443	896	[443, 11, 2, 1]	0.000627754147406
115	68	5	478	432	868	[432, 10]	0.000219747331195
116	68	5	535	417	838	[417, 8, 4, 1, 1]	0.000231051401915
117	68	4	66	415	830	[415, 1]	0.000226203177408
118	68	4	89	396	792	[396, 13, 3, 2]	0.000237436744042
119	37	4	133	179	356	[179, 161, 54, 1]	0.000949279875344
120	33	4	526	129	256	[129, 47, 1, 1]	0.0016227377715
121	26	4	141	79	156	[79, 47, 1, 1]	0.00295845108252
122	16	4	167	42	82	[42, 31, 4, 1]	0.0116113279509

Fragmentation Caused

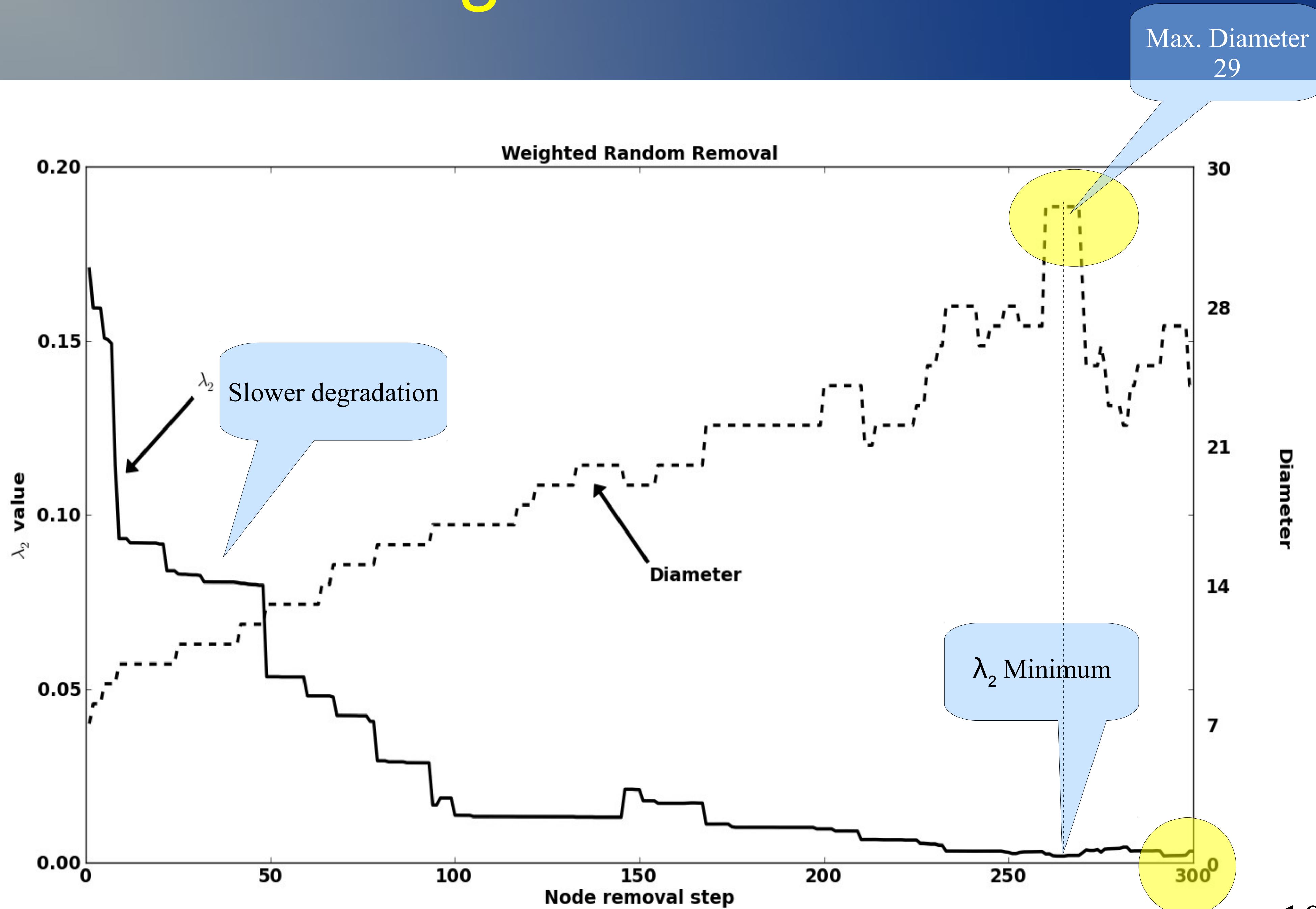
4 Components
179, 161, 54 and 1 nodes

Deleted Node

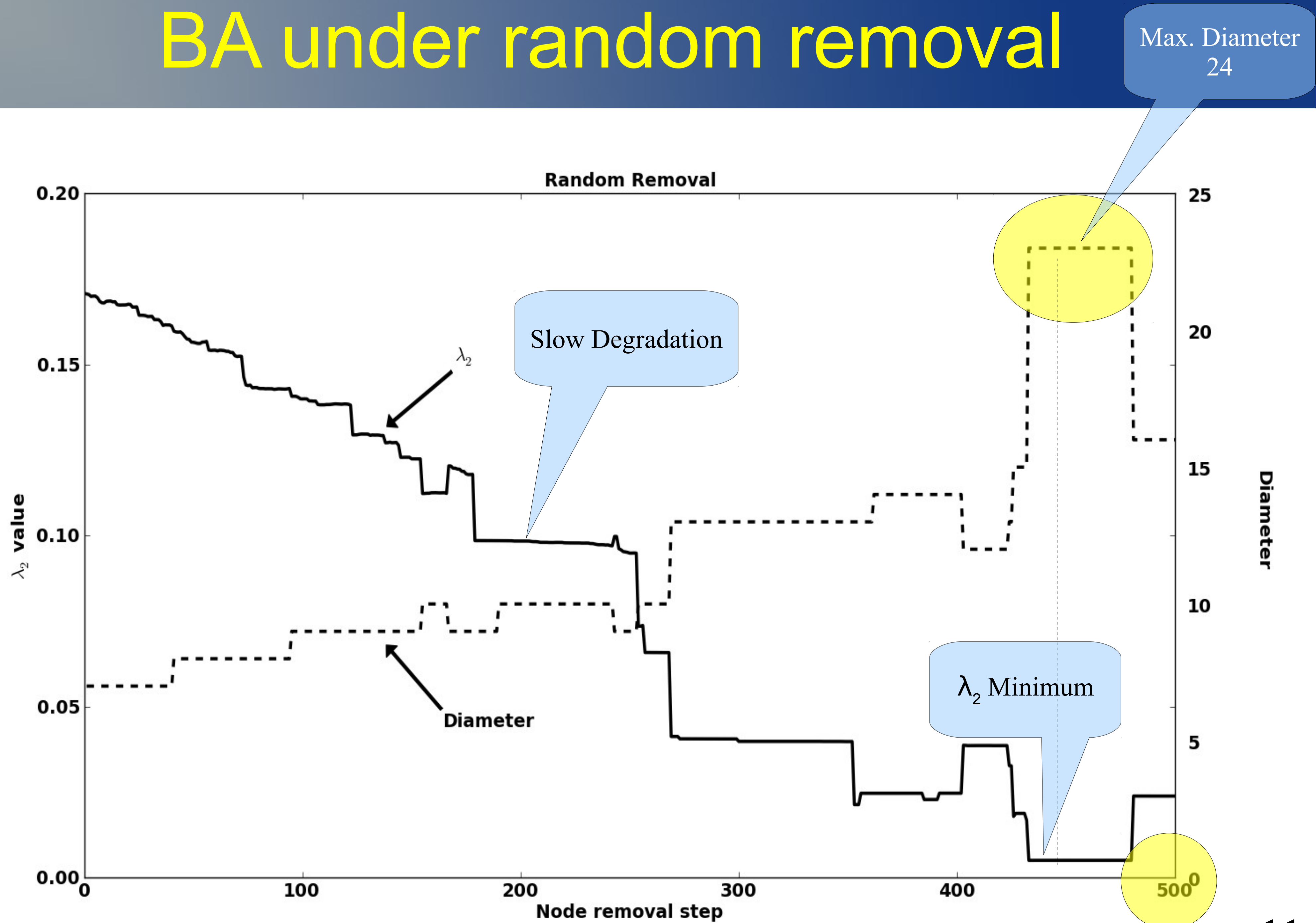
BA under strategic removal



BA under weighted random removal

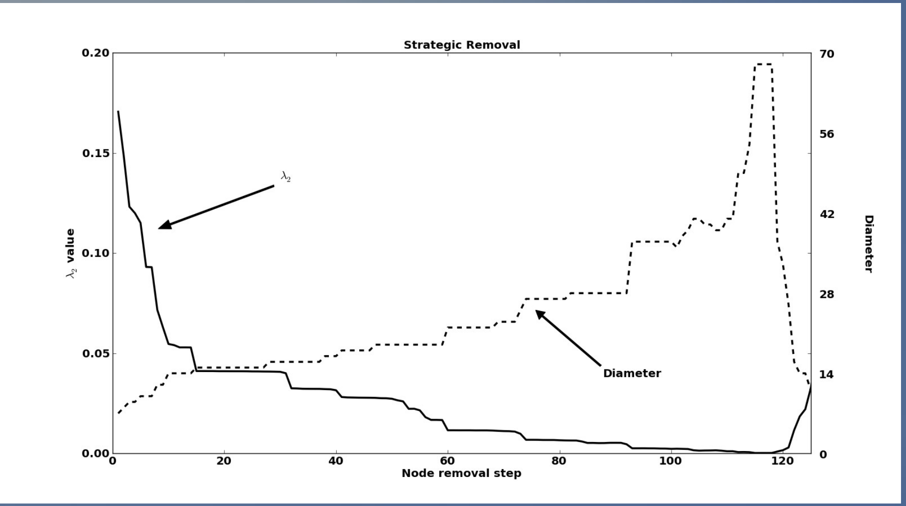


BA under random removal



Overview – BA behavior

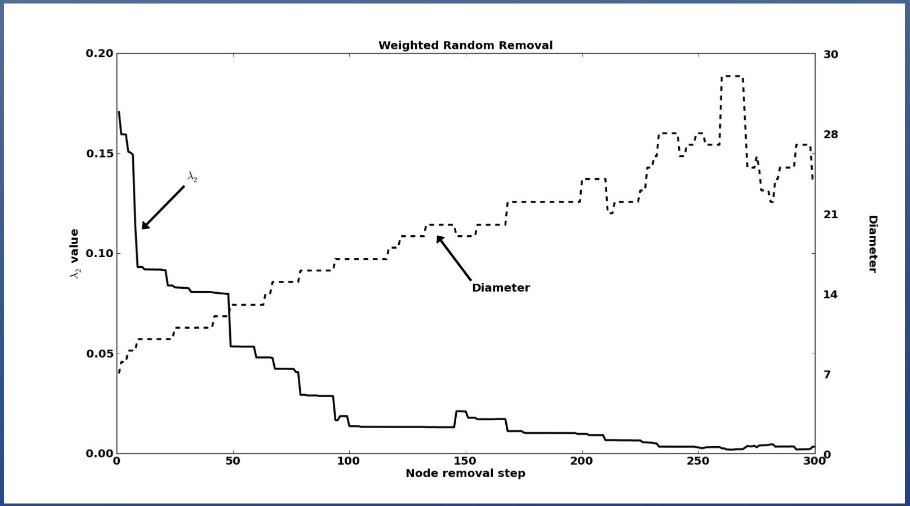
Strategic



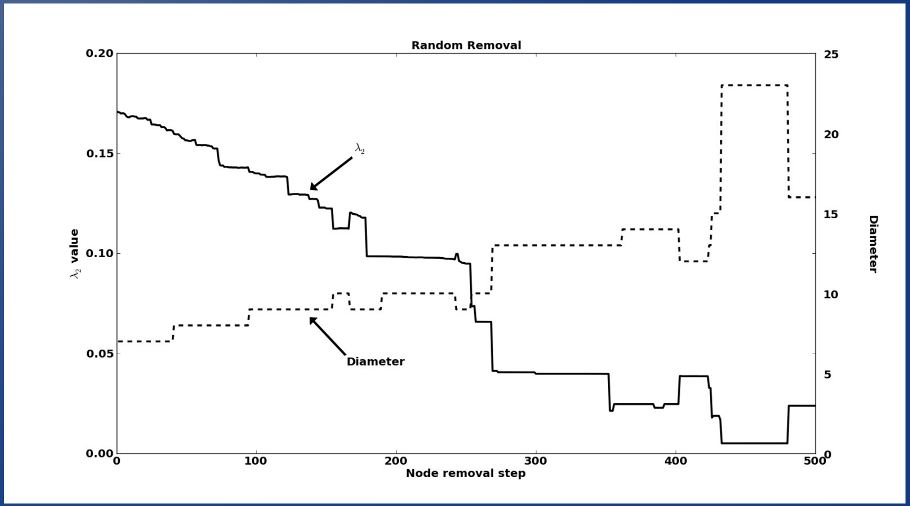
Strategic removal – Faster degradation and higher maximum diameter

