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Cooperation in costly-access environments

Carlos Gracia, Hugo Pérez, Fabio Dercole, Yamir Moreno

Why do we cooperate?

Cooperative behavior poses an evolutionary puzzle when free-riding yields a higher benefit.

Prisoner’s Dilemma

Game Theory model to study cooperative situations.

- Two strategies: cooperate (C) or defect (D).

You \ She	C	D
C	R	S
D	T	P

- For any opponent’s action, the best response is always to defect, T>R>P>S.
- The sum of payoffs is greater when both choose to cooperate (if T < 2R).

Voluntary Prisoner's Dilemma

Model to study voluntary cooperative situations.

She / You	C	D	L
C	1	0	σ
D	b	0	σ
L	σ	σ	σ

- Loners (L) and their co-players obtain σ, with $0<\sigma<1$.
- Loners perform better than two defectors but worse than cooperative pairs.

G. Szabó, C. Hauert. Phys. Rev. E **66**, 062903 (2002)

PHYSICAL REVIEW E **66**, 062903 (2002)

Evolutionary prisoner's dilemma games with voluntary participation

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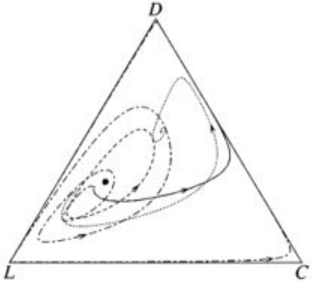

FIG. 1. Trajectories predicted by pair approximation ($\sigma = 0.3$, $K = 0.1$). (a) For $b = 1.03$ (solid line) loners go extinct; D and C survive and coexist in a stationary state. (b) All three strategies coexist for $b = 1.2$ (dotted line). (c) For $b = 1.5$ a limit cycle appears (dashed line). The bullet indicates the unstable interior fixed point. (d) All trajectories spiral toward the boundary of the simplex (dot-dashed line) ($b = 1.9$).

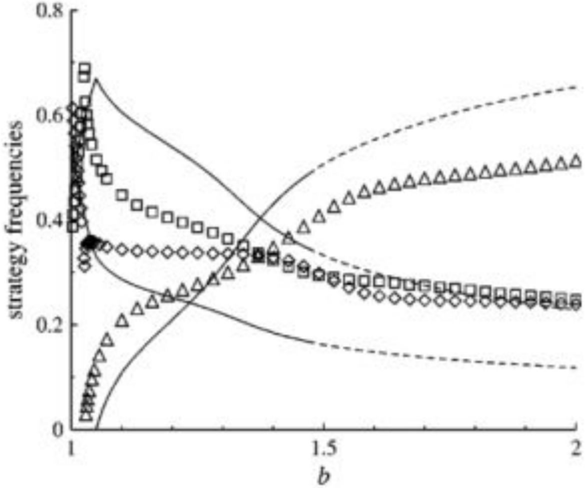

FIG. 2. Average frequency of defectors (squares), cooperators (diamonds), and loners (triangles) as a function of b for $K = 0.1$ and $\sigma = 0.3$ on square lattices. The solid (dashed) lines show the stable (unstable) stationary values predicted by pair approximation.

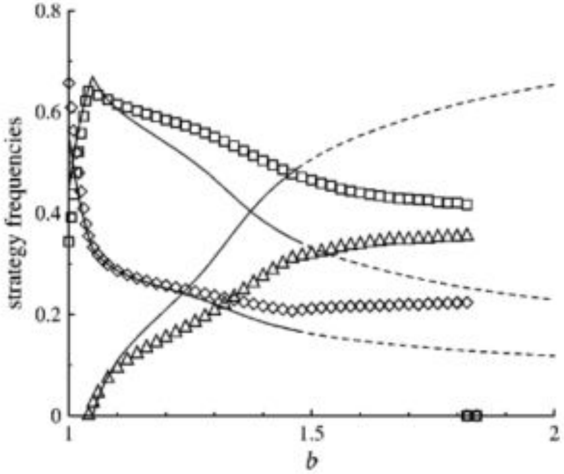

FIG. 3. Average frequencies of defectors (squares), cooperators (diamonds), and loners (triangles) as a function of b for $\sigma = 0.3$ and $K = 0.1$ on a RRG. The MC data are obtained by averaging over 10^4 MC steps per site after suitable relaxation times for $N = 10^6$ sites. The solid and dashed lines show the stable and unstable predictions of the pair approximation.

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Costly and voluntary interactions

Most real-world cooperative scenarios involve a cost to participate:

- Transportation cost
- Time investment
- Entree fees
- Costly participation in institutions…

Furthermore, many of those interactions are voluntary.



Costly and voluntary interactions: modeling

Costly-Access Prisoner's Dilemma

Costly-Access Prisoner's Dilemma

- 3 strategies: C, D, A (abstainer).
- Voluntary participation
- Participation fee t

You \ She	C	D	A
C	r-t-1	-t-1	-t
D	r-t	-t	-t
A	0	0	0

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- The participation fee t entails a cost to access the potential interaction, even if the interaction will not take place because it is refused by the other player.

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- Abstainers: nothing to win nothing to lose.

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- The participation fee t entails a cost to access the potential interaction, even if the interaction will not take place because it is refused by the other player.
- Abstainers: nothing to win nothing to lose.
- In the absence of abstainers (only C, D): Prisoner's Dilemma.

Costly-Access Prisoner's Dilemma

- The participation fee breaks the symmetry of VPD in which both agents obtain the same payoff when one of them abstains.

