Interacting social and disease dynamics in multiplex networks

Federico Vázquez

Institute of Calculus - CONICET
University of Buenos Aires

In collaboration with:

Paulo Cesar Ventura da Silva
School of Public Health, Indiana University

Fátima Velásquez
IFISC, Universitat de les Illes Balears

Colm Connaughton
Mathematics Institute, University of Warwick

Yamir Moreno
BIFI, University of Zaragoza

Francisco Rodrigues
Institute of Mathematics and Computer Science
University of São Paulo

Introduction

Main motivation of epidemic modeling:
develop and test methods to control disease transmission.

Individual prevention methods: quarantine/isolation, vaccination, face mask, social distancing, hand washing.

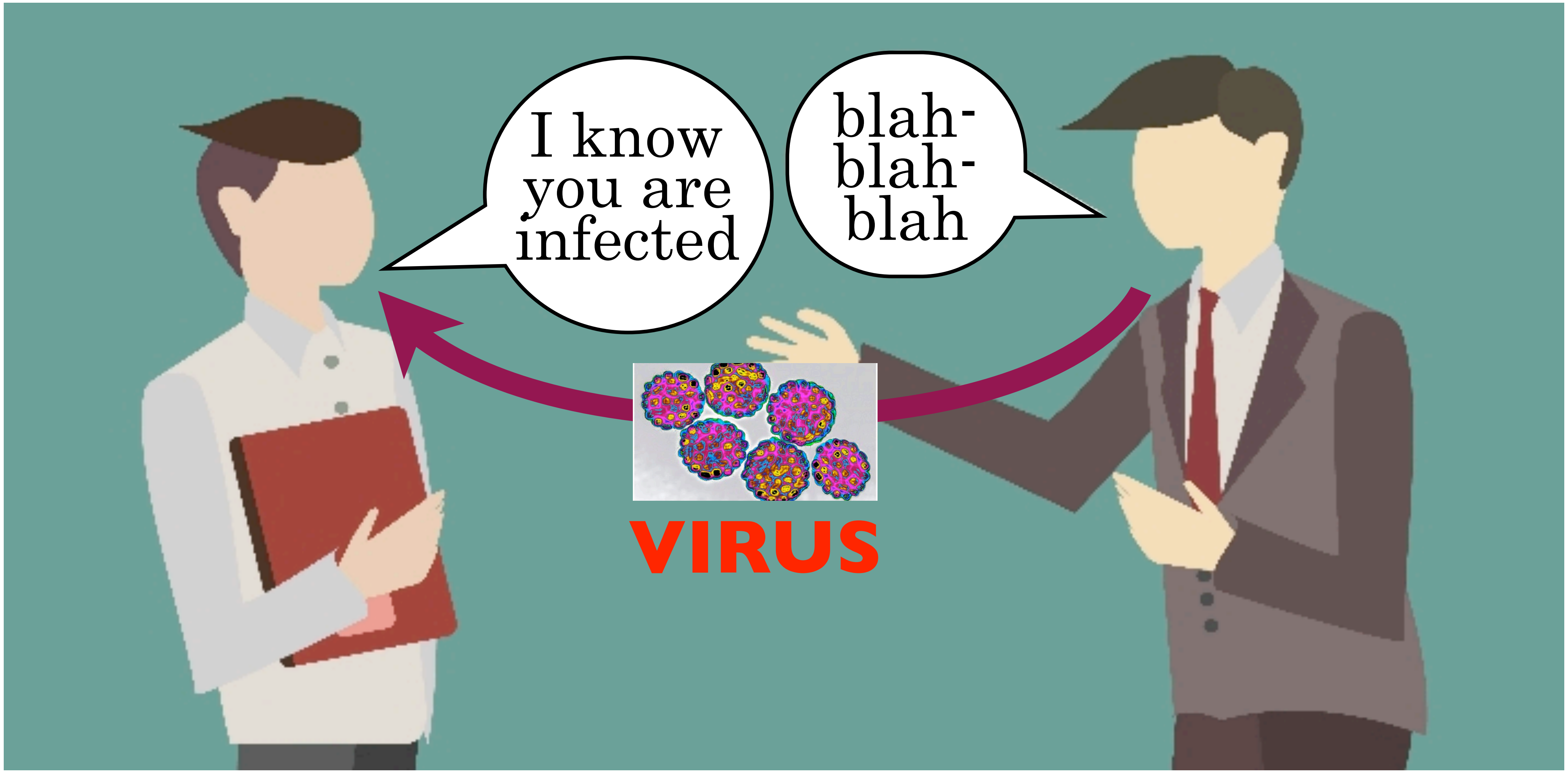
Introduction

Main motivation of epidemic modeling:
develop and test methods to control disease transmission.

Individual prevention methods:
quarantine/isolation, vaccination, face mask, social distancing, hand washing.

Decision to adopt methods depends on human behavior.

Risk perception: I become **aware** of an epidemics by noticing that a relative/friend/colleague is infected.



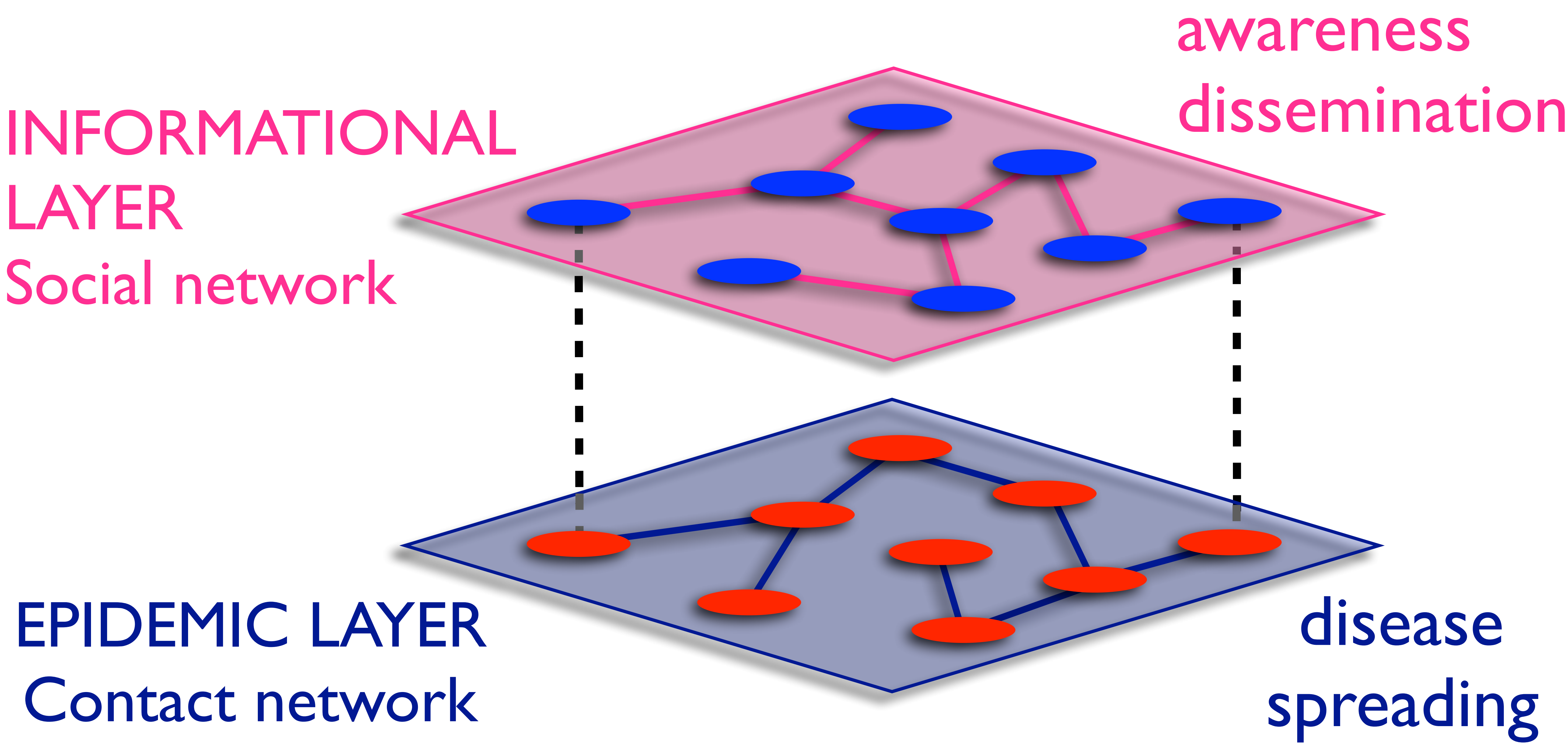
Spreading of awareness:
word-of-mouth propagation of information about the epidemics.

Aware individuals reduce their risk of infection (prevention methods).

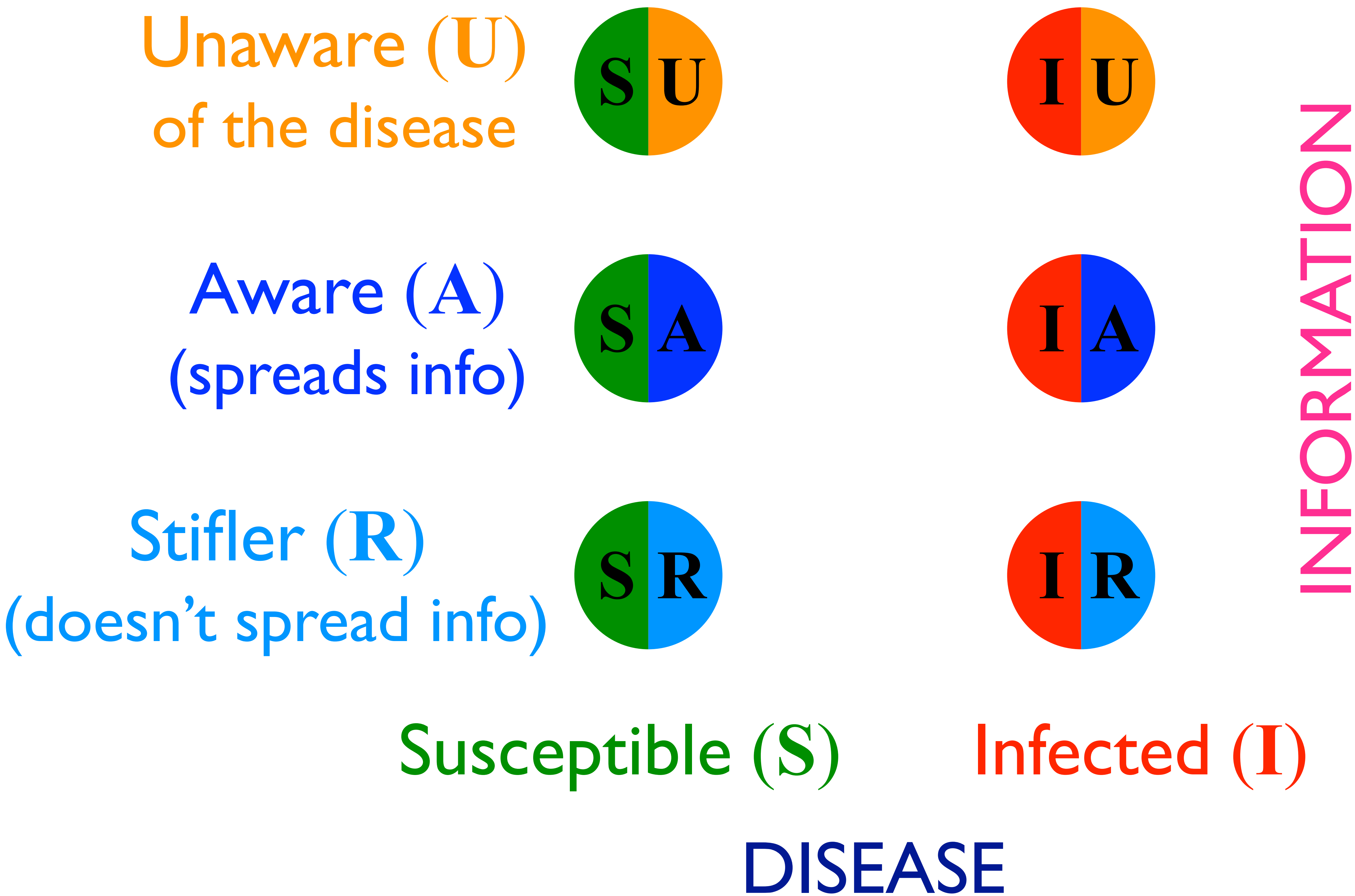
How effective is the spreading of information awareness to prevent disease transmission?

