Cooperation in costly-access environments

Hugo Pérez-Martínez[1], Carlos Gracia-Lázaro[2], Fabio Dercole[3], Yamir Moreno[2,4,5]

[1]Department of Condensed Matter Physics, University of Zaragoza. Spain.
[2]Institute for Biocomputation and Physics of Complex Systems, University of Zaragoza. Spain.
[3]Department of Electronics, Information, and Bioengineering, Politecnico di Milano. Italy.
[4]Department of Theoretical Physics. University of Zaragoza. Spain.
[5]CENTAI Institute, Turin. Italy.

Understanding cooperative behavior in biological and social systems constitutes a scientific challenge, being the object of intense research over the past decades. Many mechanisms have been proposed to explain the presence and persistence of cooperation in those systems, showing that there is no unique explanation, as different scenarios have different possible driving forces. In this regard, the Evolutionary Game Theory provides a fruitful theoretical framework for studying cooperative behavior, including cooperation in structured populations. Within this framework, the Prisoner's Dilemma (PD) constitutes the most representative and widely studied game for modeling cooperative behavior evolution.

In this work, we propose a model to study situations where the willingness to participate in a cooperative setup involves an access cost (besides the cost associated with cooperation), even if the other potential participant player refuses the interaction. The motivation is to study those scenarios where the access to the interaction place (whether physical or virtual) entails an expense, such as transport costs, entry fees, or time investment, which can be avoided by refusing interaction. The proposed model corresponds to a reciprocal Donation Game with voluntary and costly participation. By imposing a participation fee, we break the symmetry of the Voluntary PD through a payoffs difference between the player that refuses to interact (the abstainer) and her counterpart (the attendant, i.e., cooperator or defector). The proposed two-person game, hereafter Costly-Access Prisoner's Dilemma (CAPD), has three strategies: cooperation, defection, and abstention. While abstention does not involve any payoff (neither benefit nor fee), defection and cooperation entail a participation fee besides the cooperation cost associated with the latter. Note that, in the proposed CAPD, the players willing to participate in the underlying PD pay the participation fee, regardless of whether PD takes place or not. Conversely, in the Voluntary PD, the participation fee is paid only if the PD takes place. We can interpret the CAPD as a risky version of the Voluntary PD, as showing up to participate involves a risk that breaks the symmetry of the Voluntary PD.

A mean-field approach shows that, in well-mixed populations, the dynamic always leads the system to abstention. However, depending on the return parameter, numerical simulations in structured populations display an alternating behavior between mono-strategic, multi-stable, and coexistence phases. This behavior is fully explained through a theoretical analysis of the strategic motifs, the transitions being determined by the change in stability of those motifs.

The Model

Let be a population of n agents –the players– endowed with a network structure. The interaction between any pair of agents takes place through a PD with voluntary and costly participation. Specifically, agents are allowed to adopt one of the three available strategies: cooperation (C), defection (D), and abstention (A). Each agent takes one of the above strategies when playing with all her neighbors. If the agent decides to abstain, she does not pay nor receive anything. Otherwise, she must pay a participation fee t. Furthermore, cooperation has an additional cost $c = 1$ (the contribution) to the participation fee, while defection does not entail additional cost. The counterpart of a cooperator receives rc, i.e., her partner's contribution c multiplied by the enhancement factor r. Once two players decide to participate (none is an abstainer), this formulation of the PD is equivalent to a reciprocal Donation Game: in that game, each player is allowed to donate c; if she does, her counterpart receives rc.

Let σ_i be the strategy of player i; $\sigma_i^T = (1, 0, 0), (0, 1, 0), (0, 0, 1)$ for an abstainer, cooperator and defector, respectively. The payoff obtained by player i facing j is given by $\sigma_i^T M \sigma_j$, where M is the payoffs matrix:

$$M = \begin{pmatrix} 0 & 0 & 0 \\ -t & r-t-1 & -t-1 \\ -t & r-t & -t \end{pmatrix}$$

A cooperator or a defector who faces an abstainer must pay the participation fee t because she has presented himself to play, but since the game is not played (the abstainer does not appear), cooperator will not pay the cooperation cost $c = 1$. Furthermore, the highest payoff corresponds to a defector facing (and therefore exploiting) a cooperator, while the lowest to a cooperator facing a defector.

Each agent plays with the same strategy with all her neighbors. The payoff Π_i of an agent, i, will be the sum of those obtained when playing with all her neighbors. Once all the agents have played, they decide whether to keep their strategy in the next round or to change it. Specifically, each agent i randomly selects a neighbor j and compares their payoffs. Agent i adopts j's strategy with a probability given by:

$$P_{ij} = \frac{1}{1 + \exp(\frac{\Pi_i - \Pi_j}{T})} \quad . \tag{1}$$

Summary of results and discussion

In the absence of a network structure, the only evolutionarily stable strategy is abstention, which coincides with the only Nash equilibrium of the system, as defection is in the PD. We have studied the system through a mean-field approach, which resulted in the absence of inner fixed points, revealing the lack of coexistence stationary states. Furthermore, mean-field trajectories are such that any initial condition ends in a full-abstention state.