You \ She	C	D	A
C	r-t-1	-t-1	-t
D	r-t	-t	-t
A	0	0	0

Costly-Access Prisoner's Dilemma

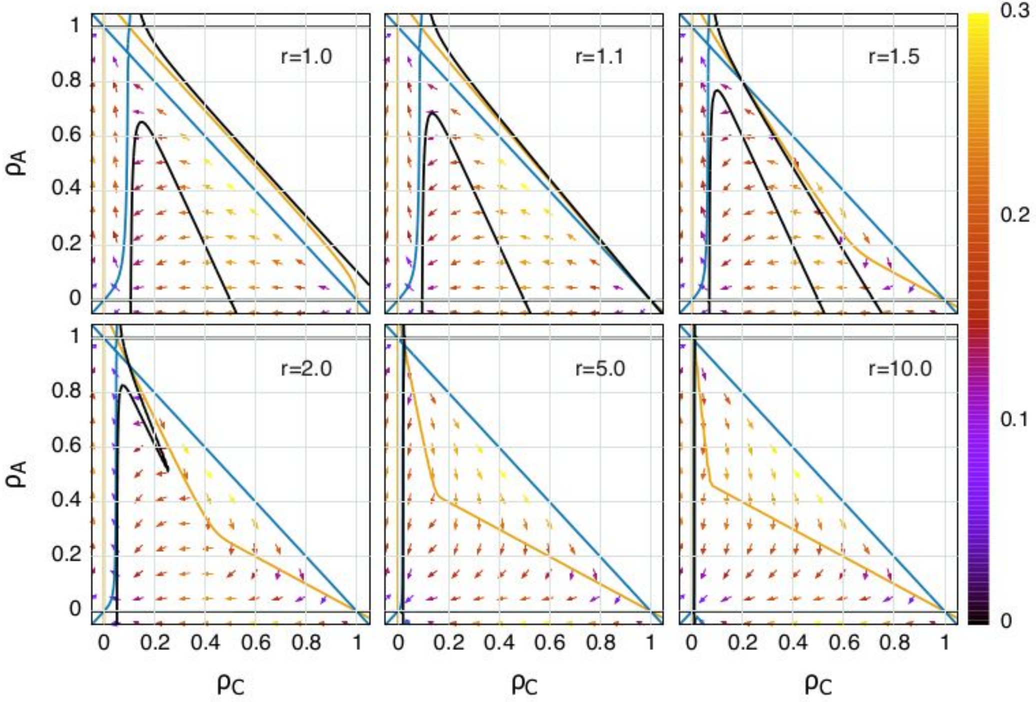
Dynamics

- Each agent plays with the same strategy (A, C, or D) with all her neighbors.
- Synchronous Fermi-like update:

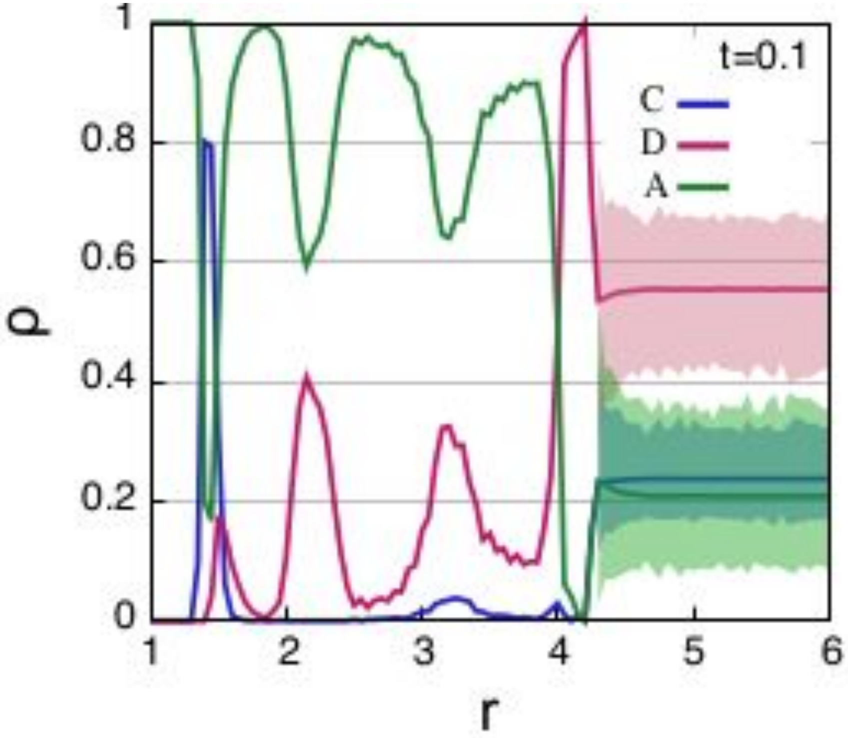
$$P_{ij} = \frac{1}{1 + \exp(\frac{\Pi_i - \Pi_j}{T})}$$

Results

Mean field approach



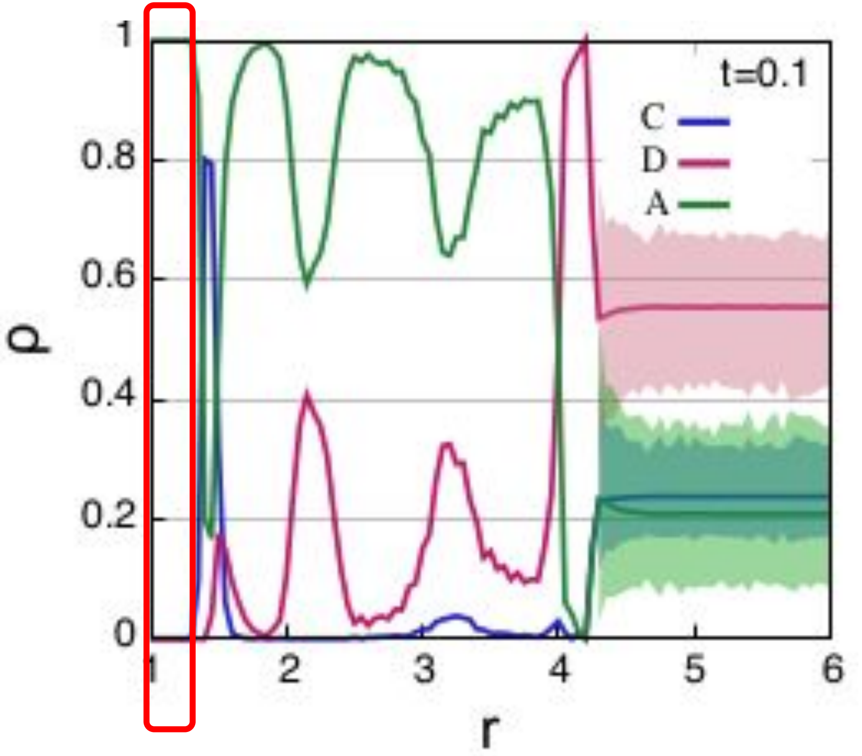
Numerical results: RRN



Numerical results: RRN

3 phases:

- $r<r_{min}$: all executions end in the full-A state (as predicted by mean-field).