Multiplex network model

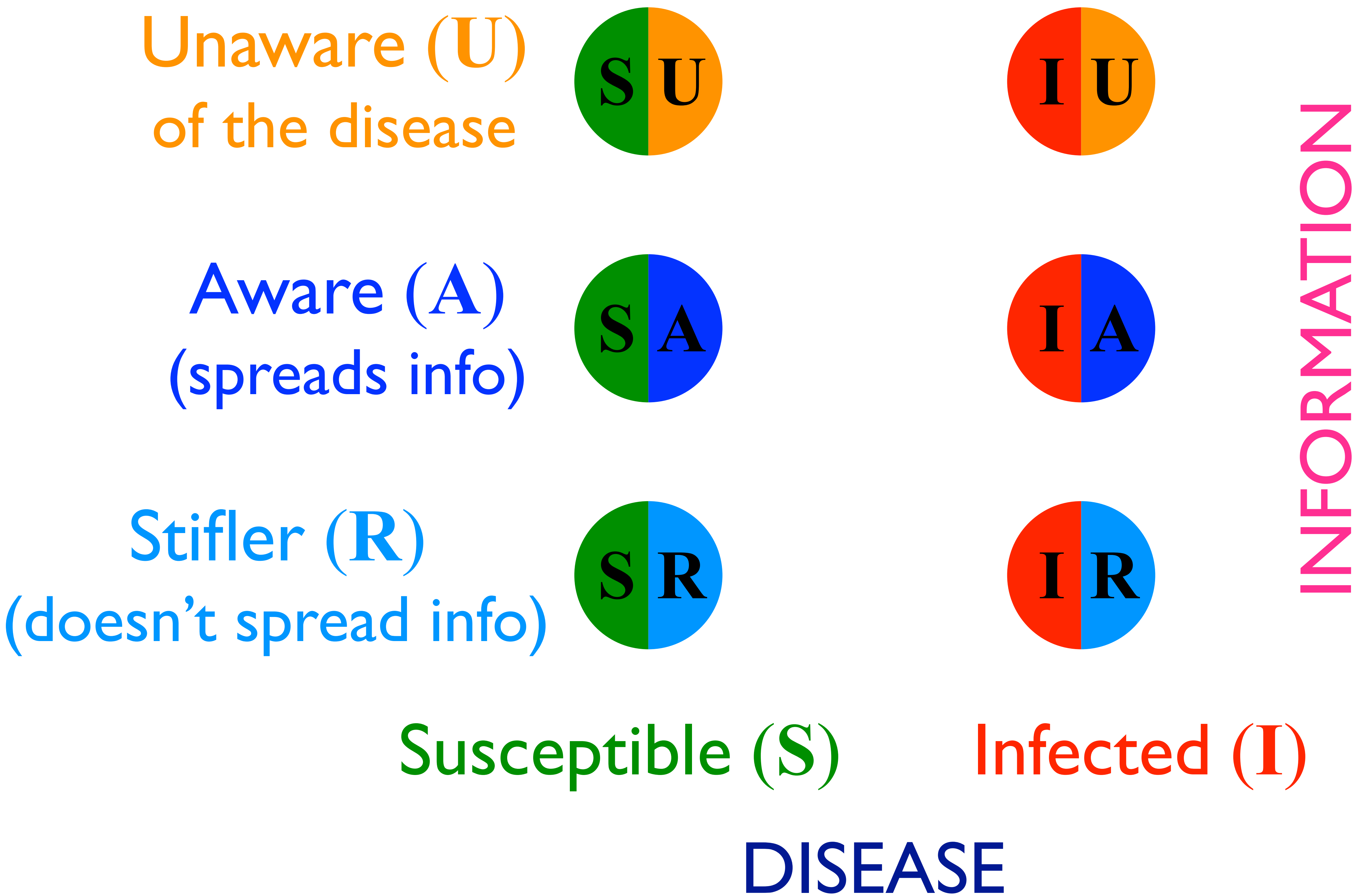
PRE **100**, 032313 (2019)



Node states

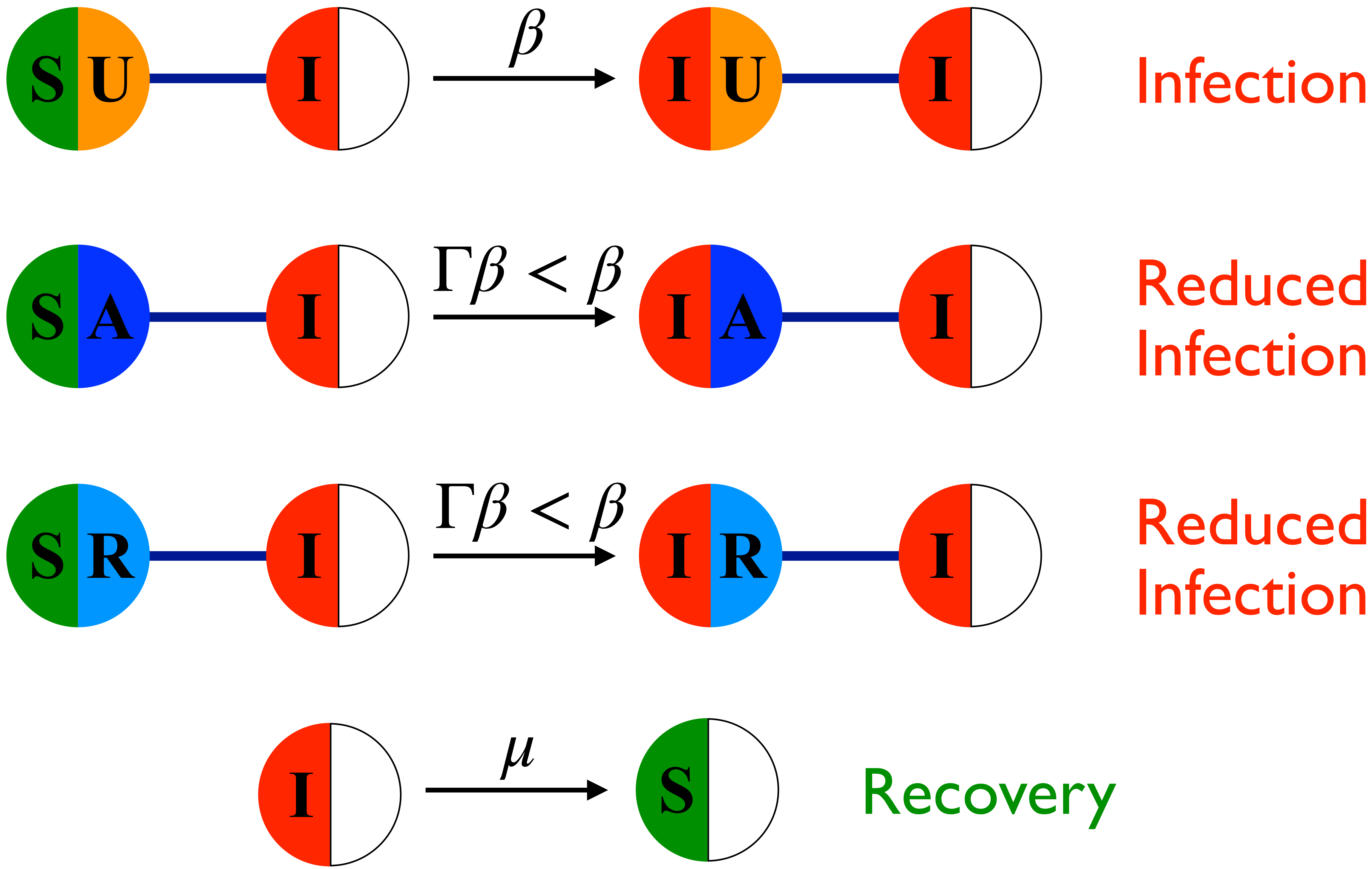

Unaware (U)
of the disease
Aware (A)
(spreads info)
Stifler (R)
(doesn’t spread info)
S U
I U
S A
I A
S R
I R
INFORMATION
Susceptible (S)
Infected (I)
DISEASE

Node states

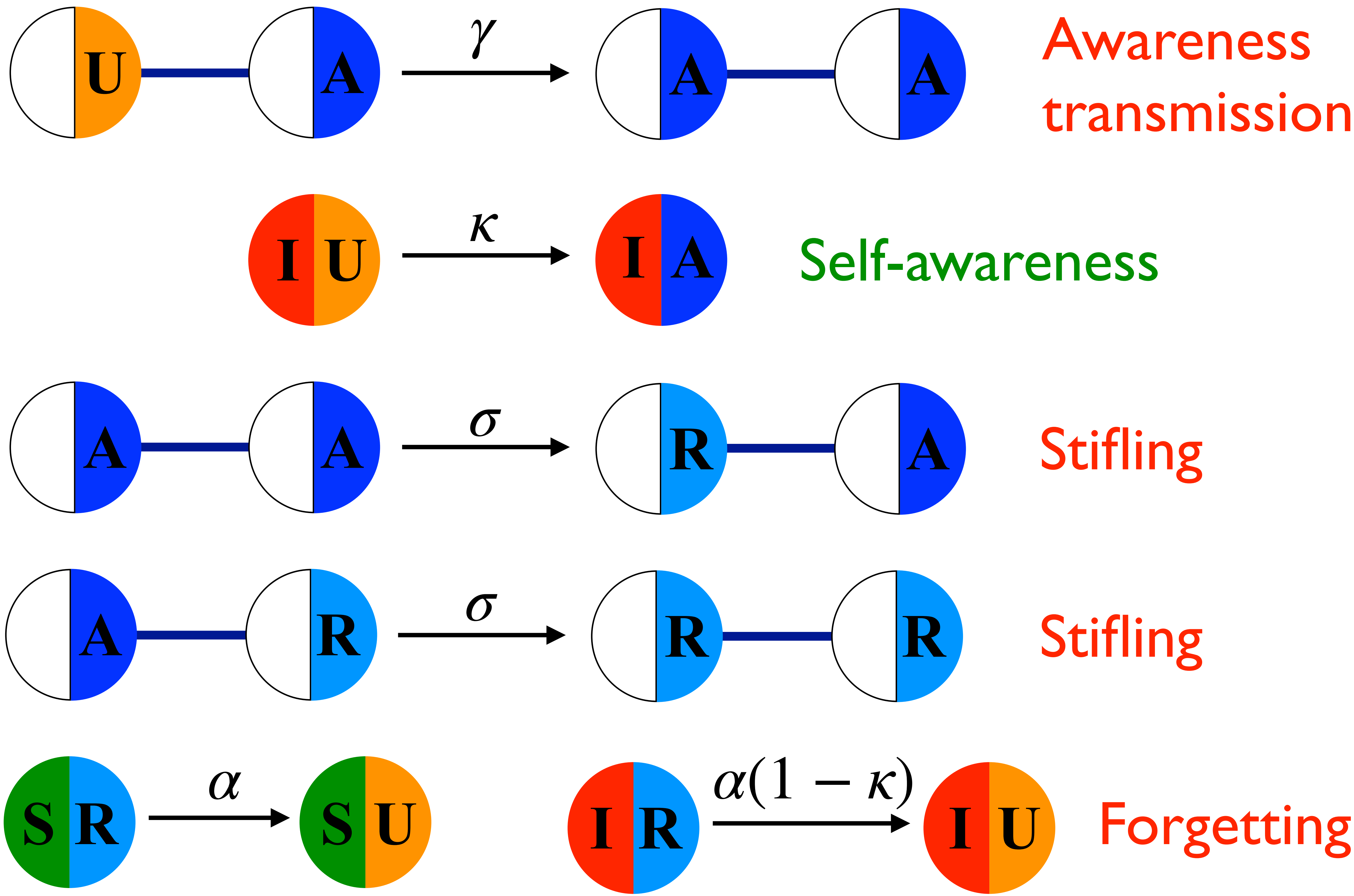


Unaware (U) of the disease
S U
I U
Aware (A) (spreads info)
S A
I A
Stifler (R) (doesn't spread info)
S R
I R
INFORMATION
Susceptible (S)
Infected (I)
DISEASE