Random removal – Slower degradation and lower maximum diameter

Weighted random – Intermediate behavior

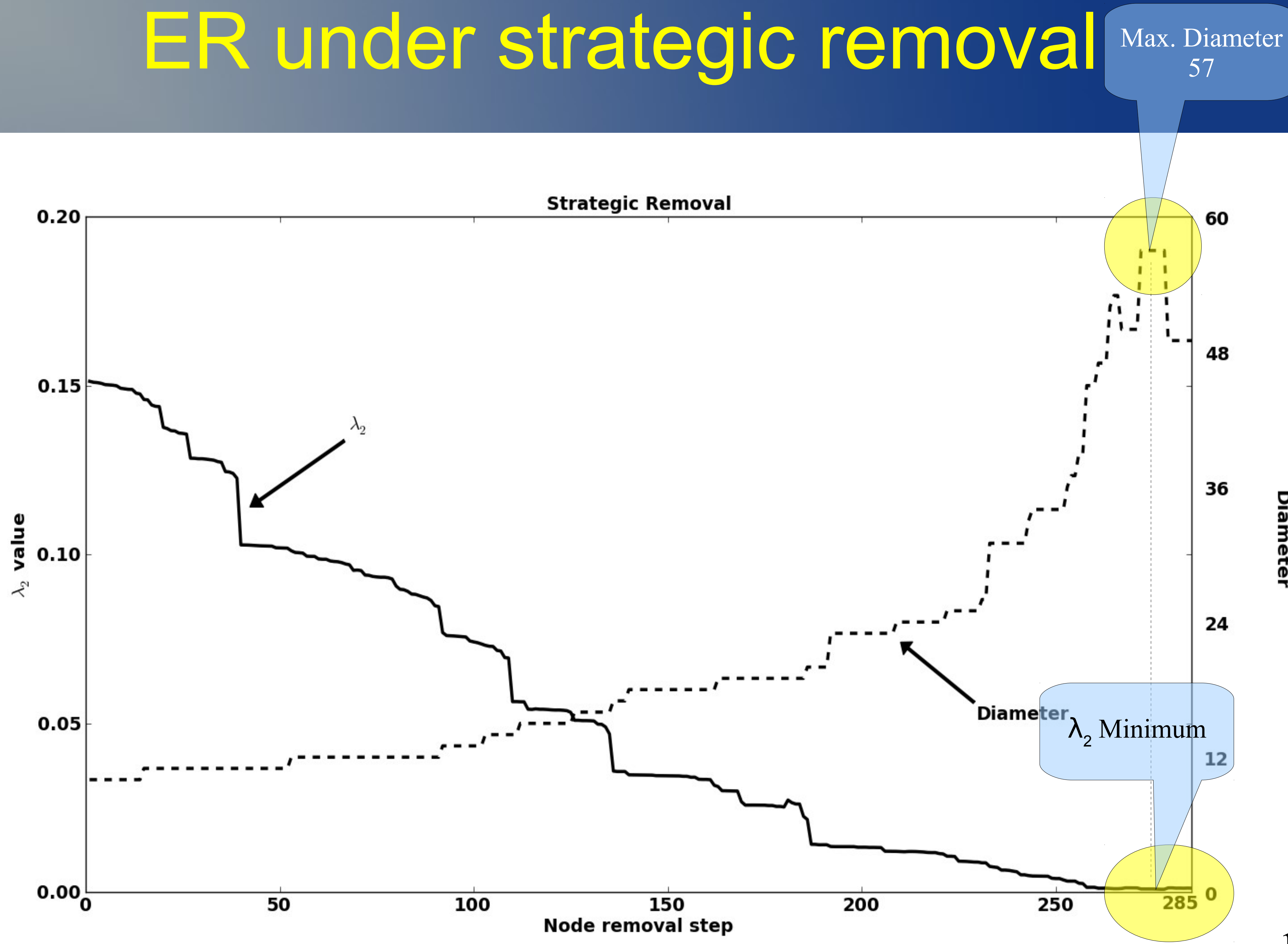


Weighted

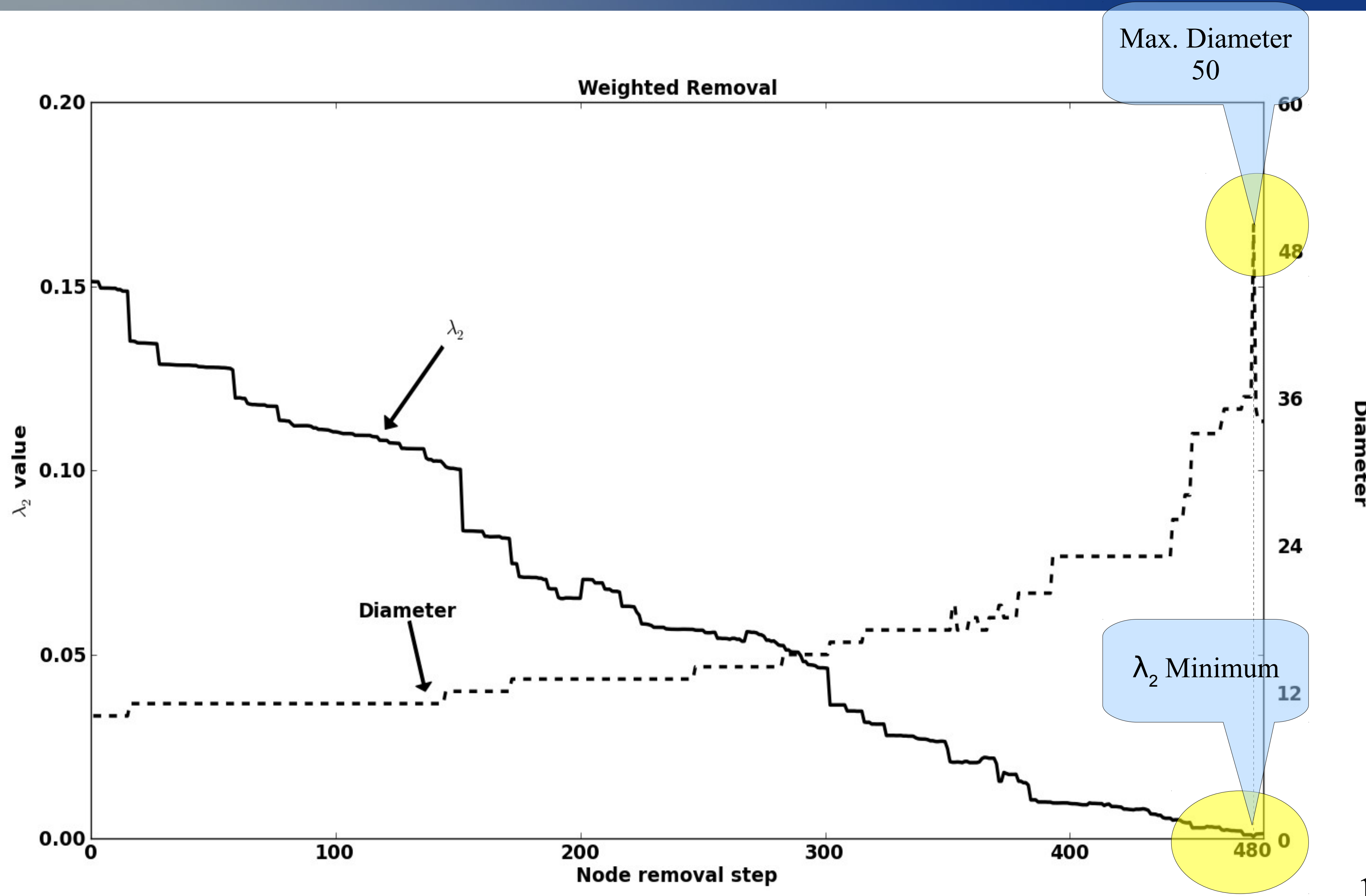


Random

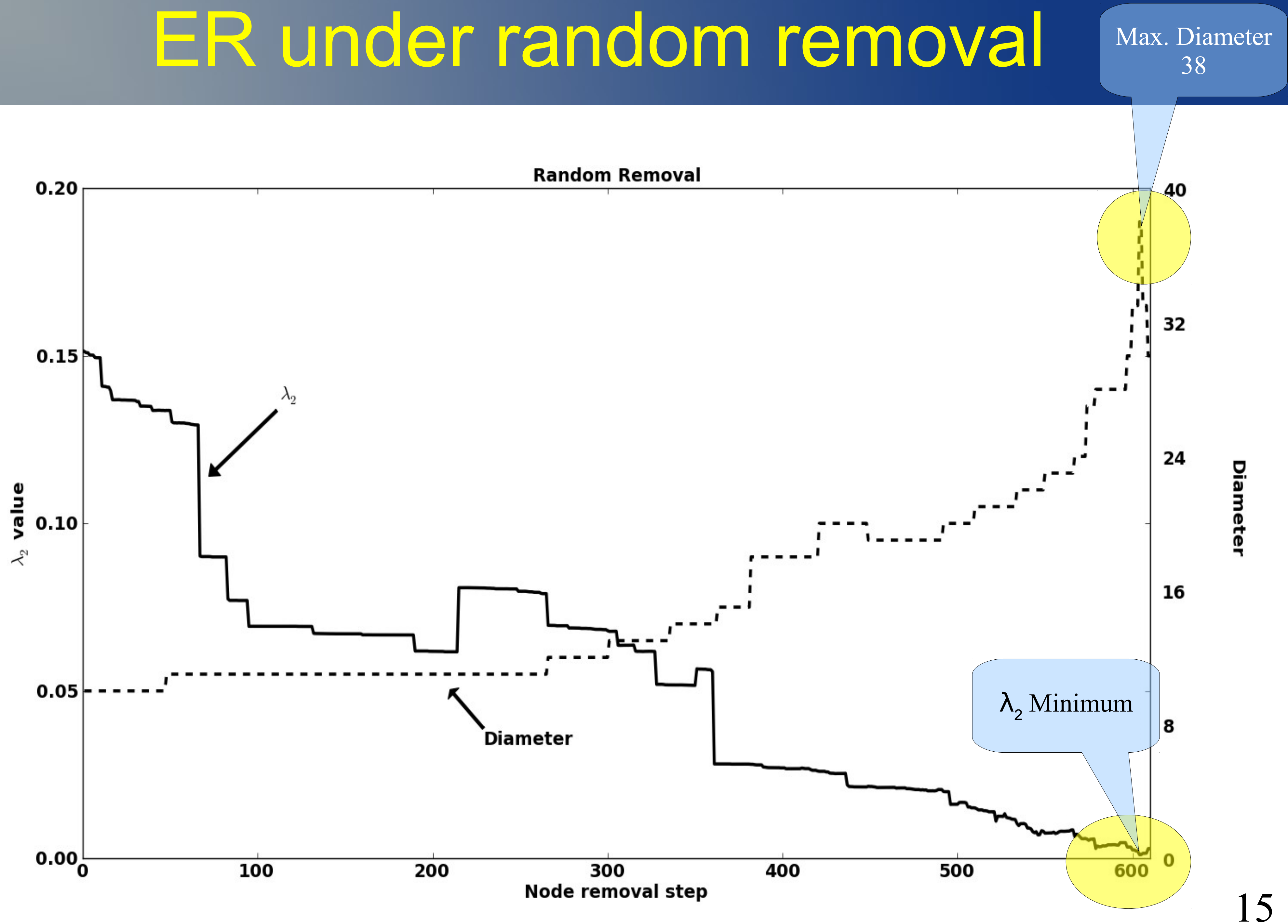
ER under strategic removal



ER under weighted removal

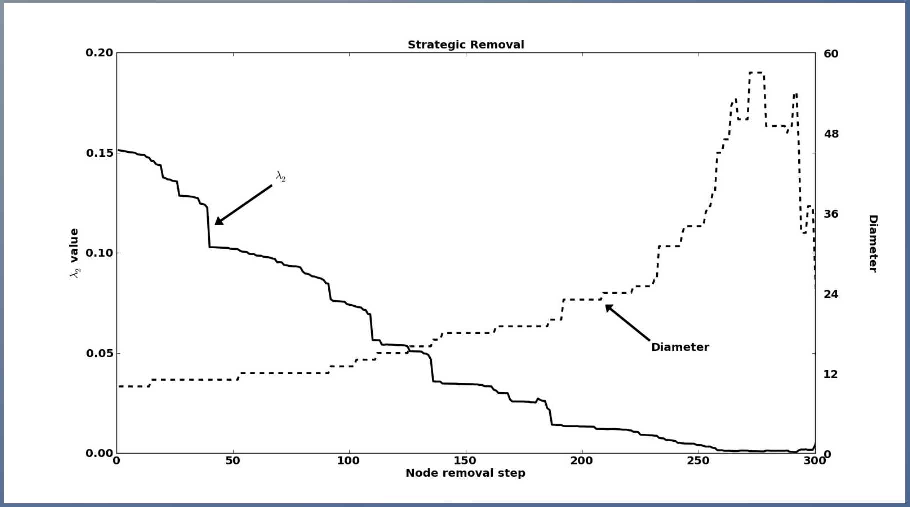


ER under random removal



Overview – ER behavior

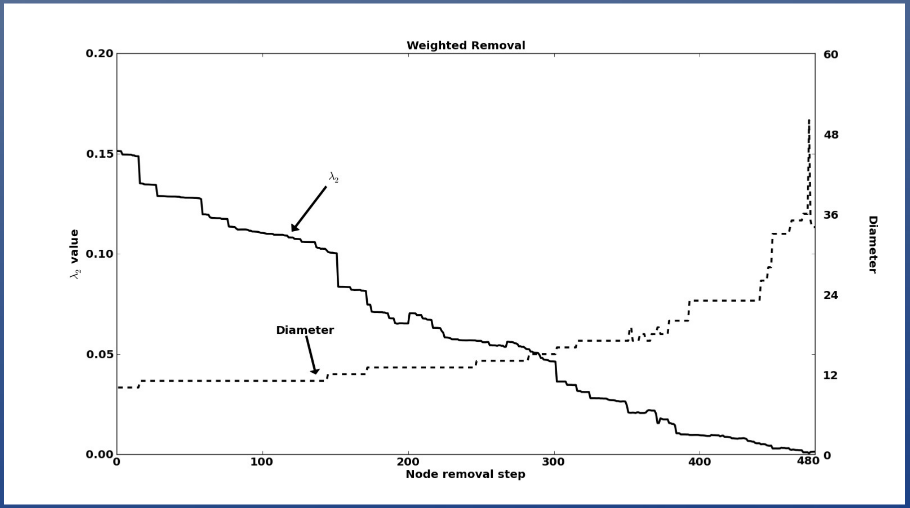