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Numerical results: RRN

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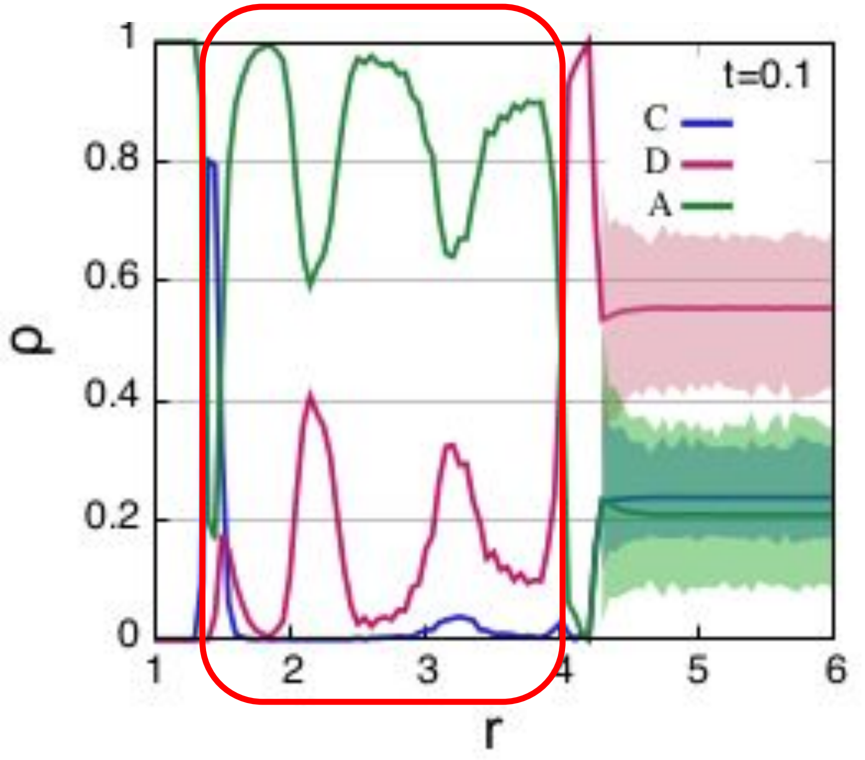
- $r<r_c$~4.2: all simulations end on one-strategy absorbing states.

D⟵A⟵C (pair).



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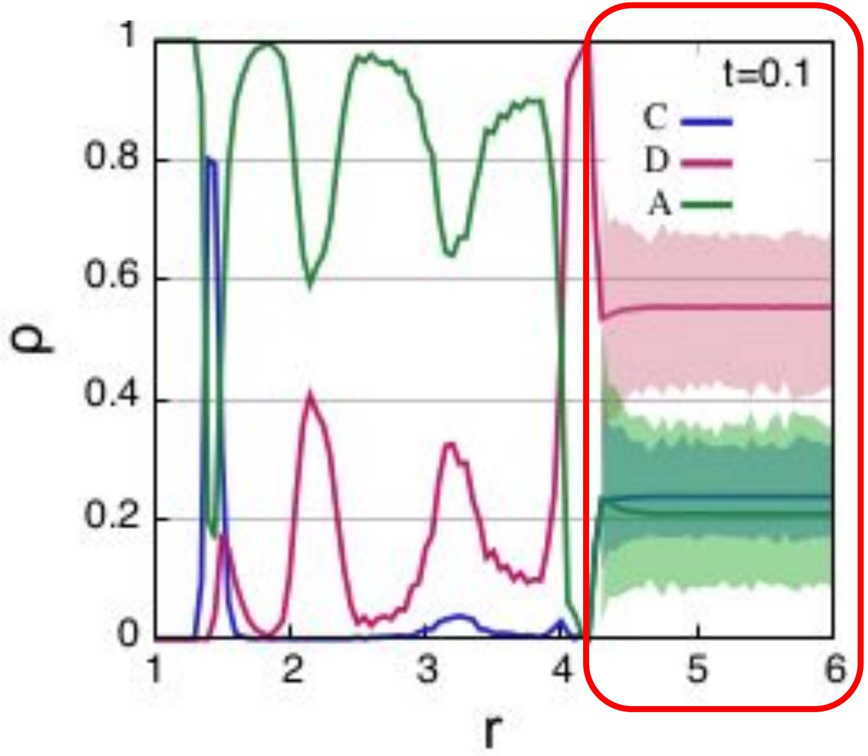
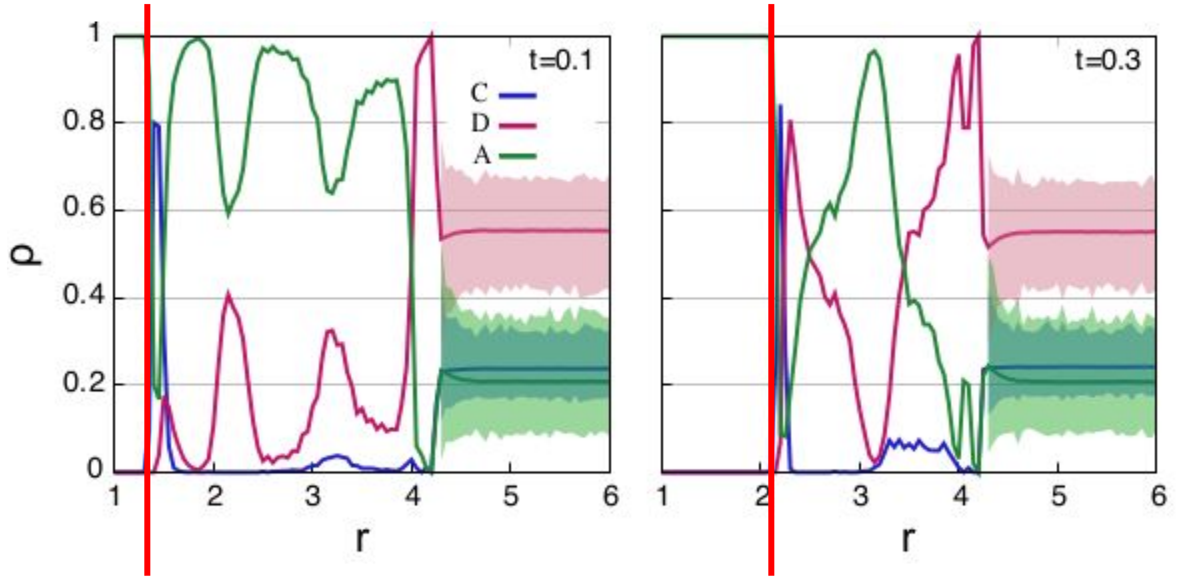
- $r<r_c$~4.2: all simulations end on one-strategy absorbing states.