SIS disease dynamics ($\Delta t = 1$):



UARU (Maki-Thompson) information dynamics ($\Delta t = 1$):



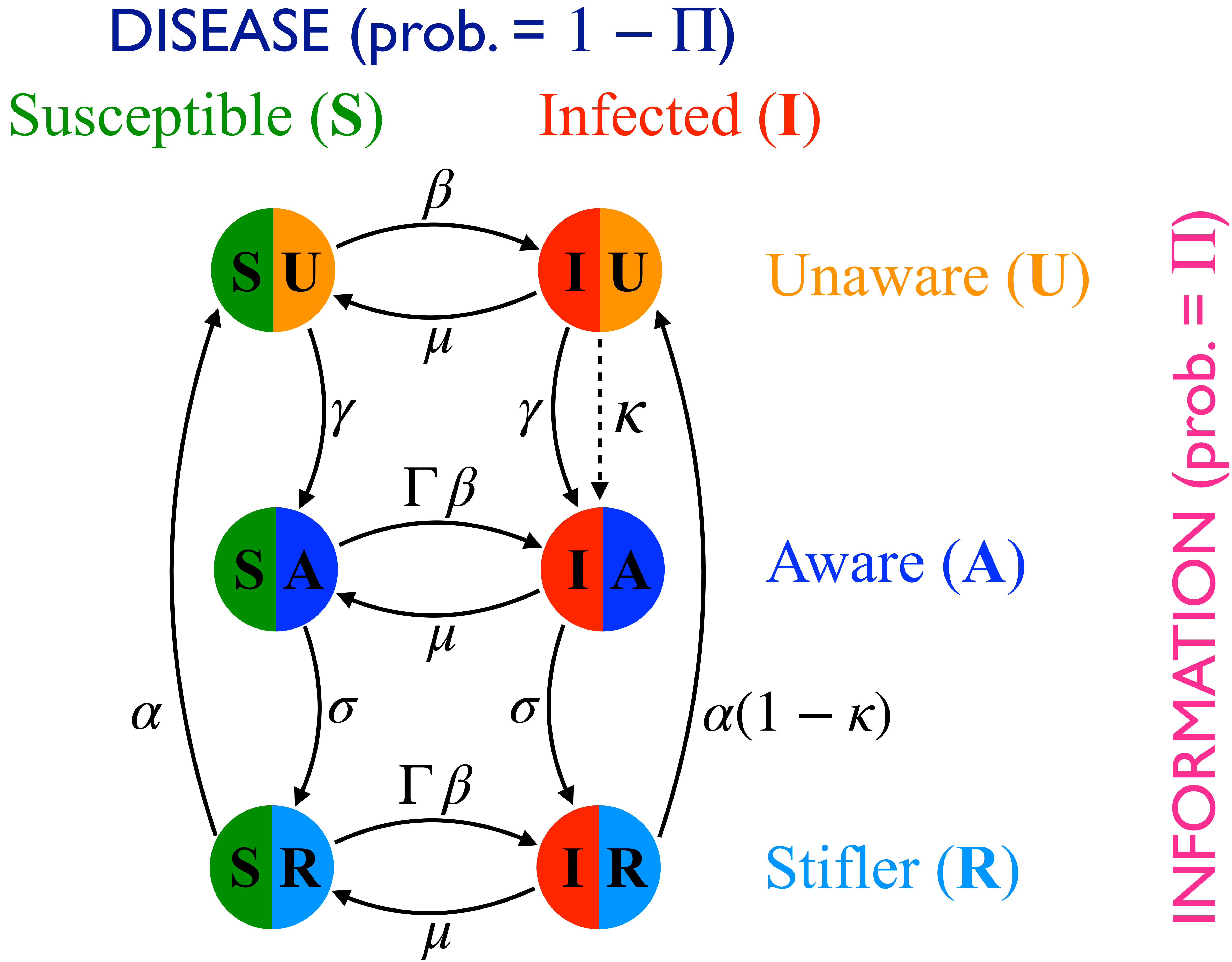
Physica A **374**, 457 (2007)

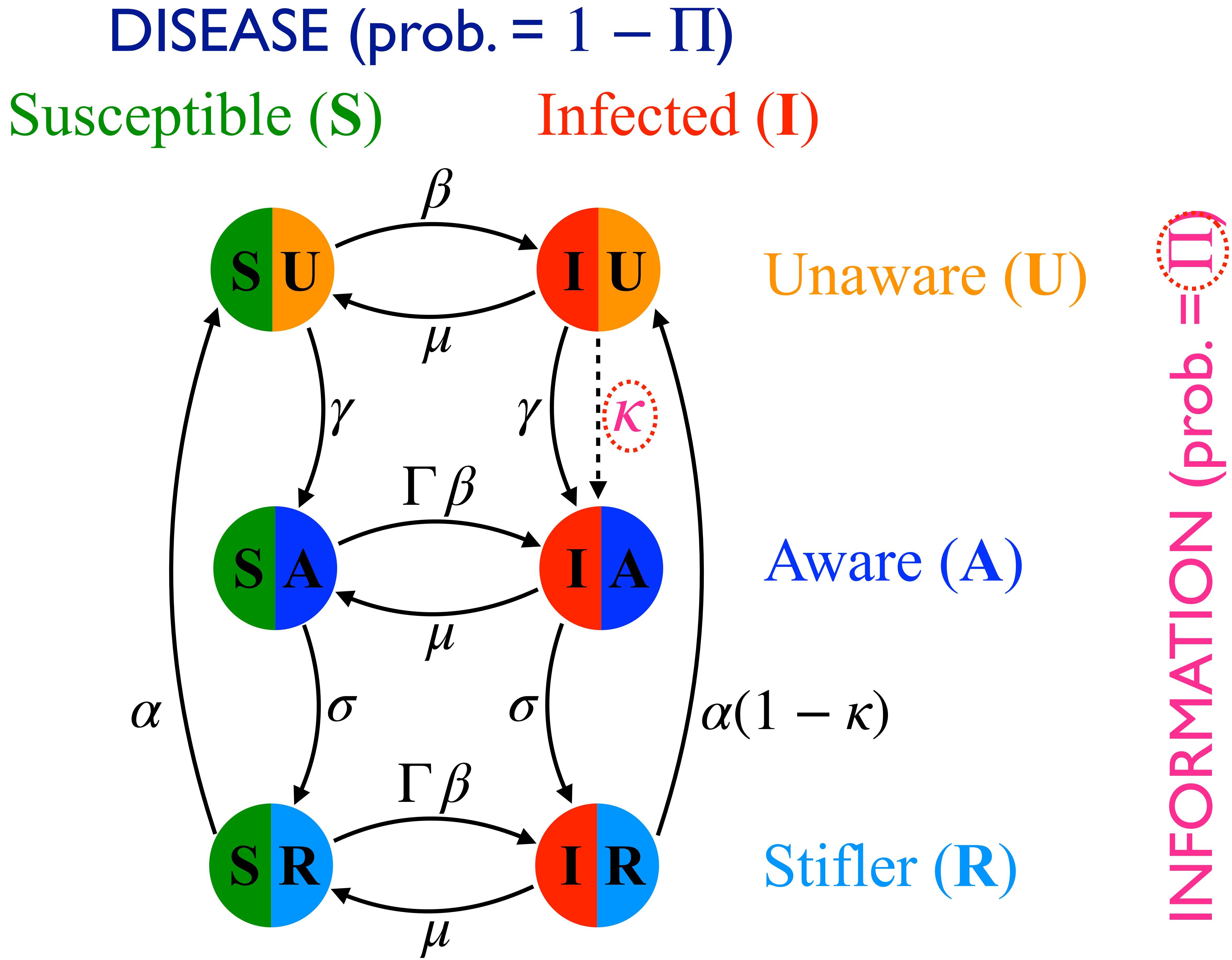
We consider that epidemic and awareness propagate and vanish at different time scales.

Relative speed of two processes controlled by parameter Π.

HIV:
life cycle (~years).
awareness spread and forgotten many times in this period.

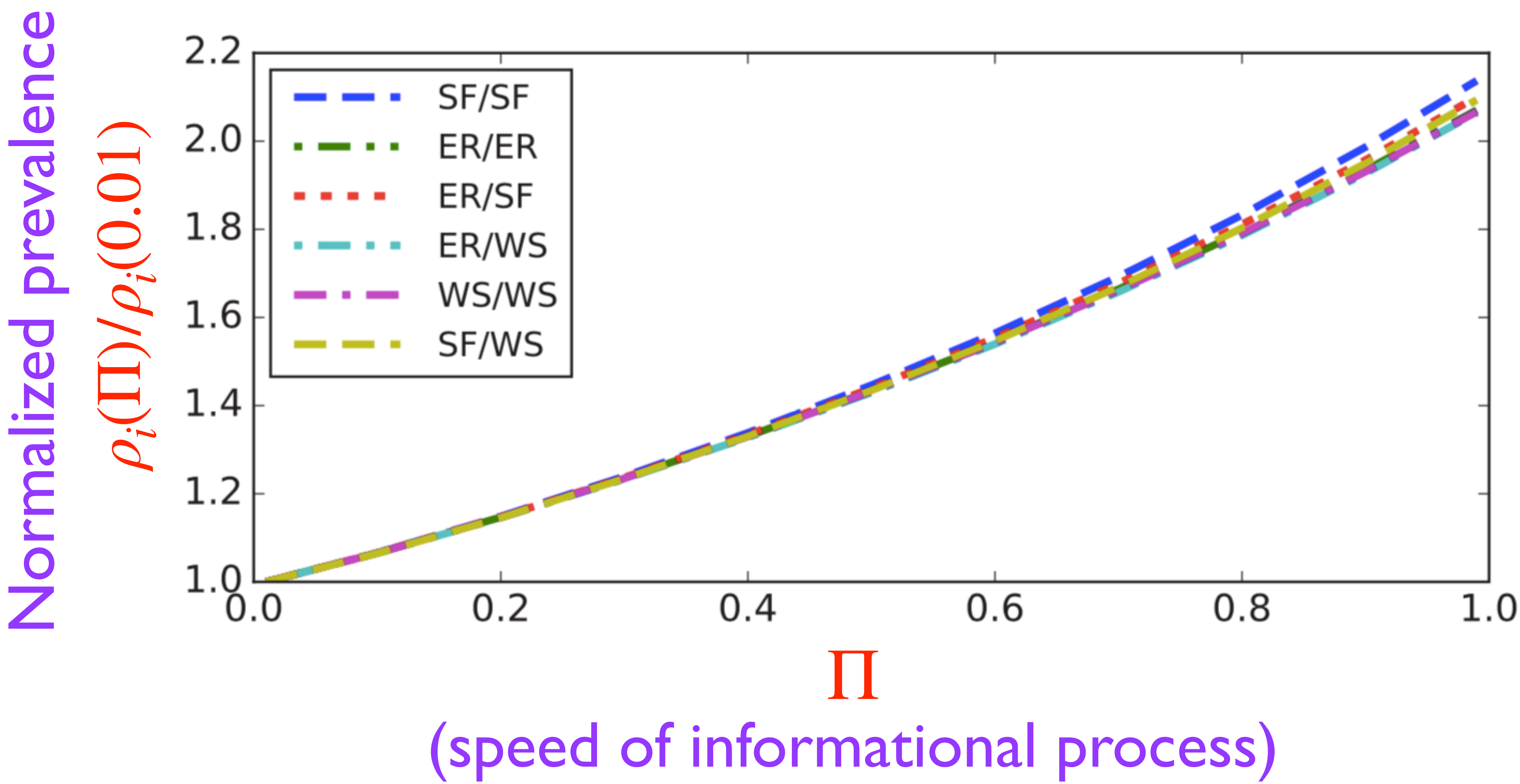
Covid-19:
Infection-recovery-infection cycle (~months).
Information cycle awareness-forgotten-awareness (~year).