Strategic



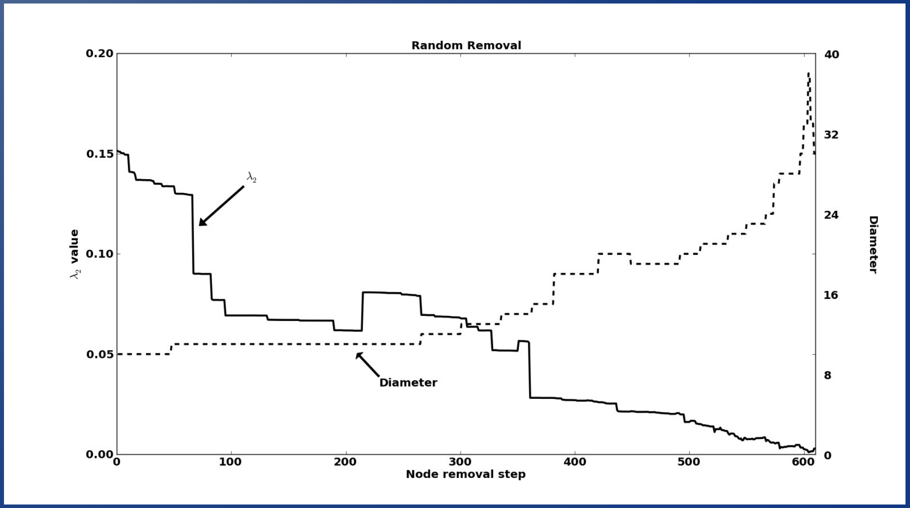
Smaller difference between the three node removal methods

But still different

Actually this is expected because of the different known behaviors of BA and ER under failures and attacks

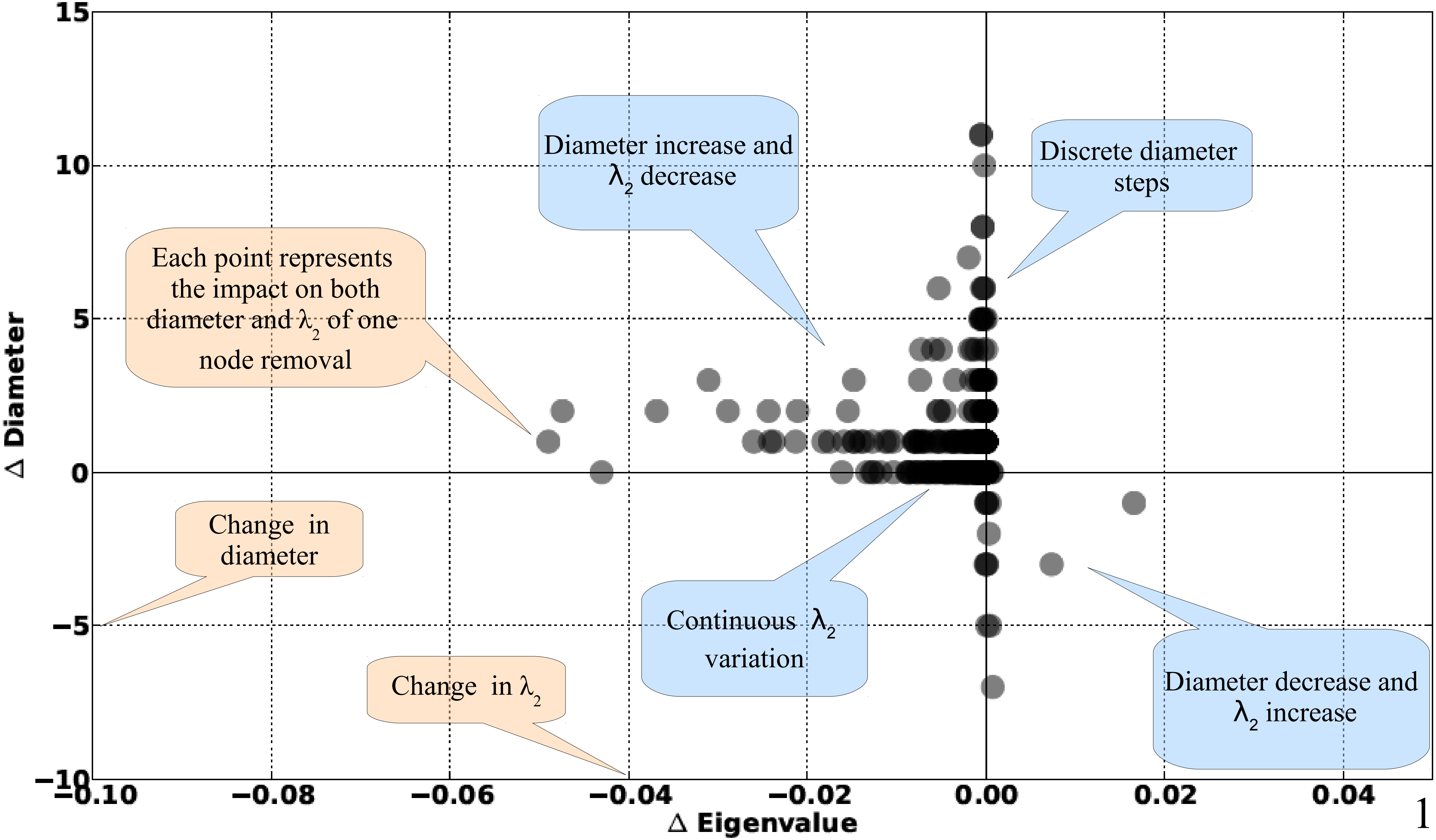


Weighted



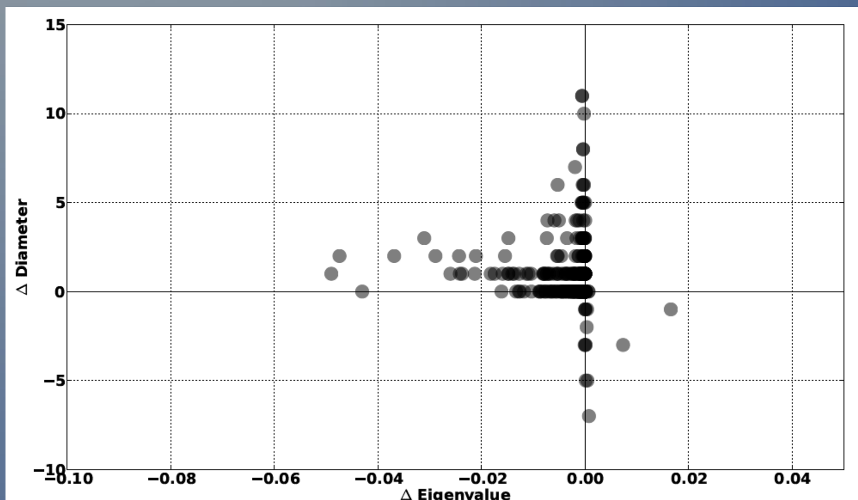
Random

Scatter Graphs



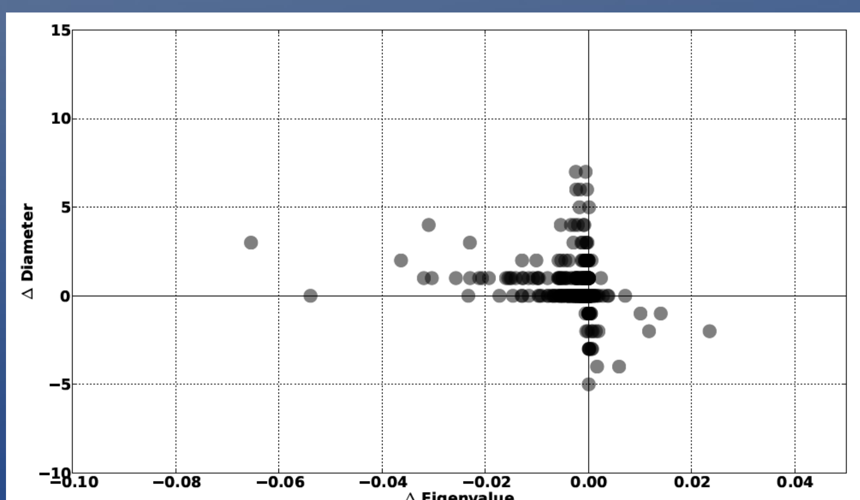
Joint dynamics of λ_2 and diameter BA case