  D⟵A⟵C (pair).
- $r>r_c$: 3-coexistent-strategies state; strategists' frequencies are independent of r.

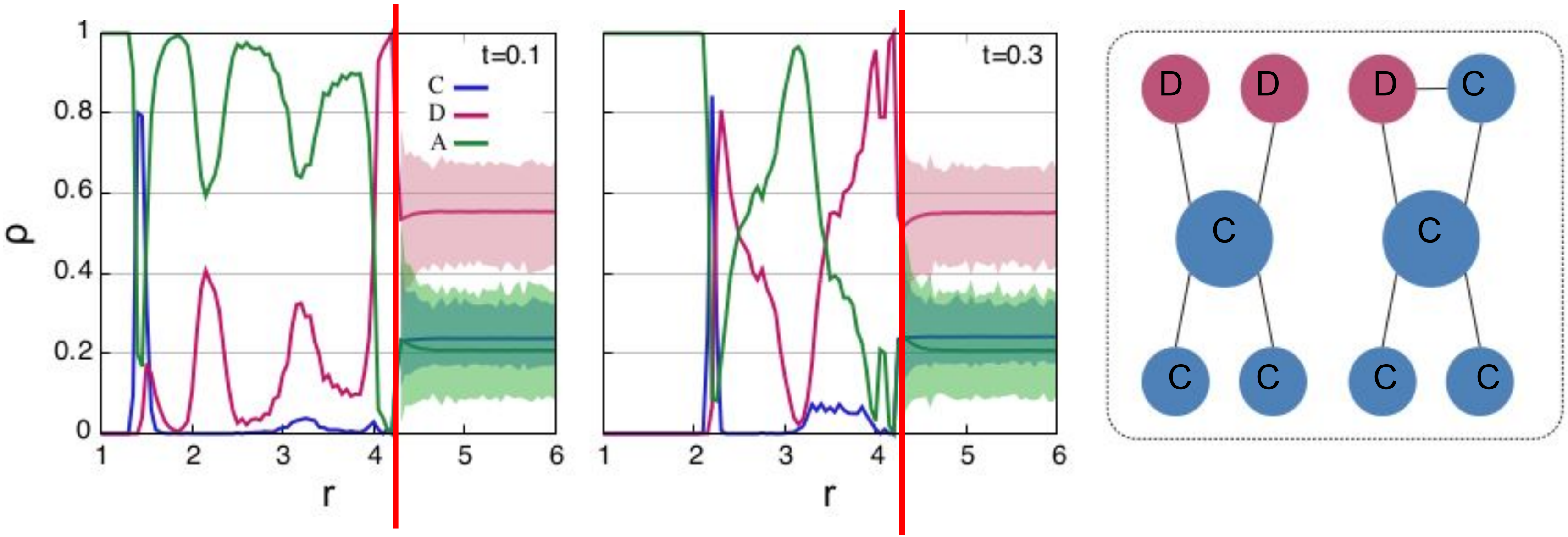


Numerical results: RRN



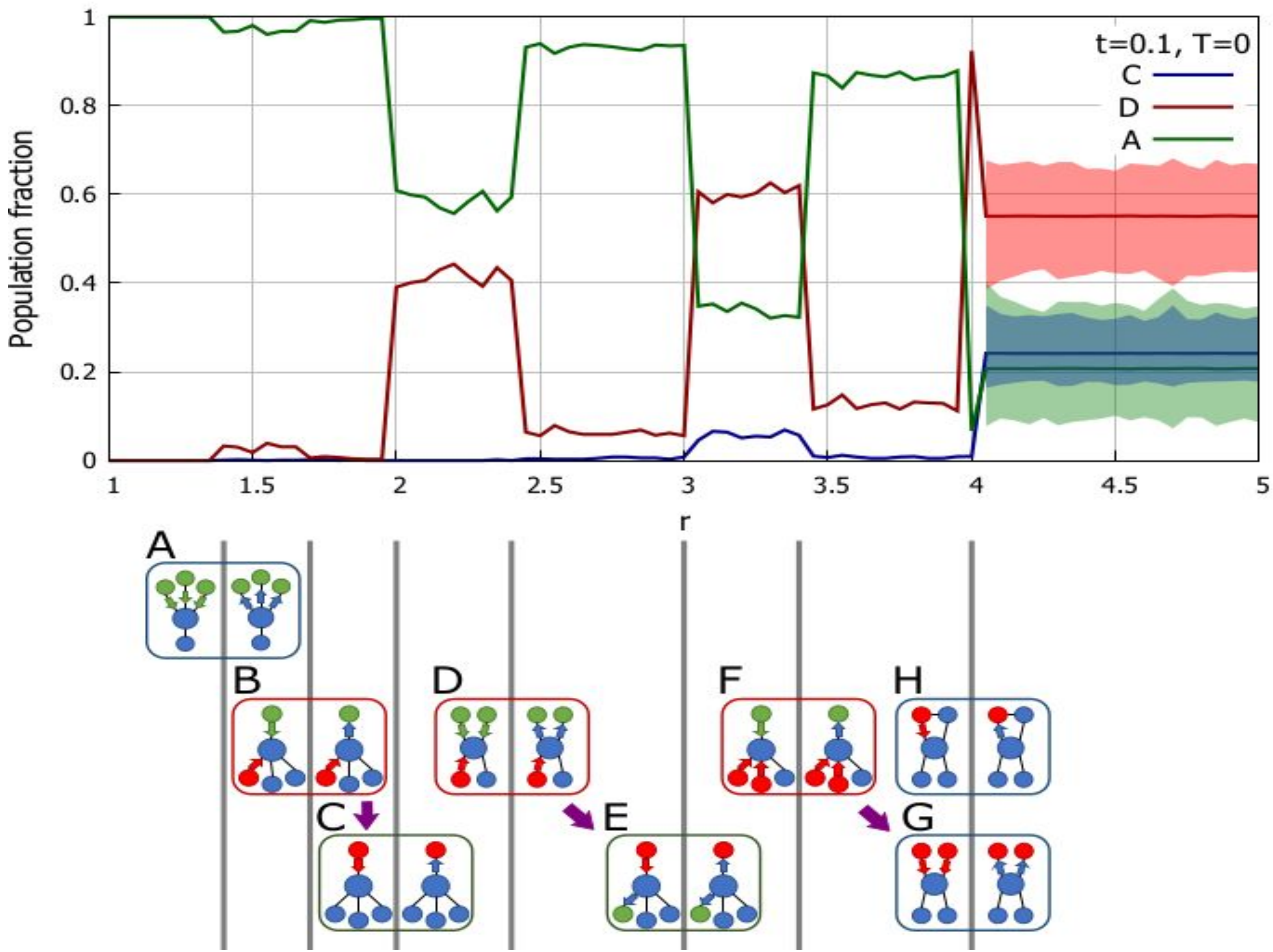
- $r > 4t+1 = \mathbf{r_{min}} \longrightarrow \Pi_{C\text{-}C} > \Pi_A$. In the (random) early stages, Cs avoid invasion.

