

# Urban segregation patterns for non-homogeneous community linkages

Victoria Arcón<sup>1</sup>, Inés Caridi<sup>1</sup>, Juan Pablo Pinasco<sup>2</sup> and Pablo Schiaffino<sup>3</sup>

<sup>1</sup> Instituto de Cálculo UBA-CONICET, Argentina

<sup>2</sup> Departamento de Matemática and IMAS UBA-CONICET, Argentina

<sup>3</sup> Departamento de Historia y Estudios Sociales, Universidad Torcuato Di Tella, Argentina

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## INTRODUCTION

Residential segregation is an urban phenomenon in which the population's households are grouped in the physical space according to some distinctive characteristics of the individuals, such as ethnicity, income level, and language, among others. This separation of the different socio-cultural groups in the territory could generate inequalities in access to education, culture, health, and work opportunities. Moreover, the homogenization of the terrain hinders social integration, and it becomes further problematic when associated with stigmatization and discrimination. However, it is also true that being surrounded by similar peers is something positive for creating and sustaining community ties and resolving daily life situations [7].

In this work, we study the mechanisms involved in residential segregation, exploring how the quality and characteristics of the housing location may relate to the importance the individuals give to being surrounded by similar neighbors. We have approached this problem from a social modeling perspective, proposing a variation of the well-known agent-based Schelling's segregation model [8] [9]. This classical model consists of agents of two types arranged in a lattice network that have a preference for being surrounded by some proportion of agents of the same type and they can move from one location of the lattice to another, in order to satisfy their preference. This simple mechanism leads to different segregation patterns, even for mildly discriminatory preferences. An extensive bibliography from economics, mathematics, physics, and computation, contributed to generate many variants of the model [2] [4].

We have introduced non-homogeneous locations in the Schelling model as a weight function over the land (nodes). This function represents objective and subjective valuations of the territory and is related to the relevance that agents give to their community ties. For example, places so prestigious or with full resources and facilities allow individuals to live regardless of their neighbors. Conversely, counting on neighbors becomes crucial under challenging contexts, and ties become relevant to survival. Thus, we define a field on the lattice nodes that modulates the weight of the links.

We study the segregation patterns that arise for different weight functions and show theoretical results that agree with computational simulations [1]. Smooth spatial variations of the weight function, with few minima and maxima, correspond to

more significantly segregated neighborhoods with clusters of large scale.

In addition, we study the phenomenon from a data analysis approach, using available Brazilian 2010 census data to visualize and quantify ethnicity-based segregation. Brazil's Sao Paulo and Rio de Janeiro cities allow us to connect the proposed model with the observed patterns. Sao Paulo shows large-scale segregation along all the city and Rio de Janeiro conglomerates near the hills with a higher number of clusters and segregation on a minor scale.

## THE WEIGHTED SCHELLING MODEL

We have  $N$  agents located as nodes of a connected network  $\Lambda_N$ , without empty sites, so we will denote indistinctly by  $i, j, \dots$  an agent and its location. A configuration is a function  $x : \Lambda_N \rightarrow \{-1, 1\}$ , assigning one of the labels  $\pm 1$  to each node (the state of the agent). We have two populations, corresponding to agents of the same state.

A weight function is a bounded function  $w : \Lambda_N \rightarrow (0, M]$  which assigns a positive real value to each node, bounded above by some  $M \in \mathbb{R}$ .

We define the neighborhood of a node  $i$  as the first neighbors on the network, thus

$$\mathcal{N}_i = \{j \in \Lambda_N \text{ such that } 0 < d(i, j) \leq 1\},$$

where  $d(i, j)$  is the usual distance between nodes  $i$  and  $j$  in a network, is defined as the number of edges of the minimum path between them.

Given a configuration  $x$ , we call  $U(x, i)$  the *happiness* or utility of an agent located at node  $i$ , that depends on the weight  $w$  and the configuration  $x$  restricted to  $i$  and its neighborhood  $\mathcal{N}_i$  in the following way:

$$U(x, i) = \frac{1}{(\#\mathcal{N}_i) M} \sum_{j \in \mathcal{N}_i} w(j)x(j)x(i),$$

as  $\#\mathcal{N}_i$  is the number of nodes in  $\mathcal{N}_i$ . The normalization factor  $(\#\mathcal{N}_i) M^{-1}$  implies that  $-1 \leq U(x, i) \leq 1$  for every  $i \in \Lambda_N$  and every configuration  $x$ .

Finally, we introduce the Hamiltonian  $\mathcal{H}(x)$ , the opposite of the mean happiness of a given configuration  $x$ , defined as

$$\mathcal{H}(x) = -\frac{1}{N} \sum_{i \in \Lambda_N} U(x, i).$$

It is useful to consider the formal correspondence between particles trying to minimize the energy of the Hamiltonian, and

agents trying to maximize their utilities to define the system’s dynamics. So, we fix some happiness threshold  $U_0$ , and we say that an agent located at  $i$  is unhappy if  $U(i) < U_0$ . Then, given some initial configuration  $x_0$ , we update the configuration by switching two unhappy agents, randomly selected, located at  $i, j$  with different signs (i.e.,  $x(i) \cdot x(j) = -1$ ).