Extensive numerical simulations on graphs have shown that the network structure has a determining influence on the system dynamics. We have identified a series of strategic motifs whose stability thresholds determine the system behavior. The motifs' analysis allowed us to explain the different phases of the system and locate the critical values of the game return parameter r that demarcate their boundaries. Also, it explains the jumps in the average strategies frequencies and provides the values of r at which they take place. In particular, the system exhibits i) a first mono-strategic phase dominated by abstention, followed by ii) a multi-stable mono-strategic phase in which dynamics leads to one of the three absorbing states, iii) a three-strategies coexistence phase, and iv) for heterogeneous networks, a phase dominated by cooperation with a residual presence of defectors.

To conclude, by breaking the symmetry of the Voluntary Prisoners' Dilemma, we have presented a model that displays a rich phenomenology, with alternating stability switches driven by a single parameter. We believe this behavior will deserve the attention of physicists and mathematicians to be extrapolated to other systems.

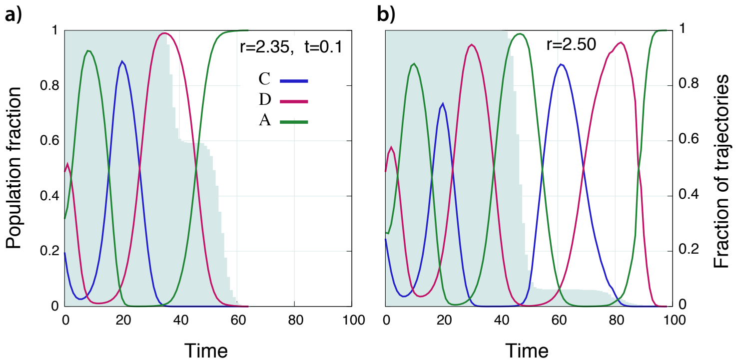

Figure 1: **Time evolution for an RRN.** Frequencies of the total fractions of each strategy as a function of time in a regular random network of $N = 10^4$ nodes and degree $k = 4$. Panel **a)** shows the results for $r = 2.35$, and **b)** for $r = 2.5$. Solid lines correspond to the average over 1000 simulations, and grey shadow to the fraction of trajectories remaining active at a certain time. $T = 0$ and $t = 0.1$.

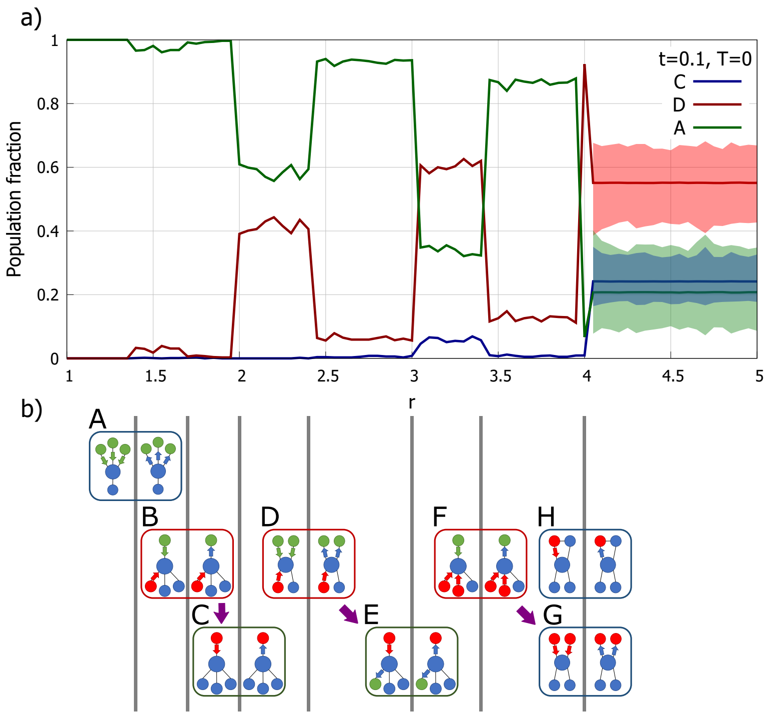

Figure 2: **Numerical results for an RRN, T=0. a)** Fraction of cooperators (C), defectors (D), and abstainers (A) versus the enhancement factor r in a RRN of 10^4 nodes, $k = 4$. **b)** Diagrams A to H correspond to the configuration transitions. Each diagram indicates a dominance transition for the corresponding motifs regarding the propagation or extinction of the central node. In these diagrams, neighbors of cooperators' D-neighbors are not cooperators. Grey arrows indicate the system's evolution when the central C-node invades an A-neighbor.

Bibliography. Hugo Pérez-Martínez, Carlos Gracia-Lázaro, Fabio Dercole, Yamir Moreno. Cooperation in costly-access environments. New Journal of Physics 24, 083005. 2022.