Numerical results: RRN



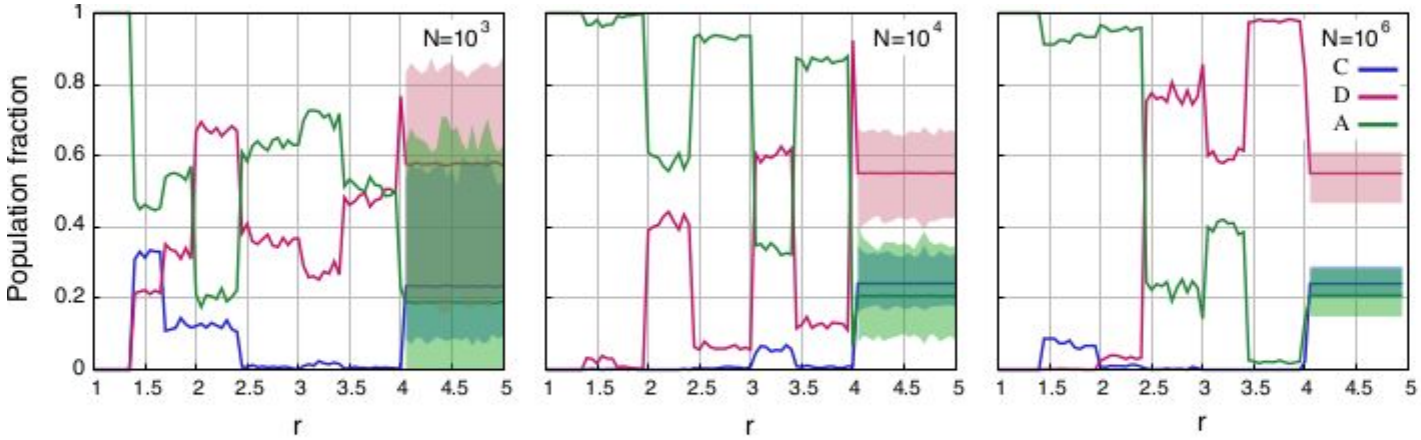
- r > 4t+1 = $\mathbf{r_{min}}$ ⟶ $\Pi_{C\text{-}C} > \Pi_A$. In the (random) early stages, Cs avoid invasion.
- Condition for stable coexistence: at C-cluster border, $\Pi_C > \Pi_D$. $\mathbf{r_c}$=k.