Strategic

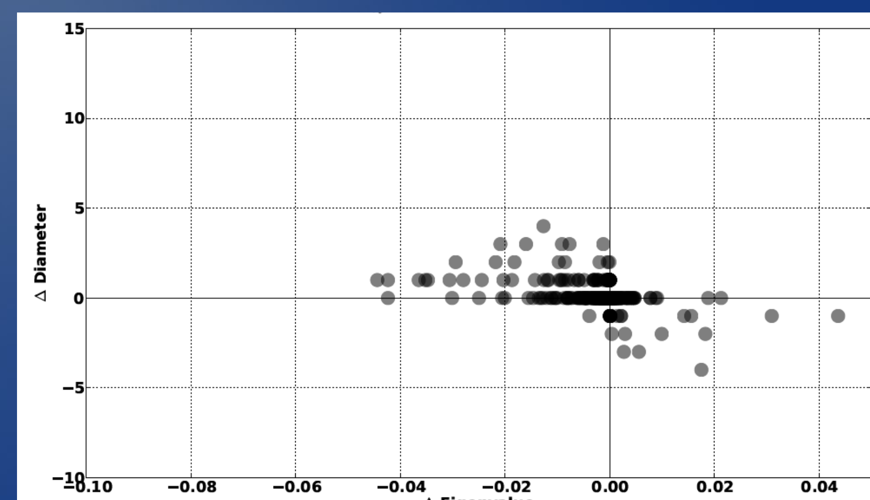


Very different behavior for the three node removal methods

Lower right quadrant identifies node removal method



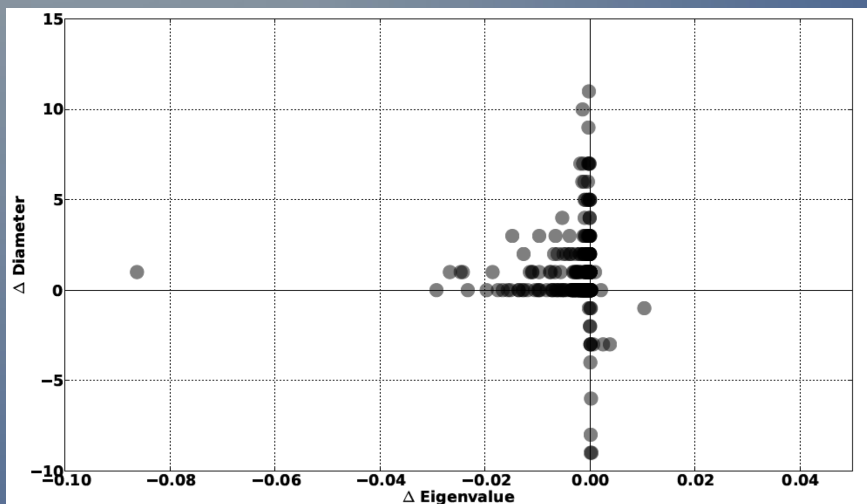
Weighted



Random

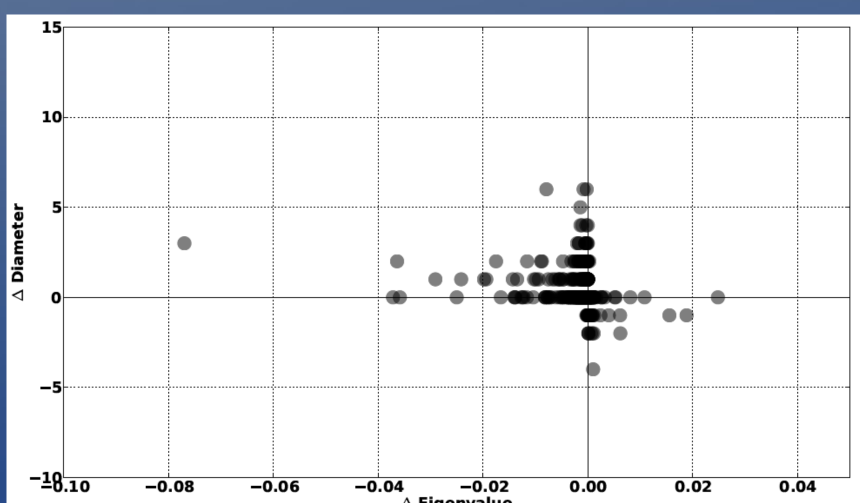
Joint dynamics of λ_2 and diameter ER case