DISEASE (prob. = 1 − Π)
Susceptible (S)
Infected (I)
Unaware (U)
Aware (A)
Stifler (R)
INFORMATION (prob. = Π)
S U
I U
S A
I A
S R
I R
β
μ
γ
γ
κ
$\Gamma\beta$
μ
σ
σ
α
$\alpha(1-\kappa)$
$\Gamma\beta$
μ



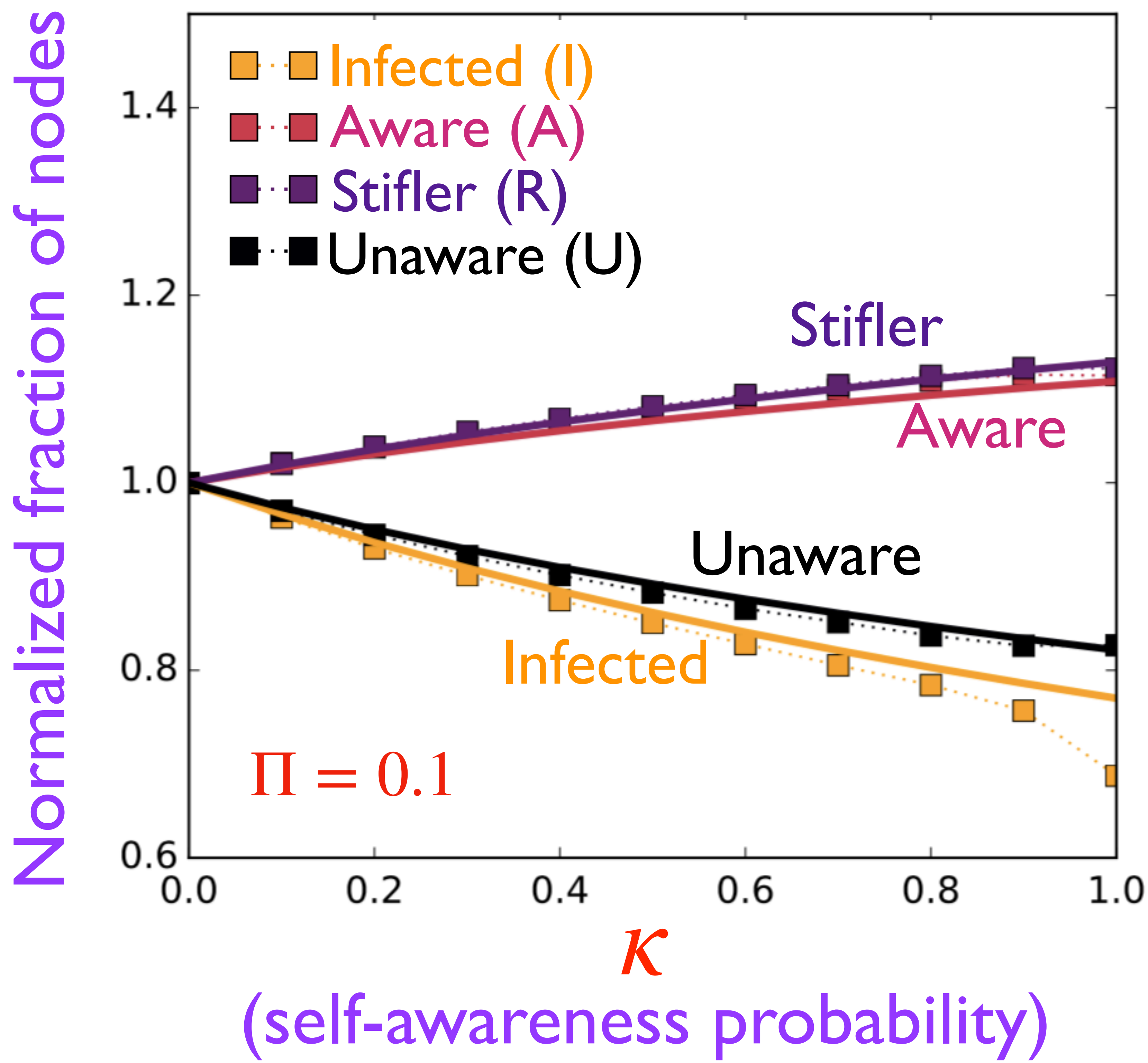
DISEASE (prob. = 1 − Π)
Susceptible (S)
Infected (I)
Unaware (U)
Aware (A)
Stifler (R)
INFORMATION (prob. = Π)
S U
I U
S A
I A
S R
I R
β
μ
γ
γ
κ
Γβ
μ
σ
σ
α
α(1 − κ)
Γβ
μ

Disease prevalence (fraction of infected nodes) increases with faster information ($\Pi \uparrow$)



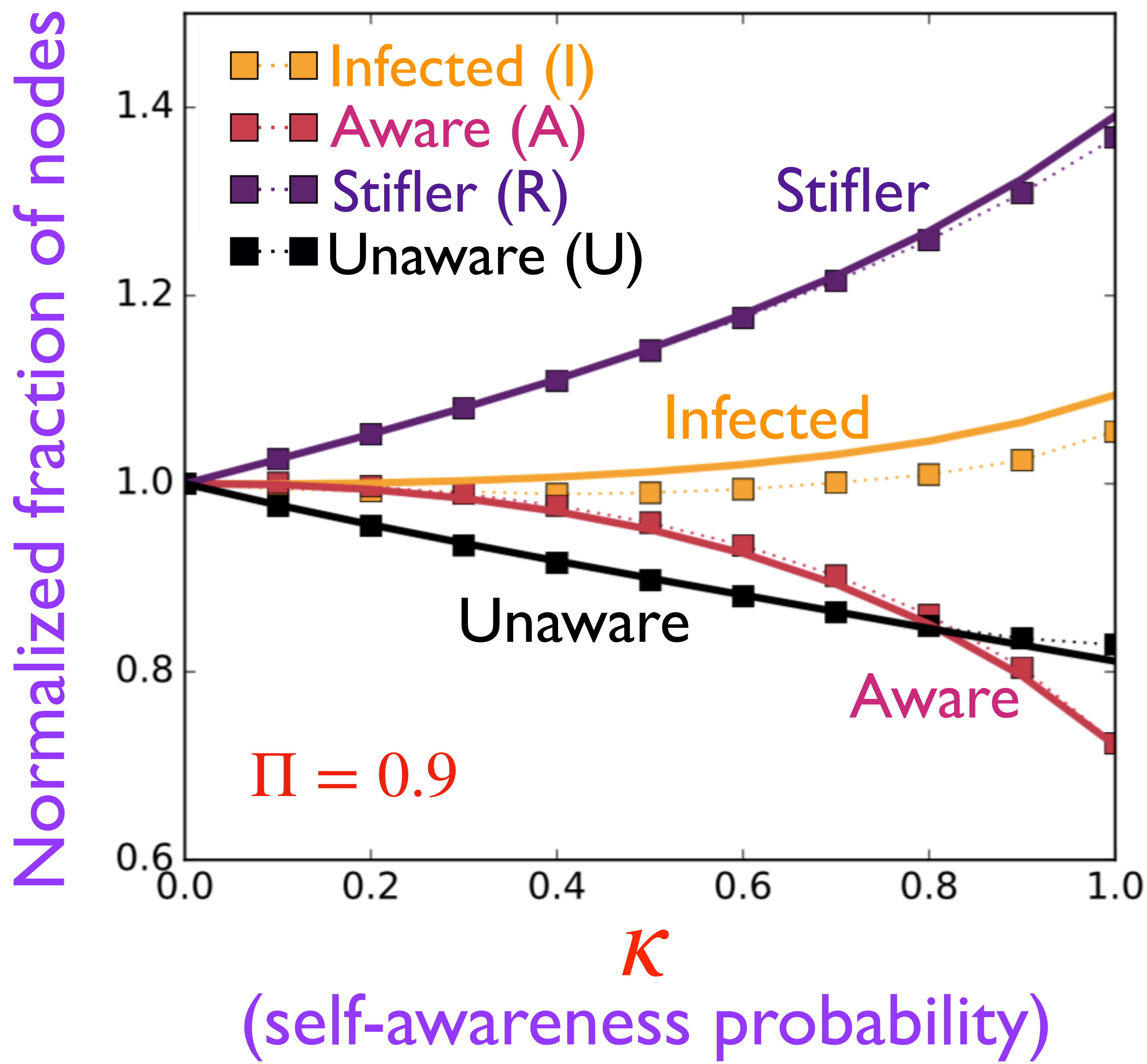
Slow information ($\Pi = 0.1$):

Disease prevalence (infected) decreases with self-awareness.