T=0



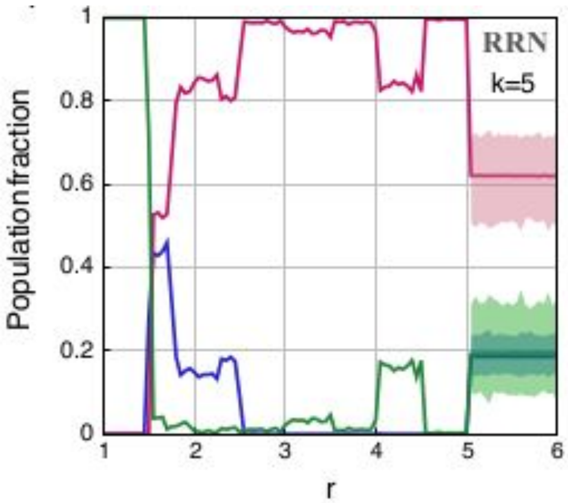
t=0.1, T=0
C
D
A
Population fraction
r
1
0.8
0.6
0.4
0.2
0
1
1.5
2
2.5
3
3.5
4
4.5
5
A
B
C
D
E
F
G
H

Effect of network size



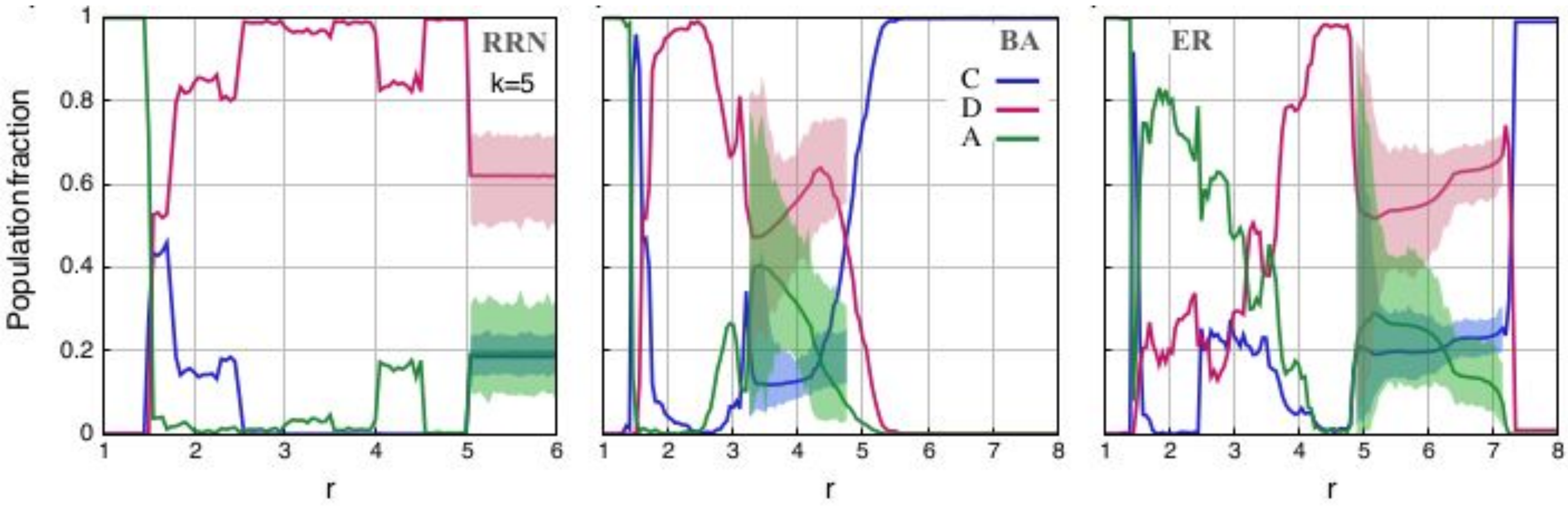
- Network size has an important effect on the final proportion of simulations that end up dominated by each strategy (although thresholds do not vary).
- Bigger networks result in longer trajectories: larger survival chance.

Effect of degree and network topology



- Fixed k: previous phases maintain. #^{n} thresholds (r_{min}> r > r_c) increase with k.

Effect of degree and network topology



- Fixed k: previous phases maintain. $\#^n$ thresholds ($r_{min} > r > r_c$) increase with k.
- Heterogeneous networks: new thresholds. Complex behavior.
- For $r > r_{all\text{-}c}$, hubs' role promoting cooperation prevails.

Take-home message

Participation fee breaks a symmetry of the VPD

and induces a reach alternating behavior.

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Thanks!   Questions?

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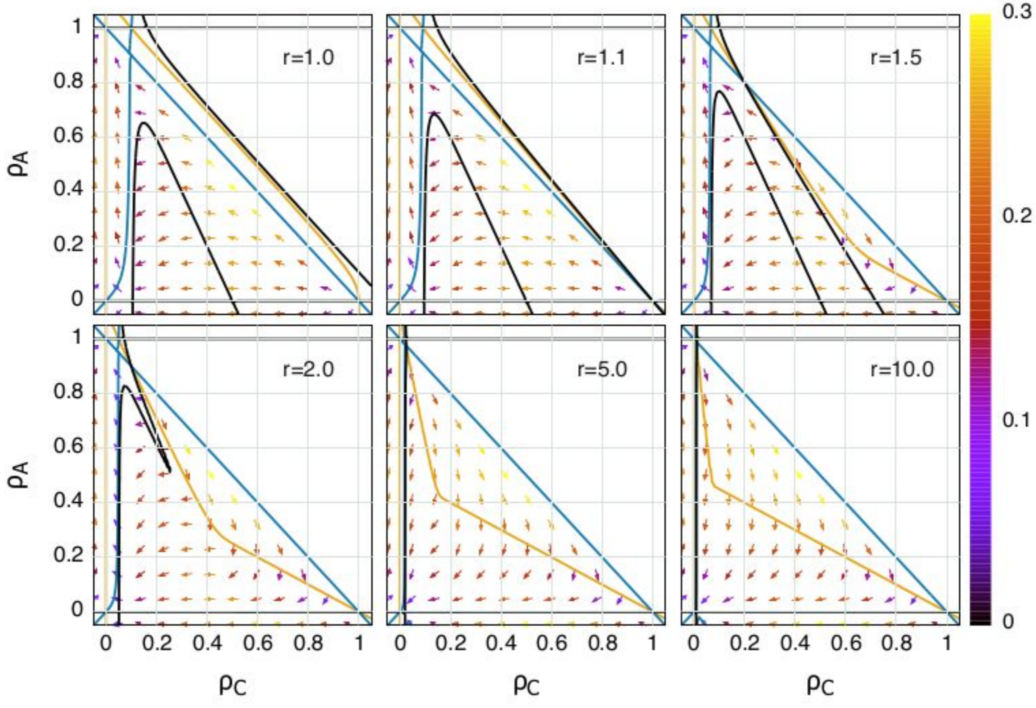
Conclusions

- Sometimes, voluntary participation involves an access cost to the cooperative interaction, besides the cost associated with cooperation.
- That participation fee breaks the symmetry between abstainers and participants.
- In well-mixed infinite populations, the dynamic always leads to abstention.
- Structured populations display an alternating behavior between mono-strategic, multi-stable, and coexistence phases.
- This behavior is fully explained through a theoretical analysis of the strategic motifs.