Strategic

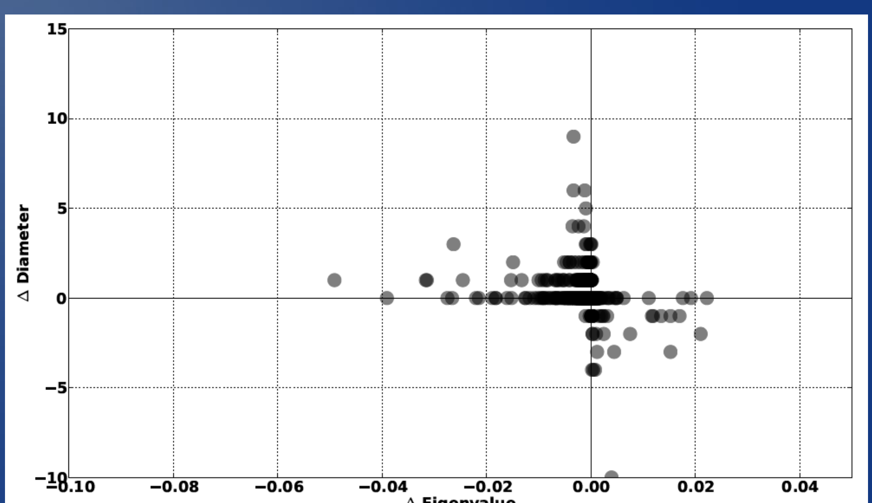


Not so much difference between the three node removal methods

Lower right quadrant still identifies node removal methods



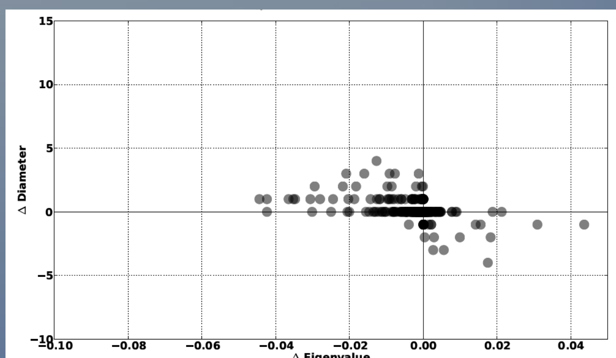
Weighted



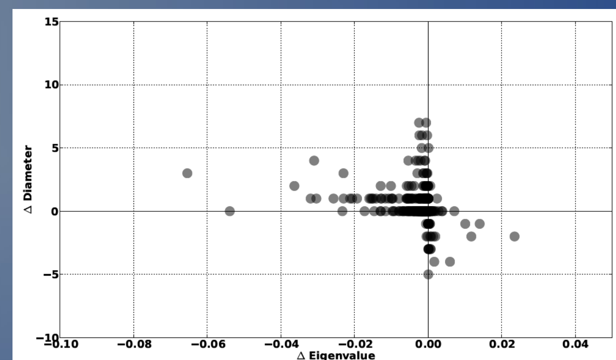
Random

Comparing BA and ER

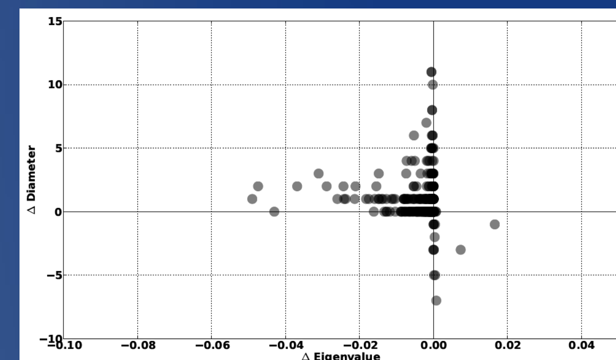
Random BA

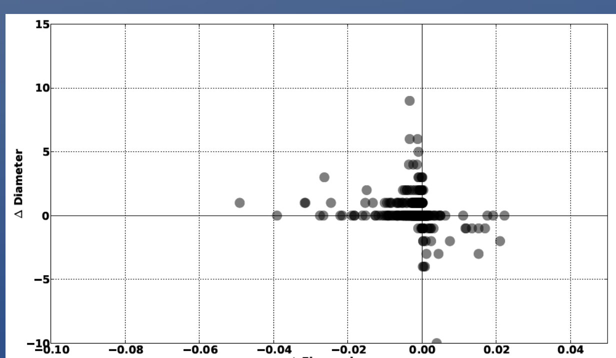


Weighted BA

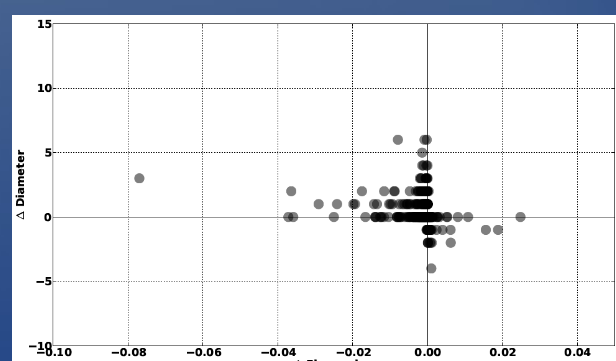


Strategic BA

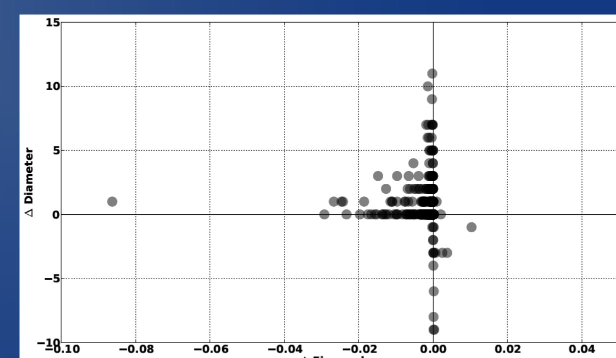




Random ER



Weighted ER



Strategic ER

Conclusions

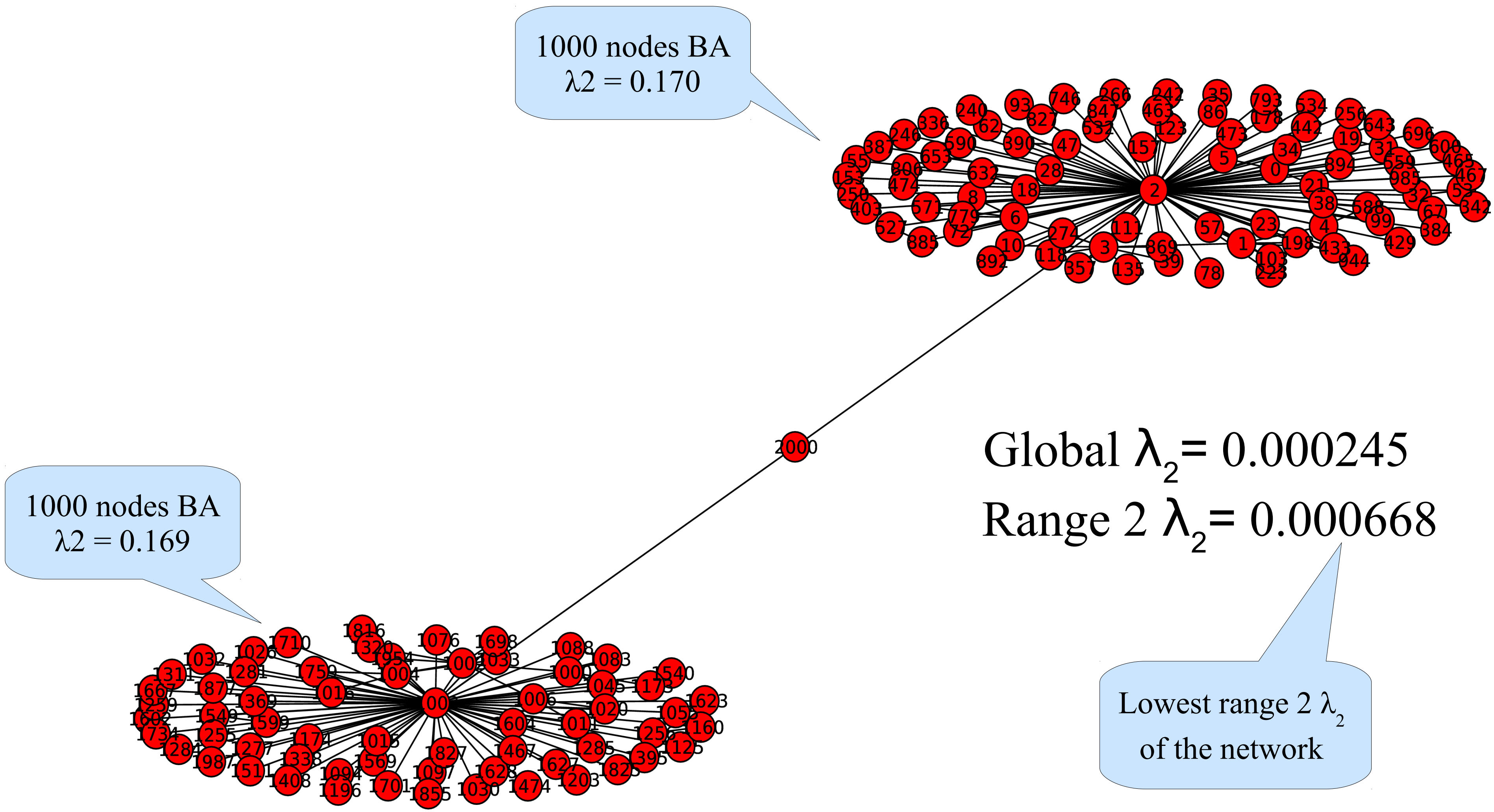
- Exposed the dynamic behavior of λ_2
- Explored the joint dynamics between λ_2 and network diameter
- Gained insight on network structures that affect λ_2

Future Work

Question: Since it is clear that a fragile structure causes λ_2 to be low, is there a way to locate the fragile part of the network?

Rationale: If a given structure causes a globally low λ_2, it should also cause a locally low λ_2 on a restricted range neighborhood...

Local λ_2 (i.e. on a subgraph)



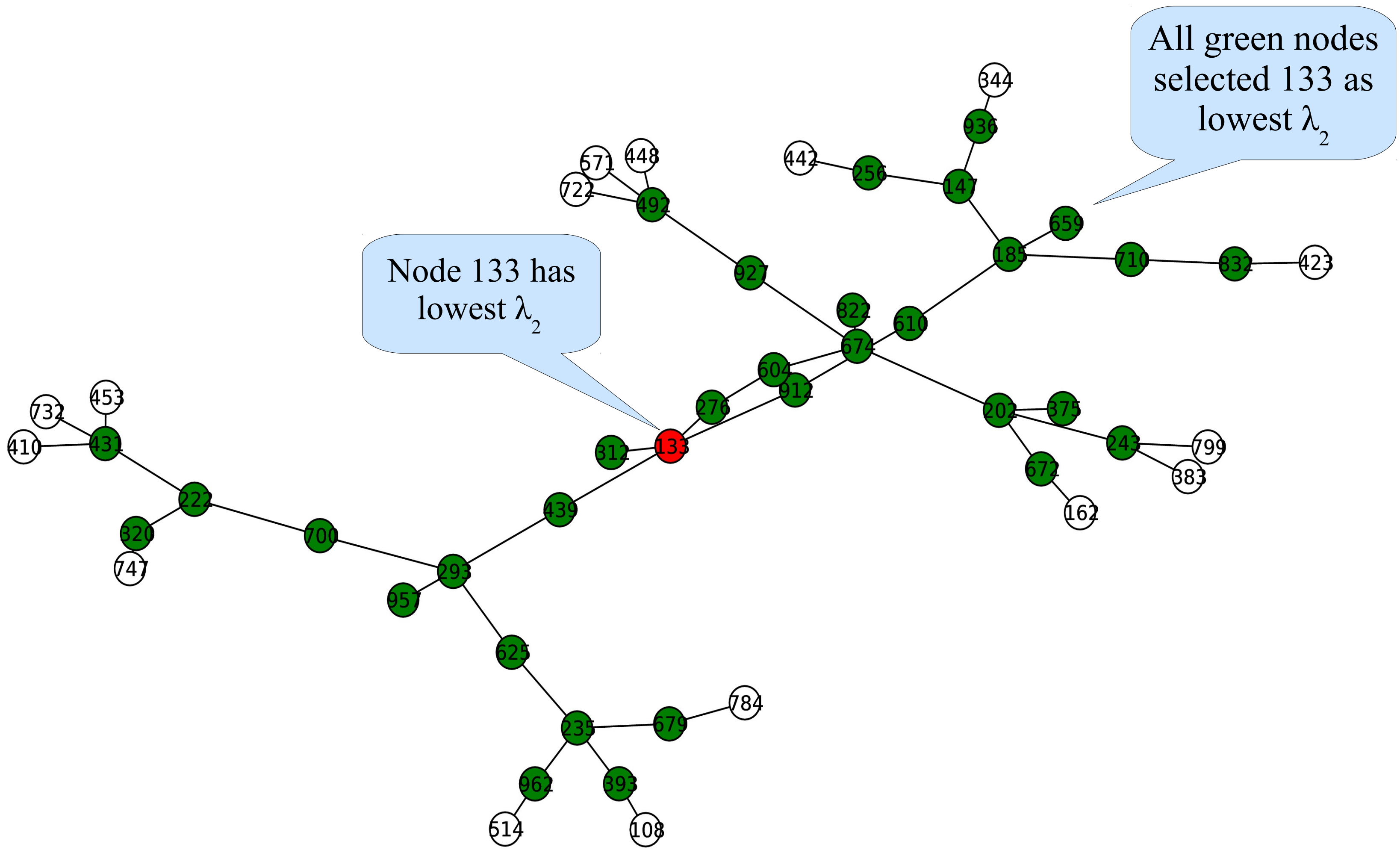
Analysis Process

- Select a neighborhood range
- For each node on the network
    - Find neighborhood of selected radius and calculate λ_2
- For each node on the network
    - Identify the lowest λ_2 node in its neighborhood

This process is very friendly for distributed implementation

Each node only has to know its neighborhood

Analysis Process



Preliminary results

- Good results on tested networks
- Still improving heuristics
- Radius setting in study

Example

BA single [396] component

Pointed nodes 133,180 and 328

Removing node 133 we get [179, 161, 54, 1] (4 components)

Removing 133, 180 and 328

[128, 116, 61, 54, 16, 15, 1, 1, 1] (9 components)

Questions?