The system will evolve until no unhappy agents can be found, or all unhappy agents have the same sign, so no partners are available for interchanging positions. We say that those final configurations are the stationary states of the model, and we can understand the stationary states as local minima of the Hamiltonian  $\mathcal{H}(x)$  for this dynamic. The final configurations will show two or more clusters grouping agents of the same type. We are interested in the kind of minima obtained for different weights  $w$ . Simulations show that lower local minima of  $\mathcal{H}$  are attained for slowly varying weights than for homogeneous weights over a region of the space.

## RESULTS

We perform computational simulations of the model on one and two-dimensional lattices with periodic boundary conditions and for different weight functions. Each realization starts from a uniformly random distribution of agents’ states. We show results obtained for the case where the populations of each type are of equal size and for a happiness threshold  $U_0 = 0$ . These results can be found in more detail in [1].

In Figure 1a we present final configurations for particular realizations of the one-dimensional case for two families of weight functions ( $w_k^1, w_k^2$ ) that exhibit distinct number of oscillations over the 2500 sites. The black curve represents the weight and, at the top, there is the stationary state where agents in state 1 are blue and in state  $-1$ , red. In both cases, we see how a higher number of oscillations (that increases with  $k$ ) relate to a larger amount of clusters in the final configuration. The segregation pattern shows small-scale clusters in this situation, similar to the classical Schelling model with constant weight.

We also implement simulations over a two-dimensional square lattice and weight functions that are constant over lattice columns. In the upper panels of Figure 1b, we show typical stationary states obtained for a square lattice of side 200, along with the corresponding weight function in greyscale. We see how few oscillations of the weight function (as in the left panel) generate larger scale segregation than a higher oscillating function (as in the right panel). In both cases, a more fragmented segregation pattern displays over the locally highly weighted sites. To quantify this effect, at the bottom of Figure 1b, we include plots of the mean number of cluster boundaries (i. e. the number of sign changes) within each lattice column, taken over 50 realizations with different initial distributions of the state of the agents. It is remarkable how the peaks of the sign changes per column coincide with the highest values of the weight.

In addition, we discuss some empirical examples. One of the variables that could give traces of the model’s weight function is the territory’s topography. The existence of hills, for instance,

favors a propensity to appreciate the relations with the neighbors and to make communities [3].

In Figure 1c we present data on the topography, along with information about the ethnic distribution, for some regions of the Brazilian cities of Sao Paulo and Rio de Janeiro. In each city, we analyze an area of equal surface, defined from a square bounding box of 0.15 degrees on the sides of latitude and longitude. Rio de Janeiro shows more variability than Sao Paulo in the topography. To visualize information about the ethnic distribution of the population, we based on the IBGE (Brazilian Institute of Geography and Statistics) Census 2010 source [5]. The census provides geo-referenced data, at the census tract level, of the self-declared ethnicity of each Brazilian citizen (over five preset categories: White, Brown/Mixed, Black, Asian, and Indigenous (see [6])). In Figure 1c, we color each census tract according to its ethnic majority and observe different segregation patterns in each city. Although a quantitative analysis requires considering the density of the different ethnicities and the size of populations, a preliminary comparison of the ethnic segregation and topography maps could illustrate the idea of the high variation of a weighting function (associated with the topology of territory), smaller clusters arise.

## DISCUSSION AND FURTHER WORK

Our model connects the traditional Schelling model where an agent wants to be surrounded by neighbors from the same group with a weight function associated with the relevance that the agent assigns to being surrounded by agents of the same type, which depends on each location. Hence, some places are relatively more important than others for establishing possible ties. Apart from the topography, another possible empirical interpretation of this weight function is the inverse of the price of land or real estate that each place represents. Another one is the inverse of the prestige of the area, the number of available resources, high provision of public goods, facilities, cultural and educational opportunities, transportation accessibility for the rest of the city, green spaces, and, from an aesthetics point of view, beauty areas. These general facilities are inverse to the weight functions of the places. And when these facilities are absent, the weight function takes a high value and the importance of ties to survive, too.

Under this rule, our model predicts two types of segregation patterns: a) higher spatially oscillations of the weighting function are associated with segregation levels similar to small patches or ghetto’s formation, with clusters of smaller size; b) oscillations of the weighting function in the space corresponds to significantly segregated neighborhoods, with clusters of big size.

In what follows, we intend to apply the model to irregular networks that are more realistic and capture relevant geographic information about specific cities. Also, we plan to advance in the characterization and quantification of residential segregation based on census tract information, considering the distribution of socio-cultural groups of each census tract, not