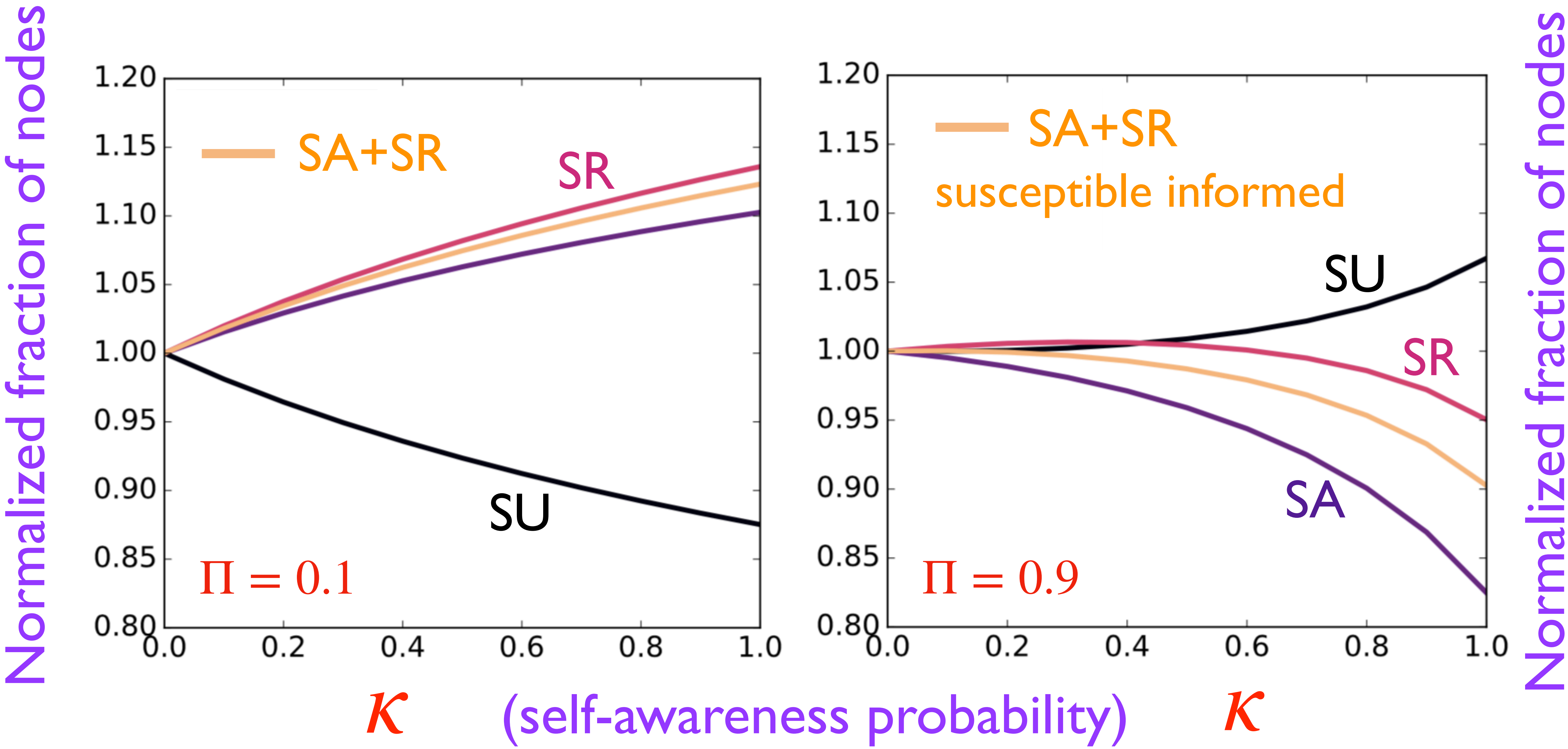


Fast information ($\Pi = 0.9$):

Disease prevalence (infected) increases with self-awareness!

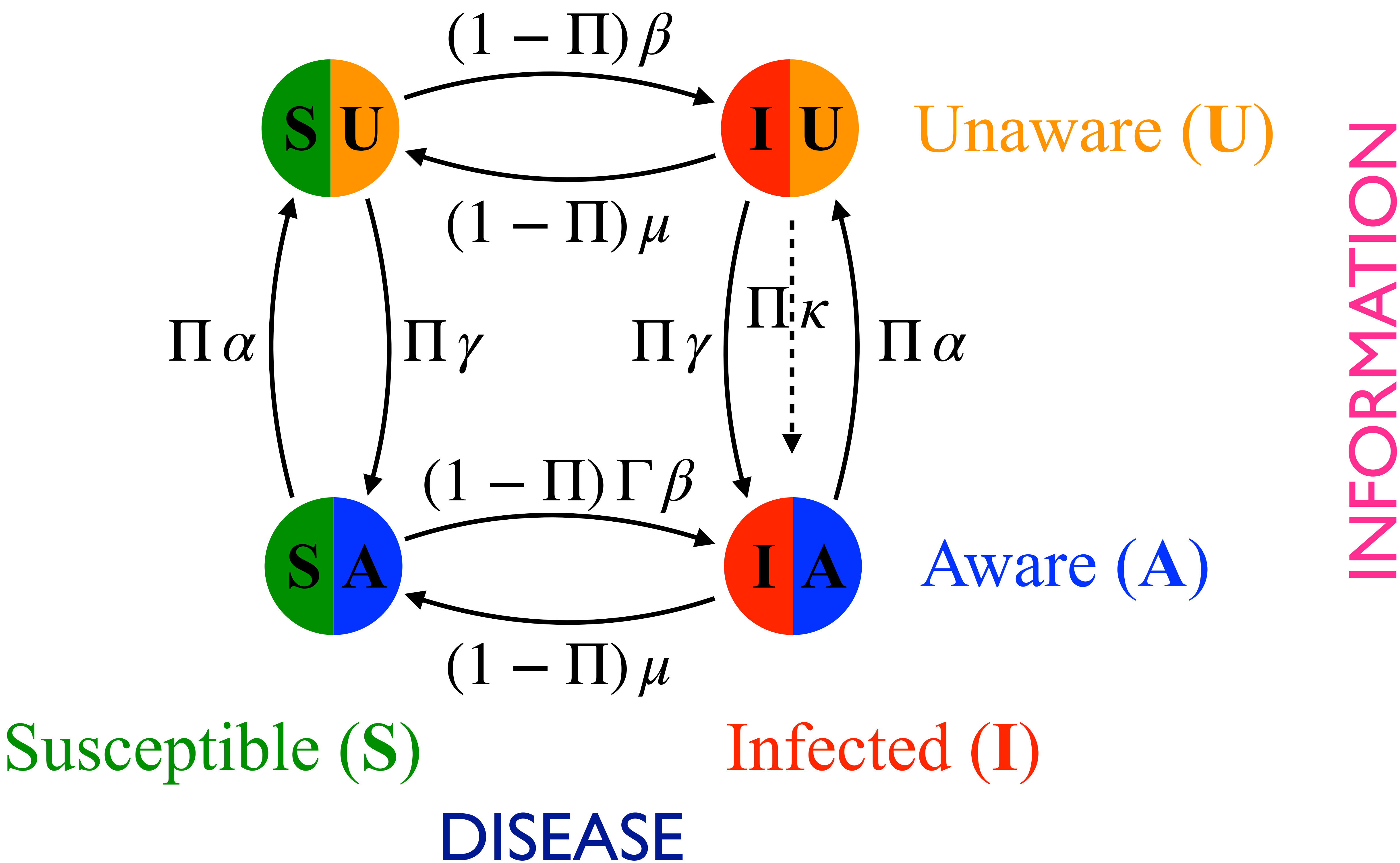


Fast information ($\Pi = 0.9$):
Fraction of susceptible informed nodes decreases with self-awareness. Less protected susceptible nodes, larger prevalence.

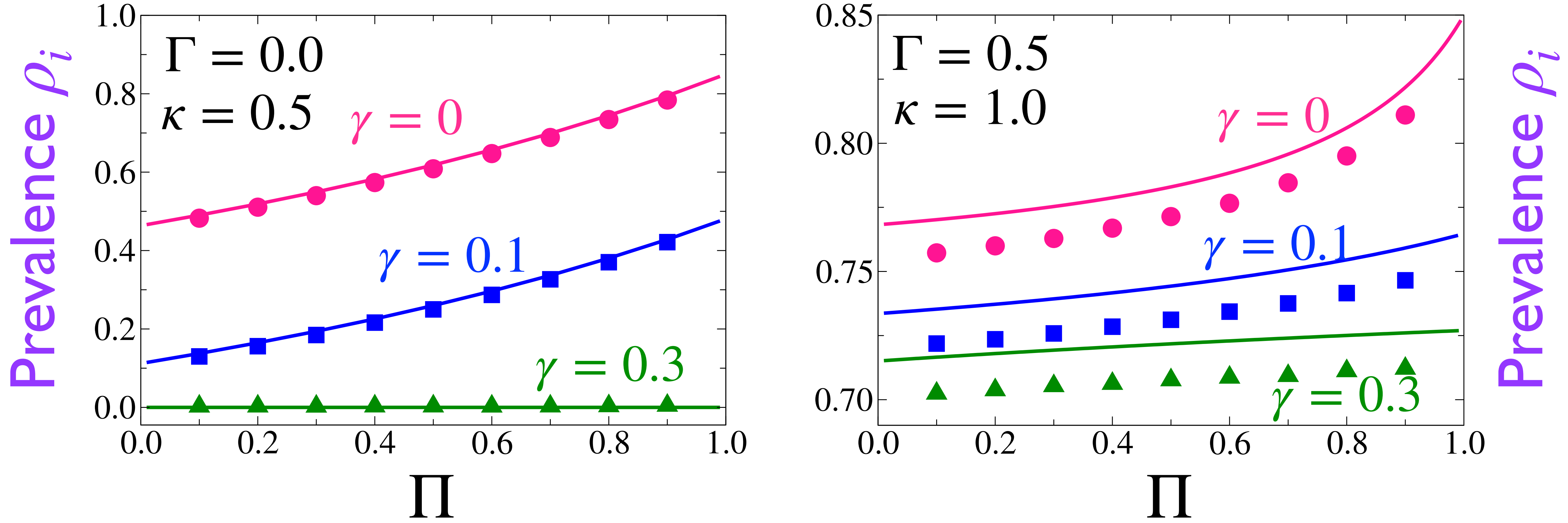


Simpler model SIS - UAU (no stiflers):

PRE **102**, 022312 (2020)