Results

Mean field approach

- Fixed points:
  - 3 absorbing states.
  - 1 on the hypotenuse (no D) unstable.
- Borders: invariant manifolds.
- Interior of the simplex: trajectories end in Full-A (Nash eq.) ∀ r.
- For r>c+t, the nullcline $\dot{\rho}_C = 0$ falls into the phase space (towards decreasing ρ_A).



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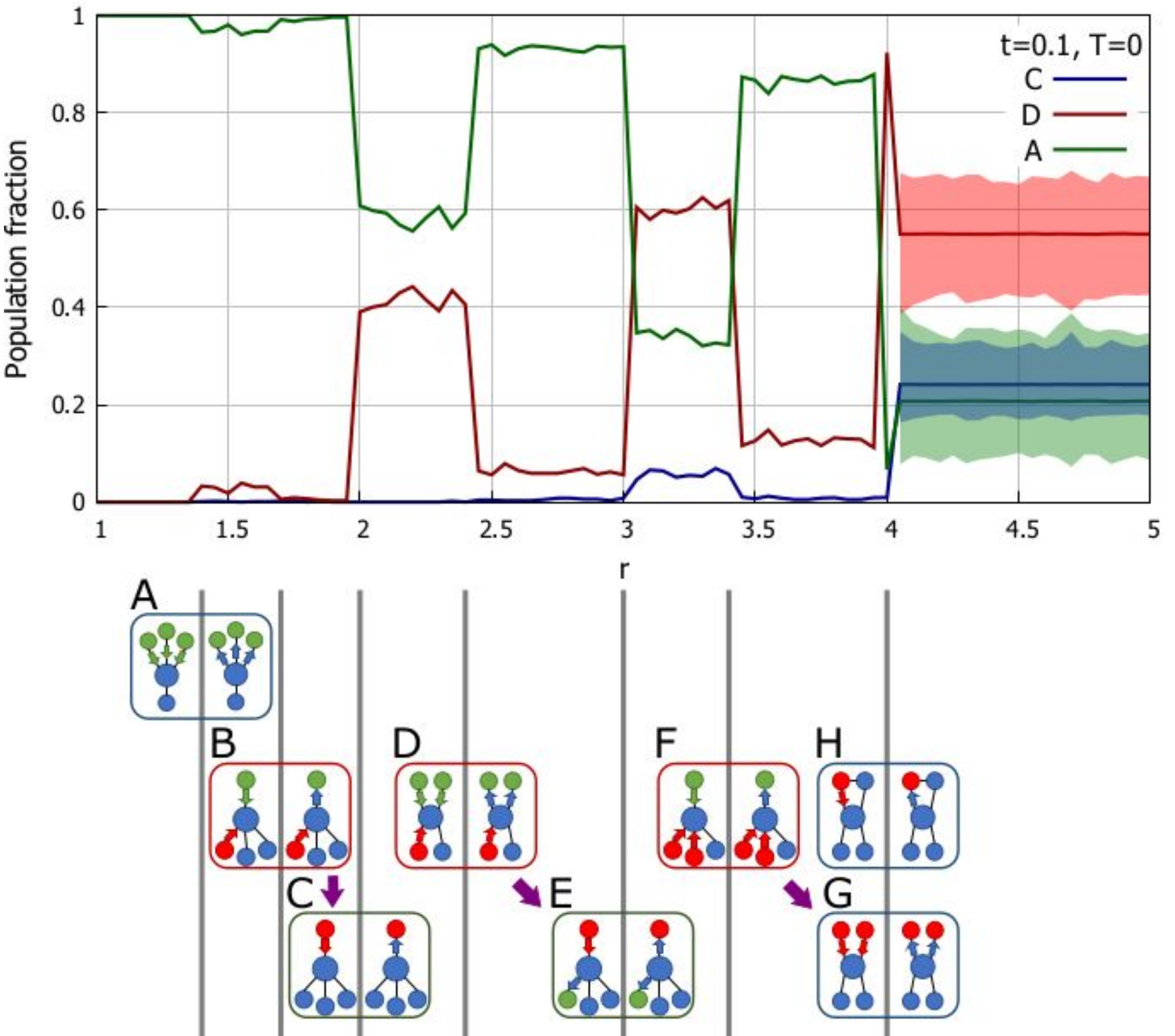
T=0

To interpret the $r_{min} < r < r_c$ region, let's remove most stochastic effects: T=0.

Transitions are explained through motifs' stability:

- Each diagram indicates a dominance transition for the corresponding motifs regarding the propagation or extinction of the central node.
- Here, neighbors of D are not C.
- Purple arrows indicate the system's evolution when the central C-node invades an A-neighbor.