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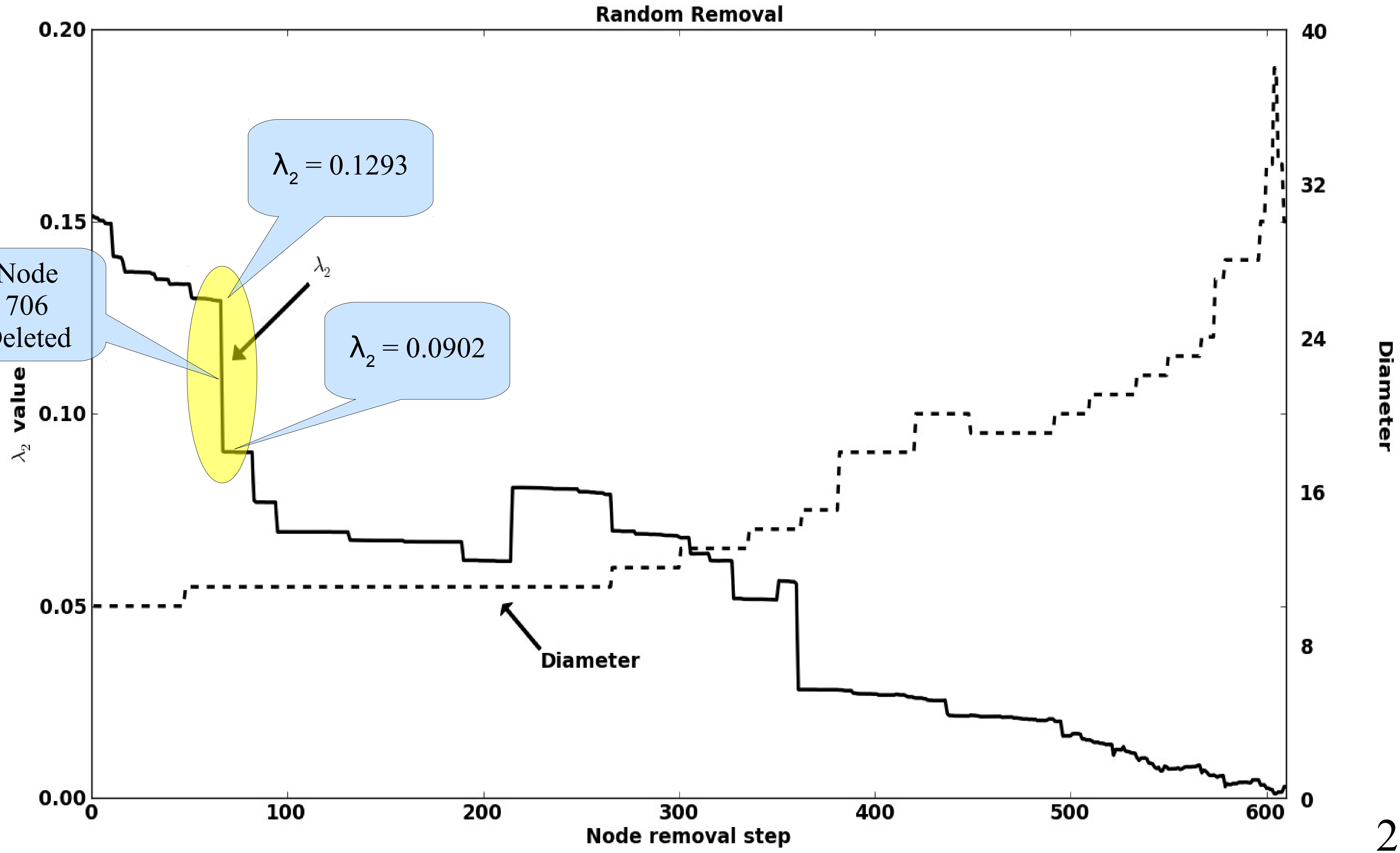
λ_2 Sharp Drops



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