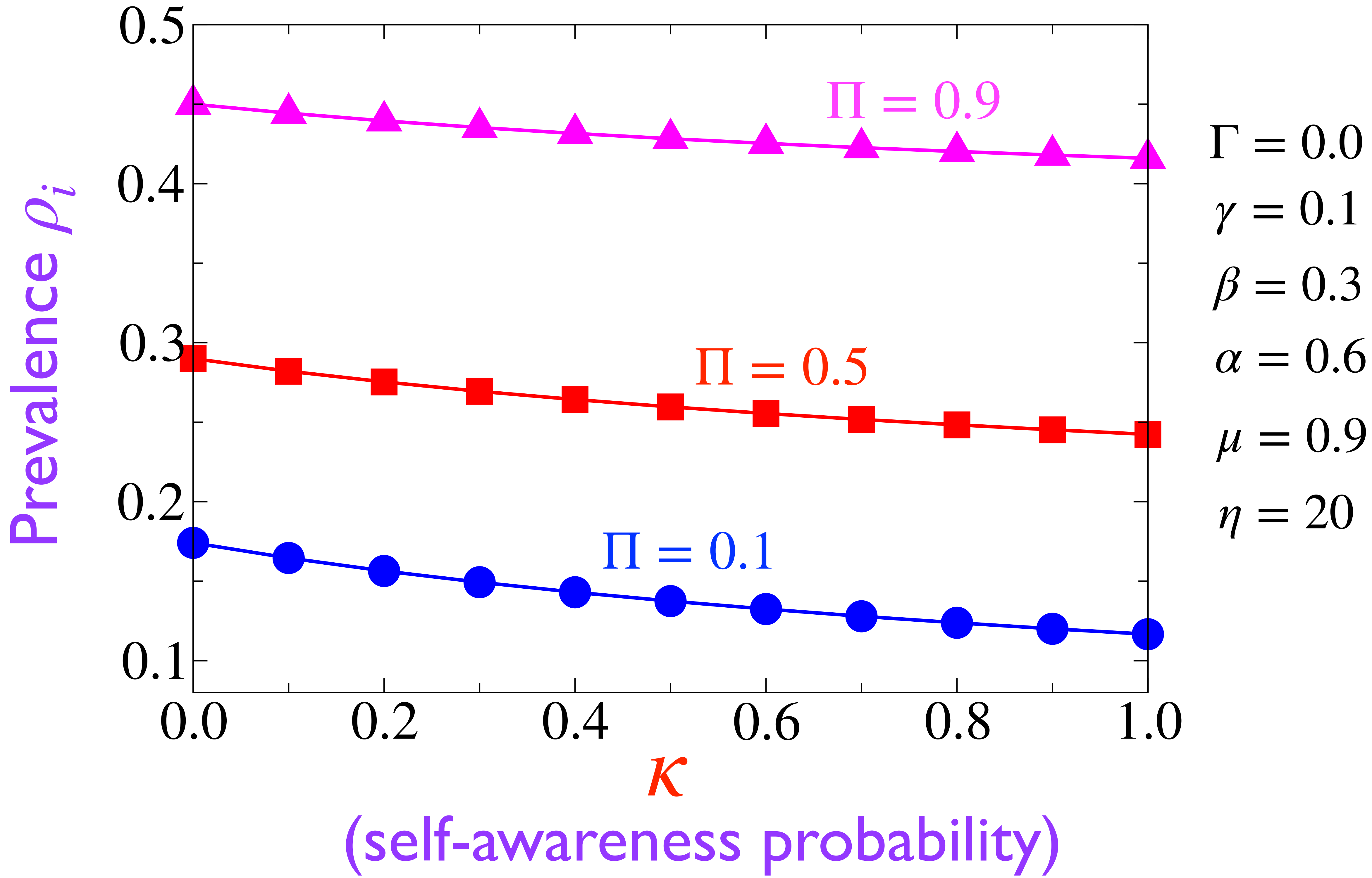


Prevalence increases with faster information, like in SIS/UAR

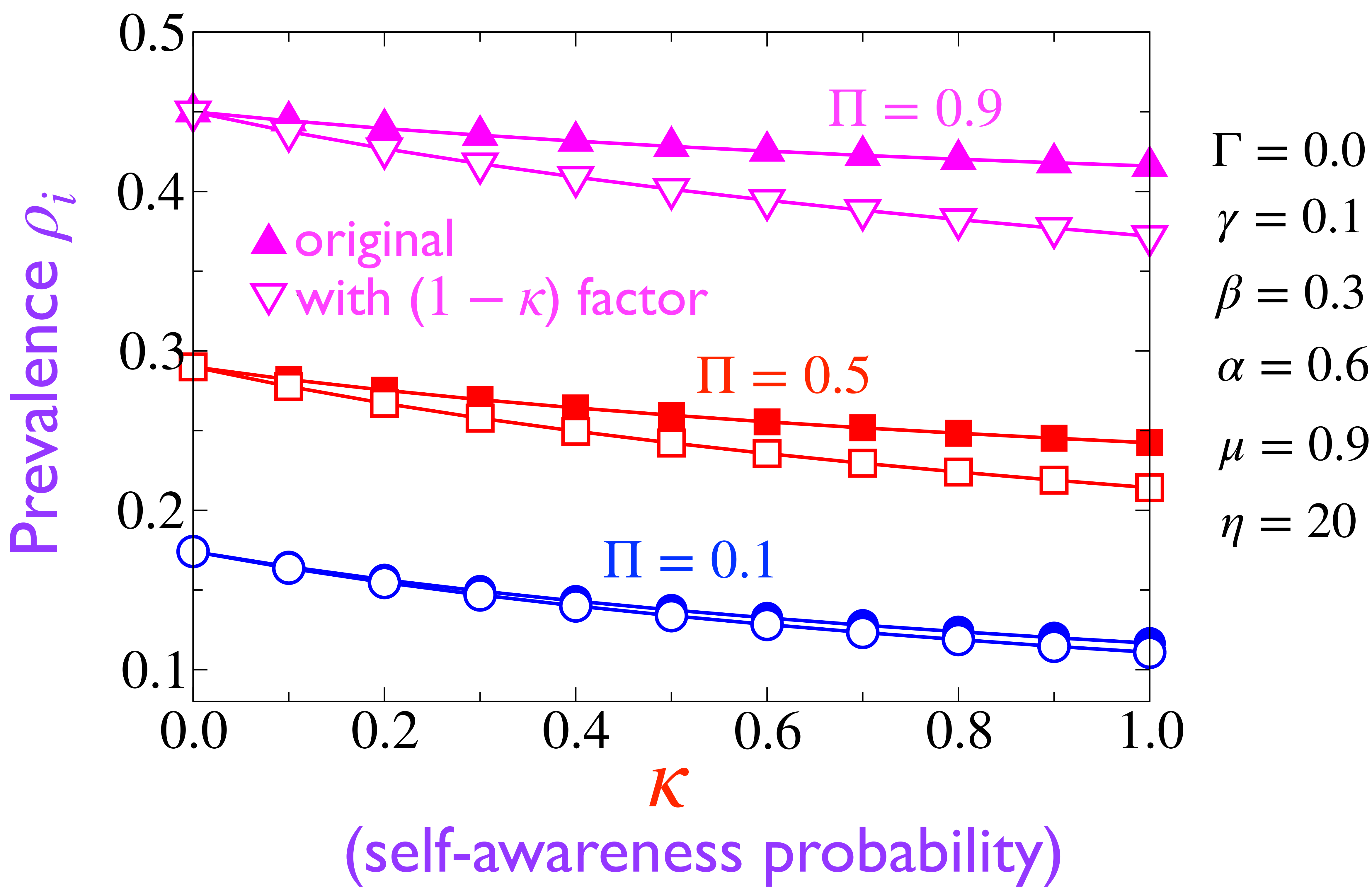


Effect of information's speed on prevalence is related to cyclic rumor dynamics of UARU or UAU.

Prevalence decreases with self-awareness for all Π, unlike SIS/UARU model



Effect of self-awareness on prevalence depends on stiflers



Mean field equations:

$$\frac{d\rho_{iu}}{dt} = (1-\Pi)\beta\eta\rho_{su}\rho_i + \Pi\alpha\rho_{ia} - (1-\Pi)\mu\rho_{iu} - \Pi\kappa\rho_{iu} - \Pi\gamma\eta\rho_{iu}\rho_a$$

$$\frac{d\rho_{su}}{dt} = (1-\Pi)\mu\rho_{iu} + \Pi\alpha\rho_{sa} - (1-\Pi)\beta\eta\rho_{su}\rho_i - \Pi\gamma\eta\rho_{su}\rho_a$$

$$\frac{d\rho_{ia}}{dt} = \Pi\gamma\eta\rho_{iu}\rho_a + \Pi\kappa\rho_{iu} + (1-\Pi)\Gamma\beta\eta\rho_{sa}\rho_i - \Pi\alpha\rho_{ia} - (1-\Pi)\mu\rho_{ia}$$

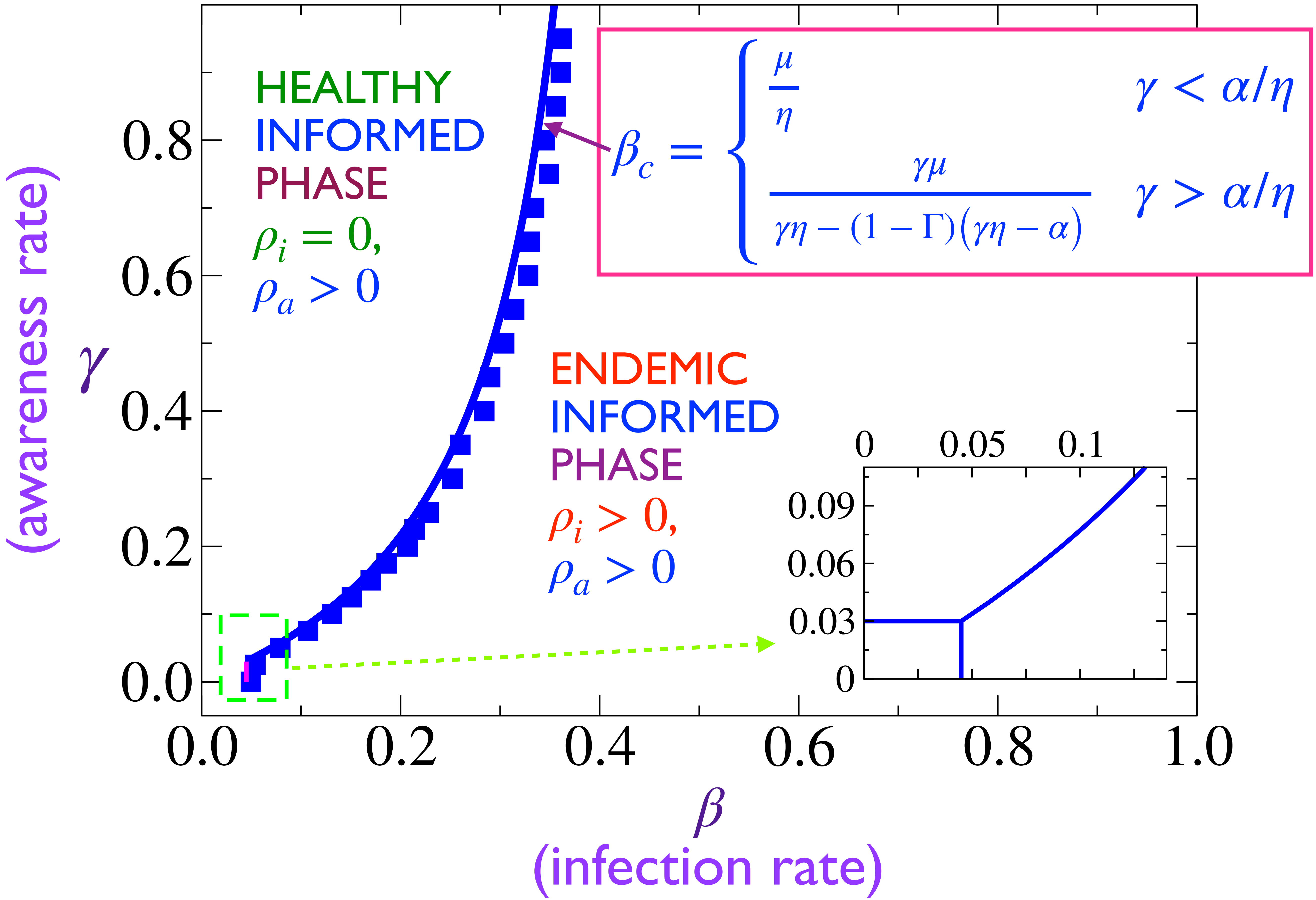
$$\frac{d\rho_{sa}}{dt} = \Pi\gamma\eta\rho_{su}\rho_a + (1-\Pi)\mu\rho_{ia} - \Pi\alpha\rho_{sa} - (1-\Pi)\Gamma\beta\eta\rho_{sa}\rho_i$$