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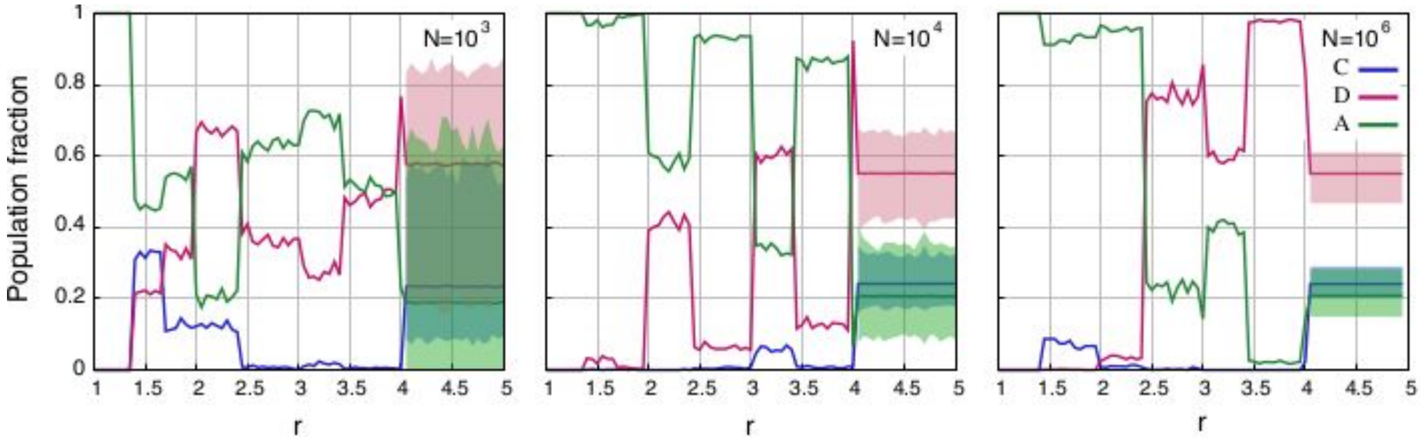

- Although network size has an important effect on the final proportion of simulations that end up dominated by each strategy, thus proving its influence on the survival probabilities, thresholds related to motifs are always present and give rise to abrupt transitions in final proportions.

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Markovian approach

Payoffs depend on the frequencies

$$\Pi_x = k(\rho_x M_{xx} + \rho_y M_{xy} + \rho_z M_{xz})$$

and yield the imitation probabilities

$$P_{xy} = \frac{1}{1 + exp\left[kT^{-1} \sum_{i \in \{A,C,D\}} \rho_i (M_{xi} - M_{yi})\right]}$$

which provide the temporal evolution

$$\begin{aligned}
\dot{\rho}_A =& \rho_A \left[\rho_C (P_{CA} - P_{AC}) + \rho_D (P_{DA} - P_{AD})\right] \\
\dot{\rho}_C =& \rho_C \left[\rho_D (P_{DC} - P_{CD}) + \rho_A (P_{AC} - P_{CA})\right] \\
\dot{\rho}_D =& \rho_C \left[\rho_D (P_{CD} - P_{DC}) + \rho_A (P_{AD} - P_{DA})\right]
\end{aligned}$$

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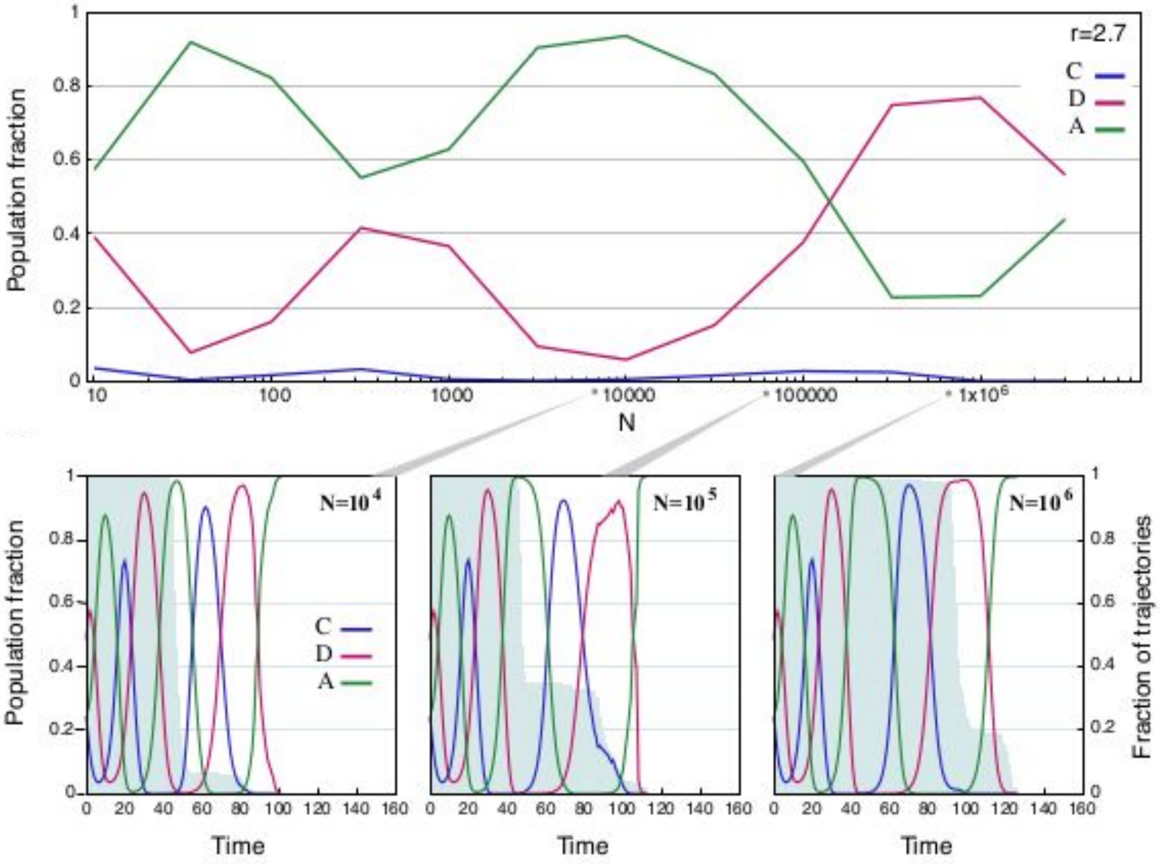

- Oscillating trajectories for all network sizes.
- Oscillation amplitude and frequency ~unaffected by N.
- Synchronous realizations.
- Bigger networks result in longer trajectories: larger survival chance.