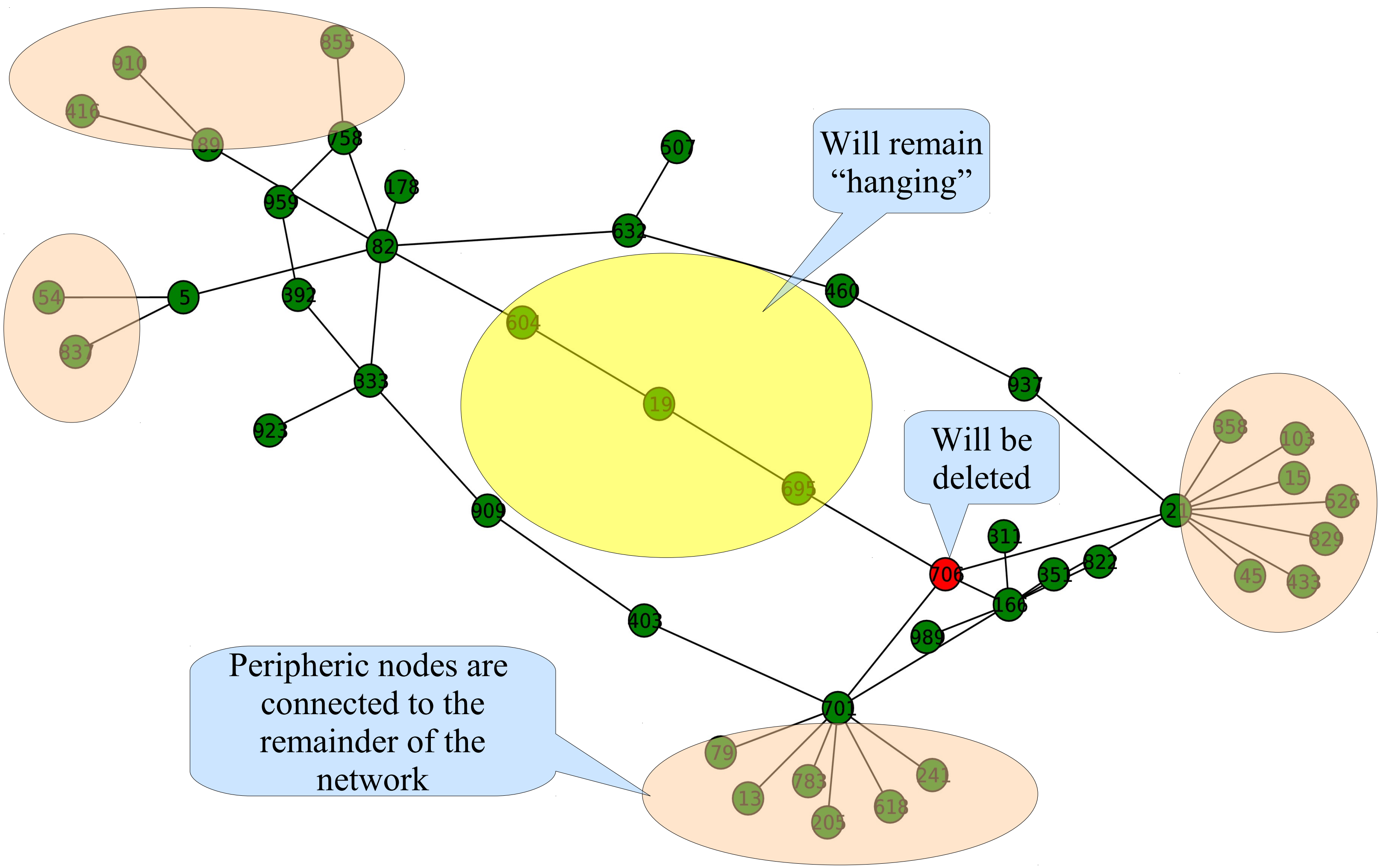
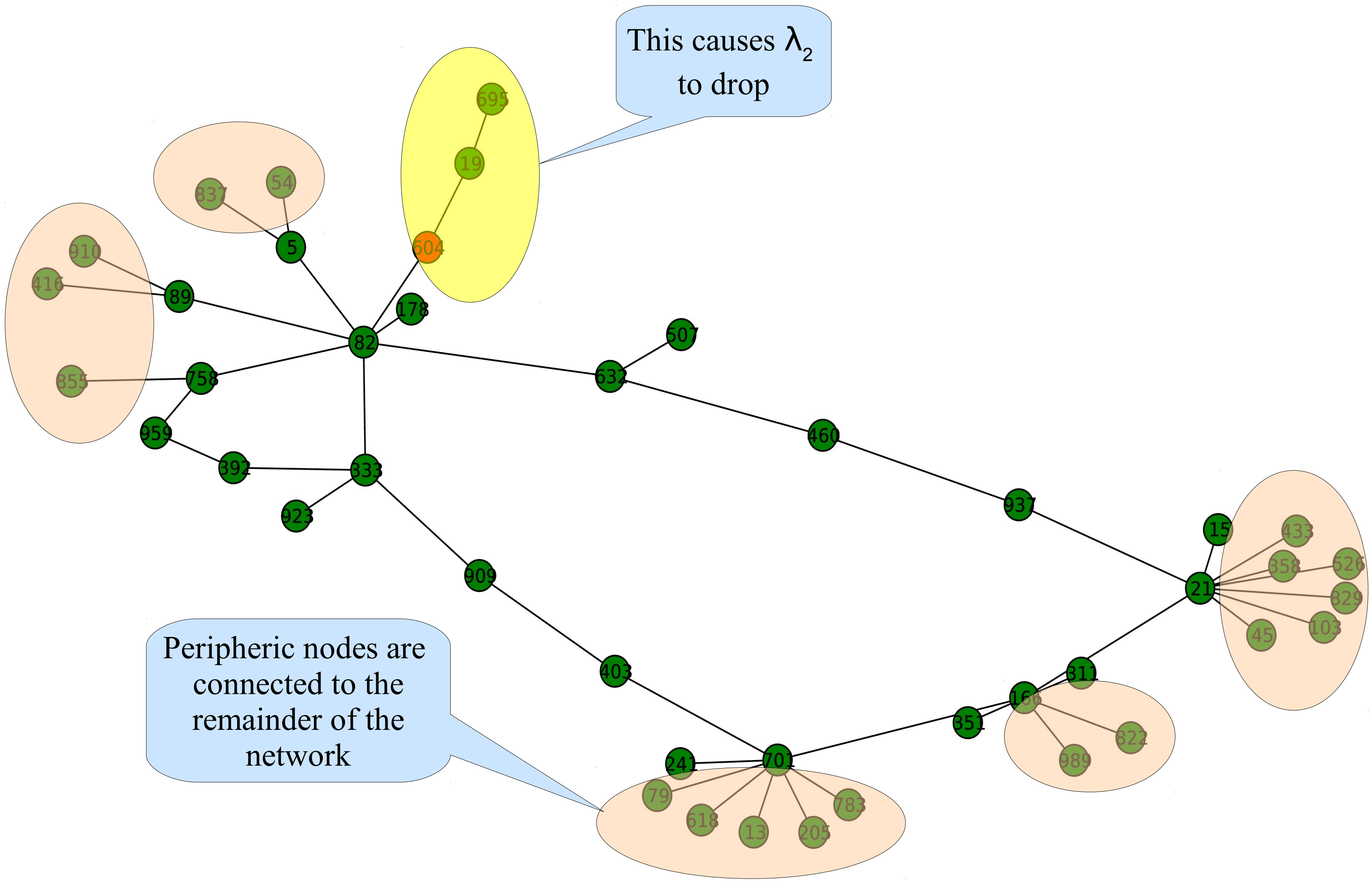
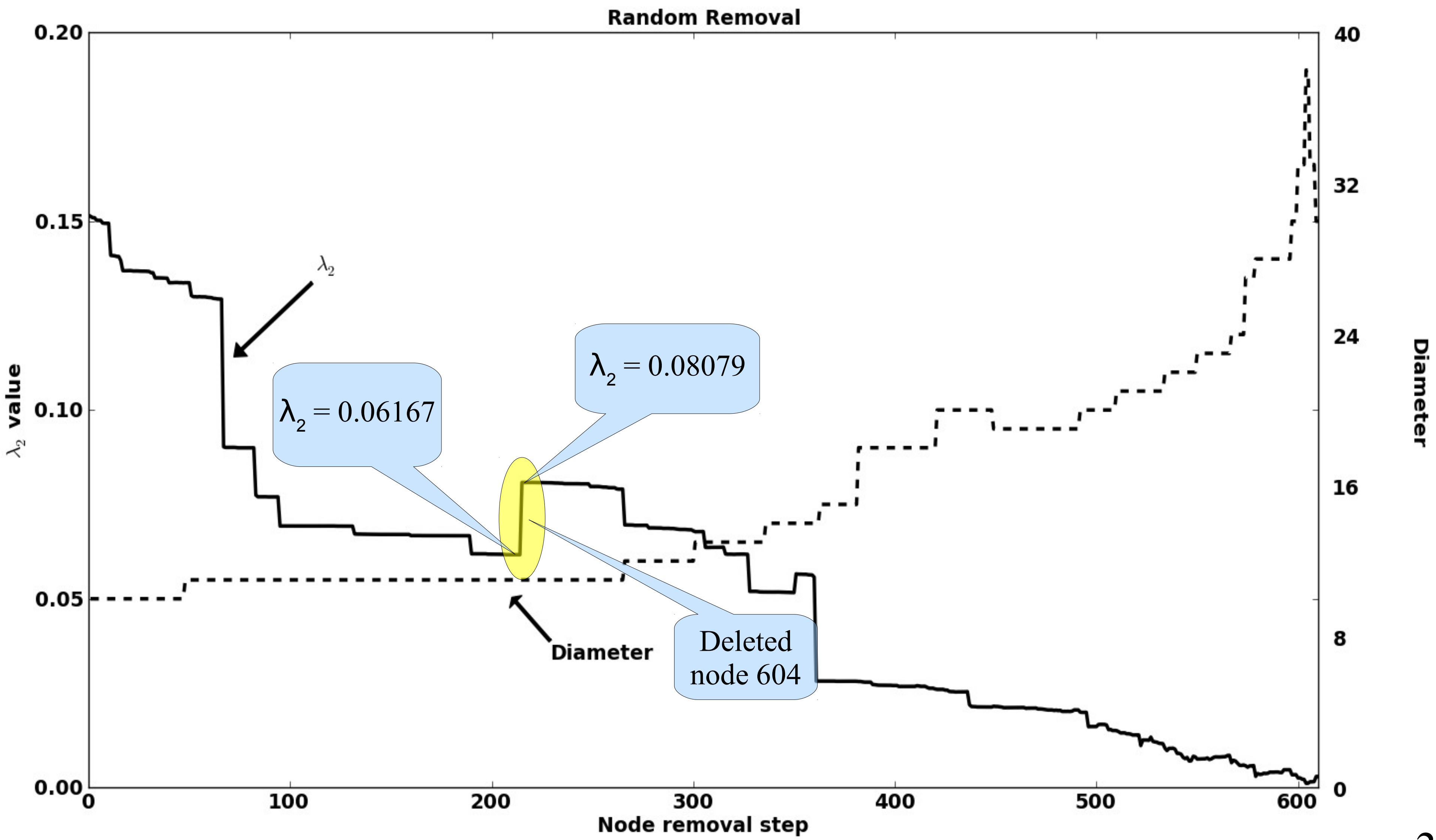