$$\rho_i = \rho_{iu} + \rho_{ia} \qquad \rho_a = \rho_{ia} + \rho_{sa}$$

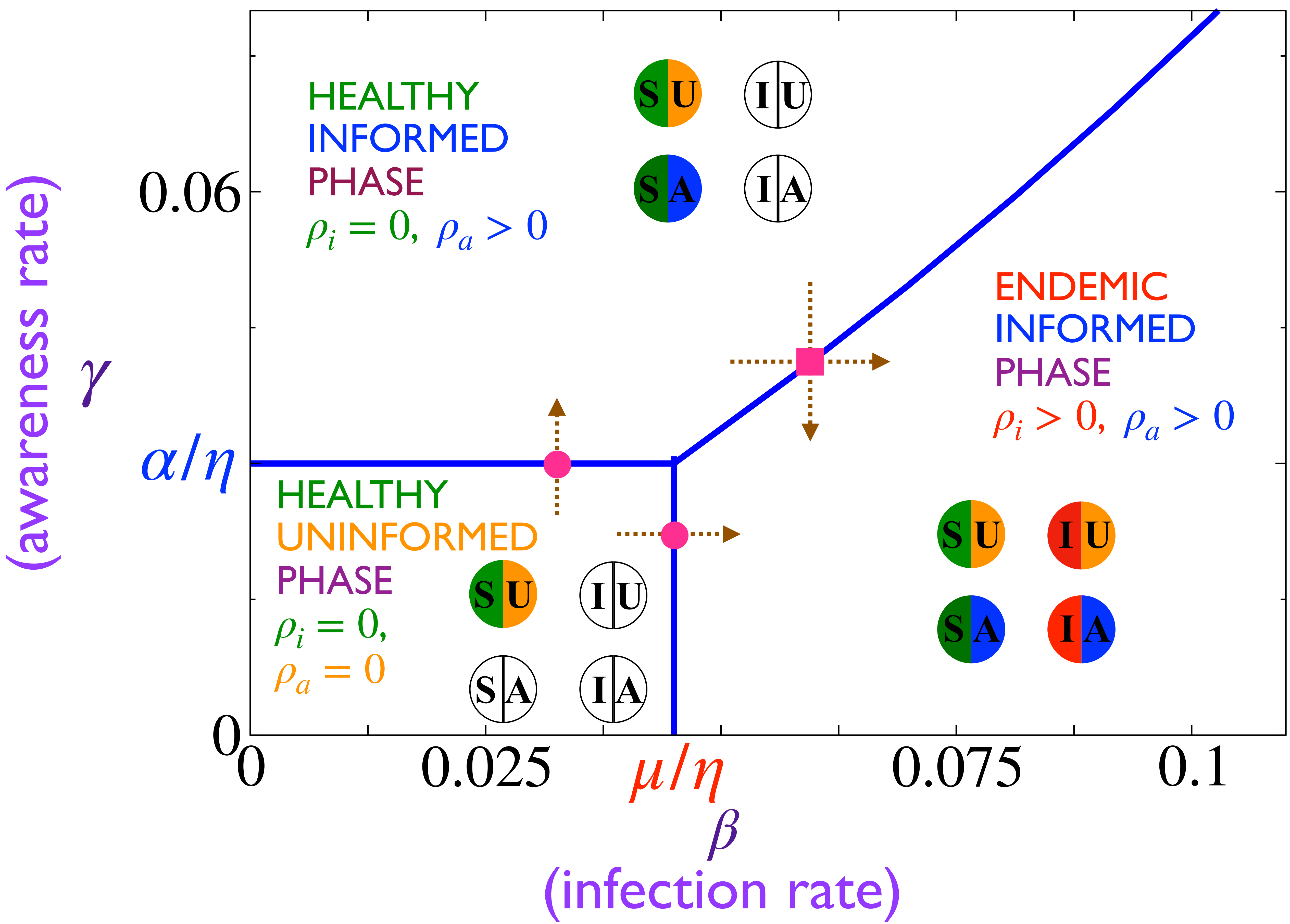
$$\rho_{iu} + \rho_{su} + \rho_{ia} + \rho_{sa} = \rho_i + \rho_s = \rho_a + \rho_u = 1$$

$\eta = \langle k \rangle$

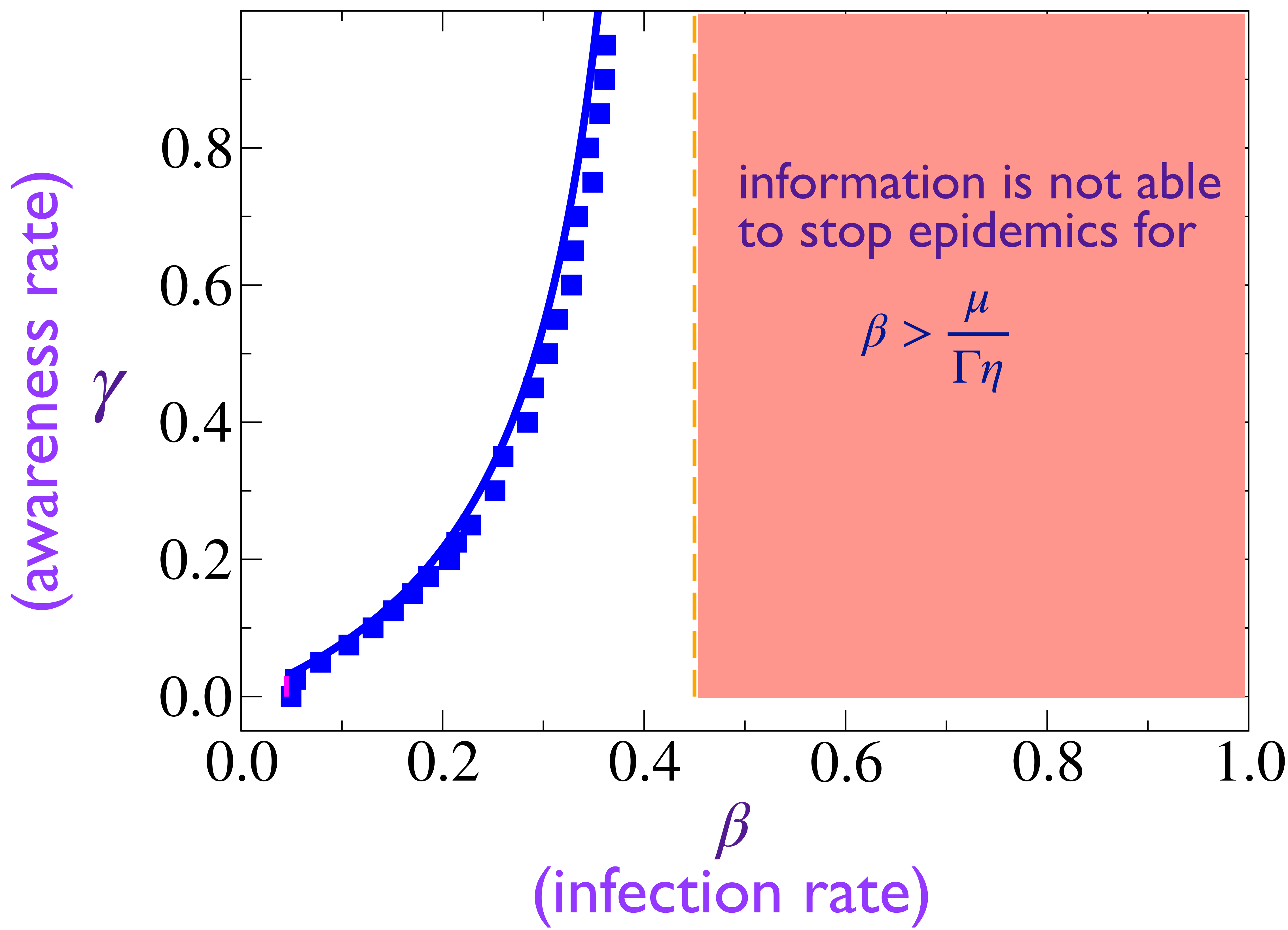
mean network degree



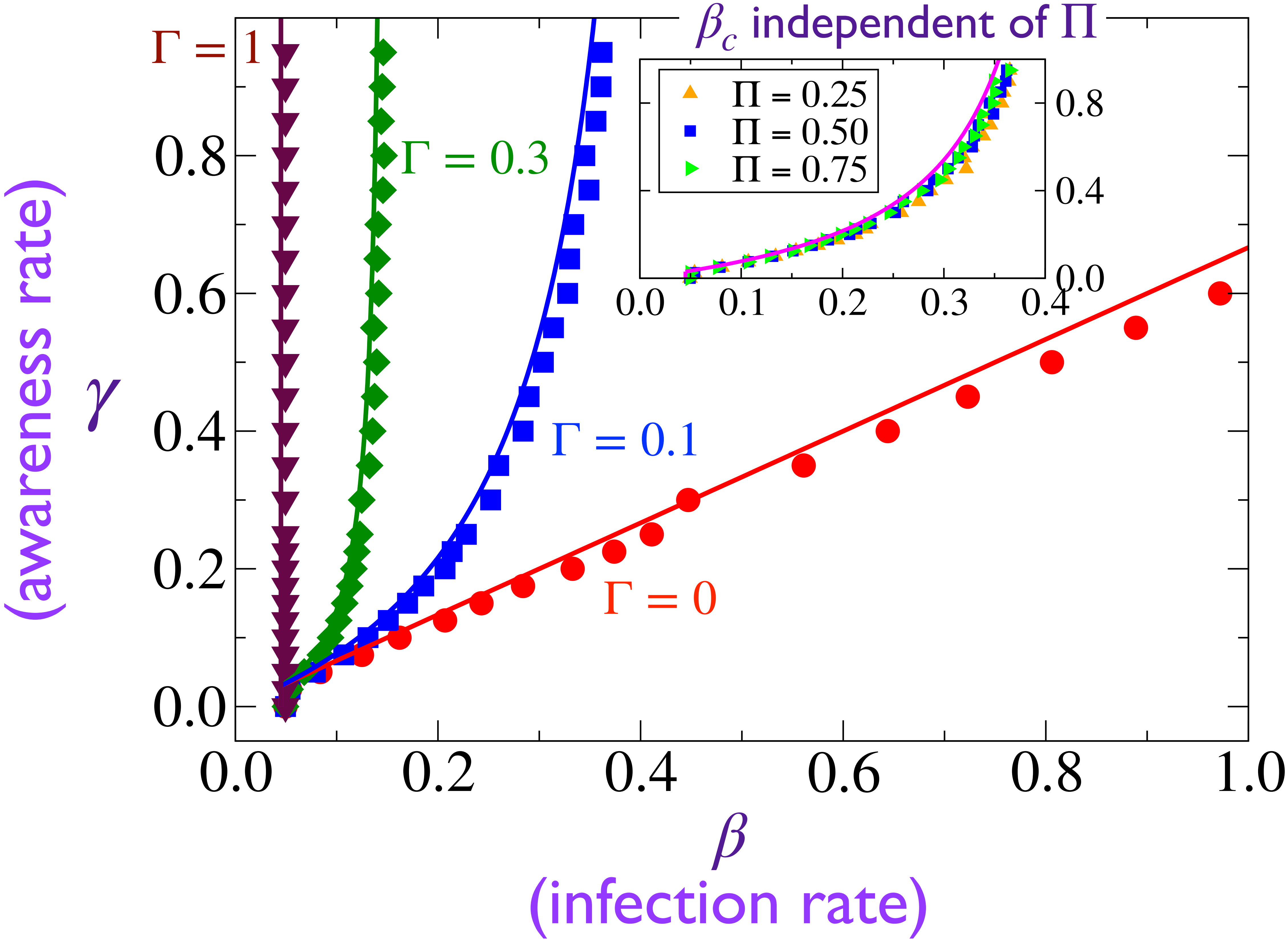
HEALTHY
INFORMED
PHASE
$\rho_i = 0,$
$\rho_a > 0$
ENDEMIC
INFORMED
PHASE
$\rho_i > 0,$
$\rho_a > 0$
$\beta_c = \begin{cases} \frac{\mu}{\eta} & \gamma < \alpha/\eta \\ \frac{\gamma\mu}{\gamma\eta - (1-\Gamma)(\gamma\eta - \alpha)} & \gamma > \alpha/\eta \end{cases}$
γ
(awareness rate)
β
(infection rate)
0.0
0.2
0.4
0.6
0.8
1.0
0
0.05
0.1
0.09
0.06
0.03



HEALTHY
INFORMED
PHASE
$\rho_i = 0, \ \rho_a > 0$
S U
I U
S A
I A
ENDEMIC
INFORMED
PHASE
$\rho_i > 0, \ \rho_a > 0$
HEALTHY
UNINFORMED
PHASE
$\rho_i = 0,$
$\rho_a = 0$
0.06
α/η
0
γ
(awareness rate)
0
0.025
μ/η
0.075
0.1
β
(infection rate)



information is not able
to stop epidemics for
$\beta > \frac{\mu}{\Gamma\eta}$
0.0
0.2
0.4
0.6
0.8
0.0
0.2
0.4
0.6
0.8
1.0
γ
(awareness rate)
β
(infection rate)



β_c independent of Π
$\Pi = 0.25$
$\Pi = 0.50$
$\Pi = 0.75$
0.0
0.1
0.2
0.3
0.4
0.8
0.4
0.0
$\Gamma = 1$
$\Gamma = 0.3$
$\Gamma = 0.1$
$\Gamma = 0$
0.0
0.2
0.4
0.6
0.8
1.0
γ
(awareness rate)
β
(infection rate)

	SIS - UARU (stiflers)	SIS - UAU
Key ingredients	• Cyclic Maki-Thompson rumor dynamics with stiflers (6 compartments). • Parameter Π controls relative speeds of two processes.	• Cyclic UAU dynamics without stiflers (4 compartments). • Parameter Π controls relative speeds of two processes.
Main results	Disease prevalence: • increases with Π. • Decreases with κ for low/ moderate Π. • Increases with κ for large Π.	Disease prevalence: • increases with Π. • Decreases with κ for all Π.
Comment	Self-awareness leads to larger pandemic when speed of information spreading is much higher than that of disease.	Self-awareness is always beneficial for disease prevention.

Conclusions/remarks

- Effect of speed of information Π is related to cyclic rumor dynamics UARU or UAU.

- Effect of self-awareness κ seems intrinsic to stiflers.

Relative speed Π between information and disease processes plays fundamental role:

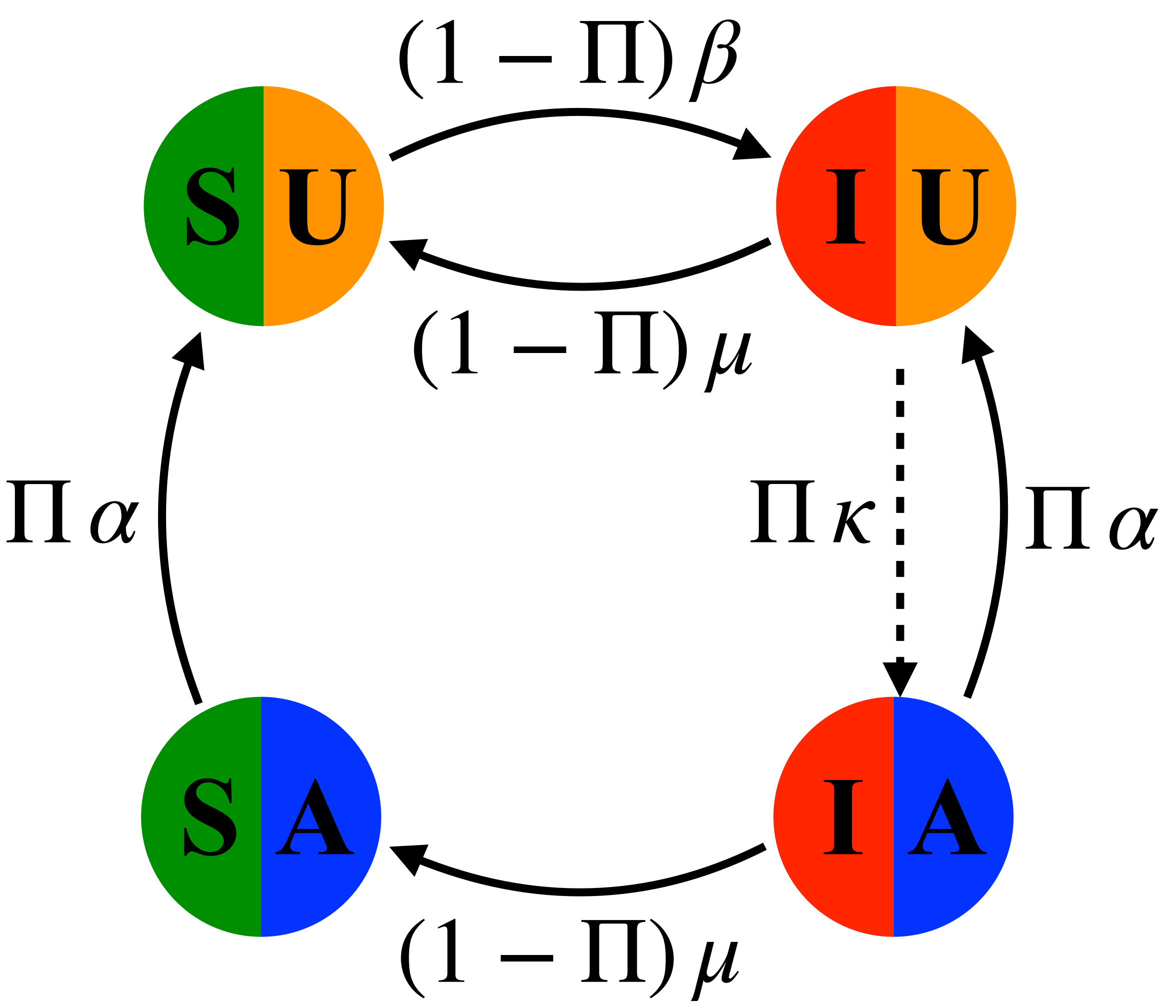
- Equal speeds $\Pi = 0.5$ (same time scales): prevalence decreases with self-awareness.

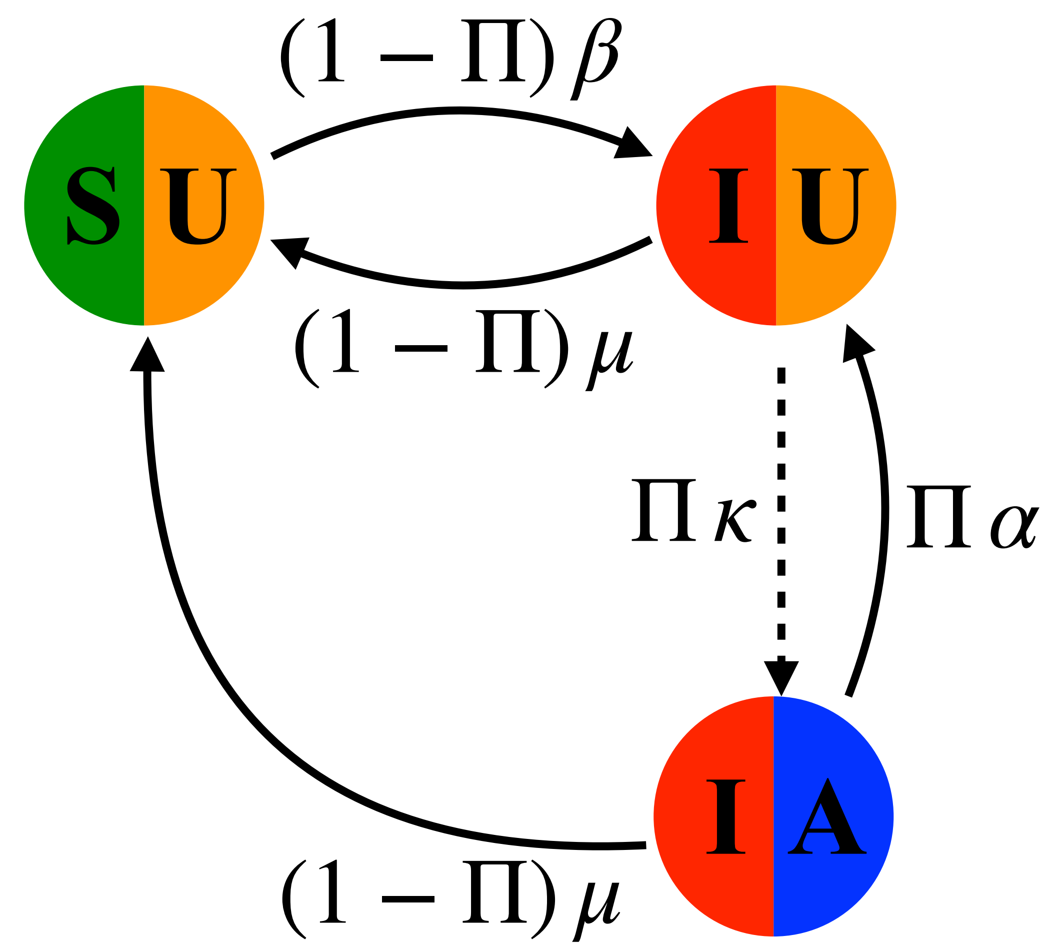
- Fast information $\Pi \gtrsim 0.8$: prevalence increases with self-awareness.

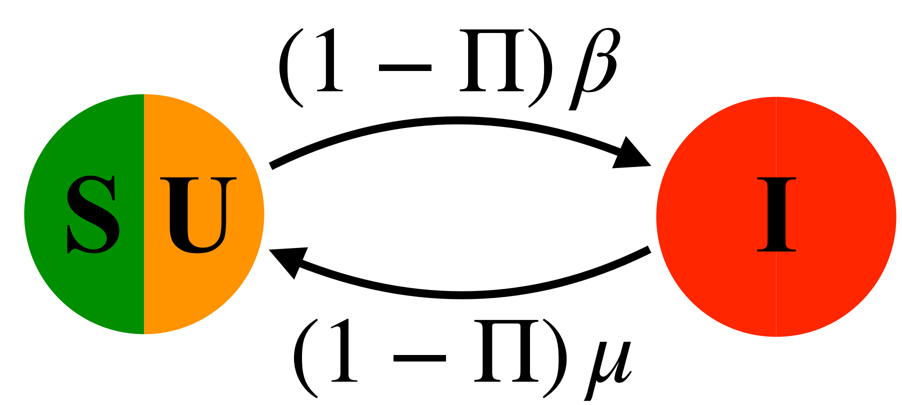
It's relevant to consider variable time scales for coupled processes.

Thank you!

Simplest nontrivial case $\Gamma = 0, \gamma = 0$: ρ_i increases with Π







$$(1-\Pi)\,\mu\,\rho_i = (1-\Pi)\,\beta\,\eta\,\rho_{su}\,\rho_i$$

$$\rho_{su} = \frac{\mu}{\beta\eta}$$

$$\rho_i \neq 0$$

when $\Pi \uparrow$:

$$\rho_s = (\rho_{su} + \rho_{sa}) \downarrow$$

$$\boxed{\rho_i = (1-\rho_s) \uparrow}$$

Stationary solution for simplest nontrivial case $\Gamma = 0, \gamma = 0$ and all $\alpha, \beta, \kappa, \mu, \eta, \Pi$

Original model

$$\boxed{\rho_i = \frac{\alpha(\beta\eta - \mu)\left[\Pi(\kappa + \alpha) + (1 - \Pi)\mu\right]}{(\kappa + \alpha)\beta\eta\left[\Pi\alpha + (1 - \Pi)\mu\right]}}$$

$$\rho_i = \frac{\alpha(\beta\eta - \mu)}{(\kappa + \alpha)\beta\eta}\left[1 + \frac{\kappa}{\alpha - \mu + \frac{\mu}{\Pi}}\right]$$

prevalence $\rho_i \uparrow$ as $\Pi \uparrow$

$$\rho_i = \frac{\alpha(\beta\eta - \mu)}{\beta\eta\left[\Pi\alpha + (1 - \Pi)\mu\right]}\left[\Pi + \frac{(1 - \Pi)\mu}{(\kappa + \alpha)}\right]$$

prevalence $\rho_i \downarrow$ as $\kappa \uparrow$

Stationary solution for simplest nontrivial case
$\Gamma = 0, \gamma = 0$ and all $\alpha, \beta, \kappa, \mu, \eta, \Pi$

Original model

$$\boxed{\rho_i = \frac{\alpha(\beta\eta - \mu)\left[\Pi(\kappa + \alpha) + (1 - \Pi)\mu\right]}{(\kappa + \alpha)\beta\eta\left[\Pi\alpha + (1 - \Pi)\mu\right]}}$$

Modified model

$$\boxed{\rho_i^{stat} = \frac{\alpha(\beta\eta - \mu)\left[\Pi(\kappa + \alpha - \kappa\alpha) + (1 - \Pi)\mu\right]}{\beta\eta\left[\Pi\alpha(\kappa + \alpha - \kappa\alpha) + (1 - \Pi)\mu(\kappa + \alpha)\right]}}$$

prevalence $\rho_i \uparrow$ as $\Pi \uparrow$ prevalence $\rho_i \downarrow$ as $\kappa \uparrow$

Original and modified models have the same behavior