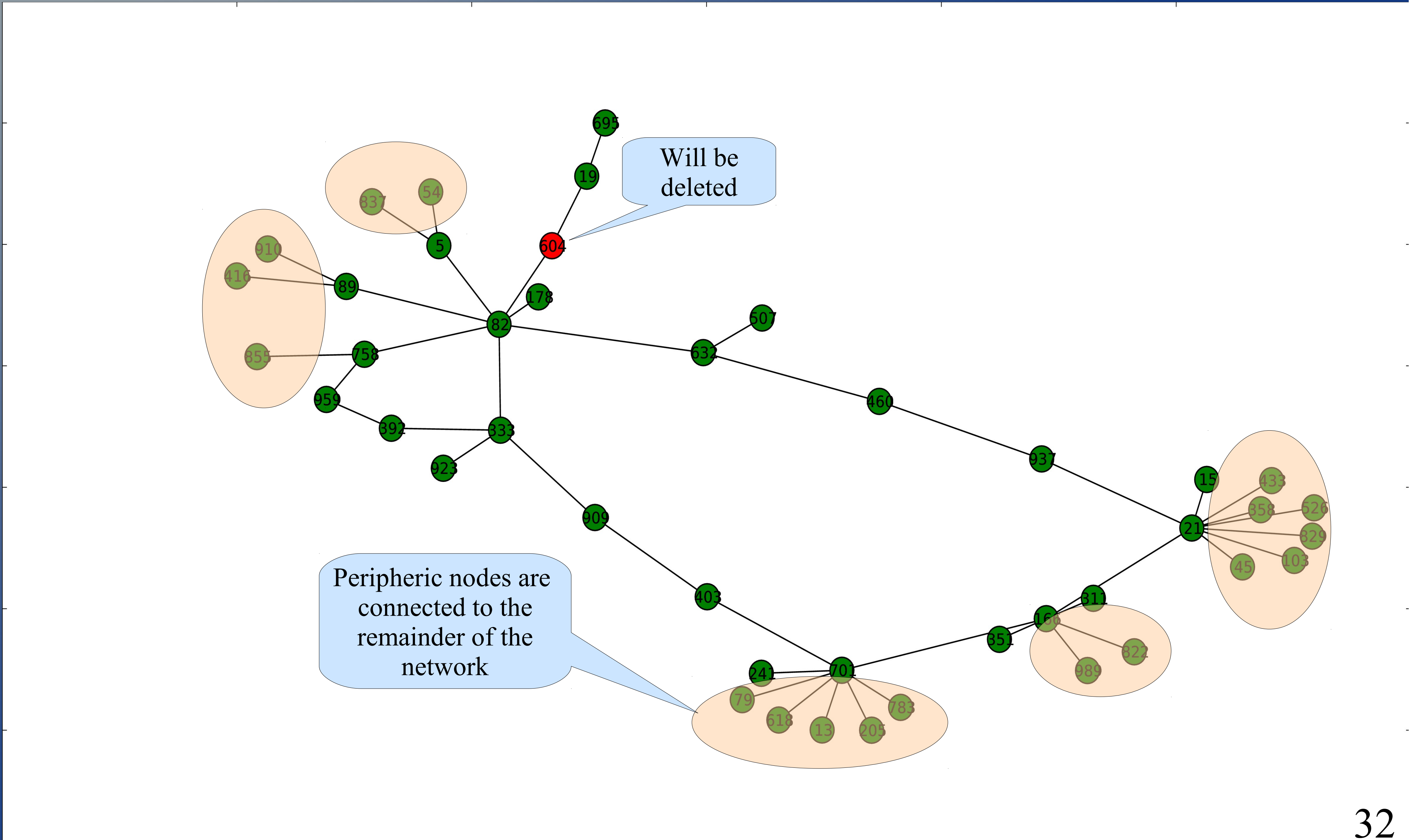
λ_2 Sharp Drops



λ_2 Sharp Raise



λ_2 Sharp Raise



λ_2 Sharp Raise