RANDOM MULTIPLAYER GAMES

Natalia Kontorovsky
Juan Pablo Pinasco
Federico Vazquez

instituto de cálculo
UBA - CONICET

NetSci-X 2023, 7-10 February, Buenos Aires, Argentina.

What is a game?

- A set of N players;
- A set $\mathbf{S}$ of m pure strategies;

Rock, paper and scissors:

Different games with paired interactions and multiple interactions were studied.

Different games with paired interactions and multiple interactions were studied.

Many situations cannot be reduced to pair interactions.
We propose to study the case where in each round:

- With probability p: 2 agents play.
- With probability 1-p: 3 agents play.

Duel matrix:

$$\begin{array}{ccc} & s_1 & s_2 \\ s_1 & (1/2, 1/2) & (3/4, 1/4) \\ s_2 & (1/4, 3/4) & (1/2, 1/2). \end{array}$$

- s_1 is playing the *perfect* strategy:
  Proba of killing = 1
- s_2 is playing the *mediocre* strategy:
  Proba of killing = 0.5

(The perfect strategy is preferred)

Truel matrices:

$$\begin{array}{ccc} & s_1 & s_2 \\ s_1 & (1/3, 1/3, 1/3) & (1/4, 1/4, 1/2) \\ s_2 & (1/4, 1/2, 1/4) & (7/24, 17/48, 17/48) \end{array}$$

when player *III* plays s_1 , and

$$\begin{array}{ccc} & s_1 & s_2 \\ s_1 & (1/2, 1/4, 1/4) & (17/48, 7/24, 17/48) \\ s_2 & (17/48, 17/48, 7/24) & (1/3, 1/3, 1/3) \end{array}$$

when player *III* plays s_2 . (The mediocre strategy is preferred)

σ proportion that plays the strategy s_1 (perfect).
P_1 expected payoff of the strategy 1.
$(\sigma P_1 + (1-\sigma)P_2)$ is the average expected payoff of the population.

The evolution of the proportion of agents that play the perfect strategy is given following ordinary differential equation:

$$\frac{d}{dt}\sigma(t) = \sigma\{P_1 - (\sigma P_1 + (1-\sigma)P_2)\},$$

which is known as the replicator equation.

Nash Equilibria (NE): It is not convenient for any player - unilaterally - to change what they are playing.

Every Nash equilibria is a rest point of the replicator system.

A NE that is stable in time is called an Evolutionary Stable Strategy (ESS)

Replicator equation:

$$\frac{d}{dt}\sigma(t) = \sigma\left\{P_1 - (\sigma P_1 + (1-\sigma)P_2)\right\},$$

σ proportion that plays the strategy s_1 (perfect)

$$\frac{d\sigma}{dt} = \sigma(1-\sigma)\left[\frac{p}{4} - (1-p)\left(\frac{1}{24} + \frac{1}{8}\sigma\right)\right], \tag{1}$$

where p is the probability of playing Duel.

Replicator equation:

$$\frac{d}{dt}\sigma(t) = \sigma\left\{P_1 - (\sigma P_1 + (1-\sigma)P_2)\right\},$$

σ proportion that plays the strategy s_1 (perfect)

$$\frac{d\sigma}{dt} = \sigma(1-\sigma)\left[\frac{p}{4} - (1-p)\left(\frac{1}{24} + \frac{1}{8}\sigma\right)\right], \tag{1}$$

where p is the probability of playing Duel.
Interior fixed point:

$$\sigma^* = \frac{7p-1}{3(1-p)}. \tag{2}$$

Nash equilibria with respect to p :

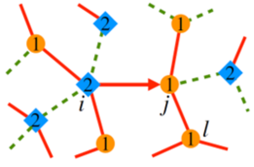
$$\sigma_{NE} = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{7}, \\ \sigma^* & \text{if} \quad \frac{1}{7} < p < \frac{2}{5}, \\ 1 & \text{if} \quad \frac{2}{5} \leq p. \end{cases}$$

- The pure Nash equilibria are ESS.
- The mixed Nash Equilibria are ESS for $\frac{1}{7} < p < \frac{2}{5}$.

In a single time step $\Delta t = 1/N$:
Agent i is choosen at random.
State $\theta_i = 2$ (perfect) or $\theta_i = 1$ (mediocre)

With proba p (duel):
One neighboring agent j is randomly choosen.

$$\theta_i \to \theta_j \quad \textit{with proba} \quad P_{\theta_j}$$



With proba $1 - p$ (truel):
Two neighboring agents j and k are randomly choosen.

$$\theta_i \to \theta_j \quad \textit{with proba} \quad \frac{\tilde{P}_{\theta_j}}{2}$$

$$\theta_i \to \theta_k \quad \textit{with proba} \quad \frac{\tilde{P}_{\theta_k}}{2}$$

Expected payoffs of agent i are given by:

$$P_1^i = \frac{3}{4} - \frac{1}{4}\sigma_i \quad \text{si } \theta_i = 1$$

$$P_2^i = \frac{1}{2} - \frac{1}{4}\sigma_i \quad \text{si } \theta_i = 2,$$

when i plays duel, and by:

$$\tilde{P}_1^i = \frac{1}{3}\sigma_i^2 + \frac{1}{2}\sigma_i(1-\sigma_i) + \frac{7}{24}(1-\sigma_i)^2 \quad \text{si } \theta_i = 1,$$

$$\tilde{P}_2^i = \frac{1}{2}\sigma_i^2 + \frac{17}{24}\sigma_i(1-\sigma_i) + \frac{1}{3}(1-\sigma_i)^2 \quad \text{si } \theta_i = 2,$$

when i plays truel.

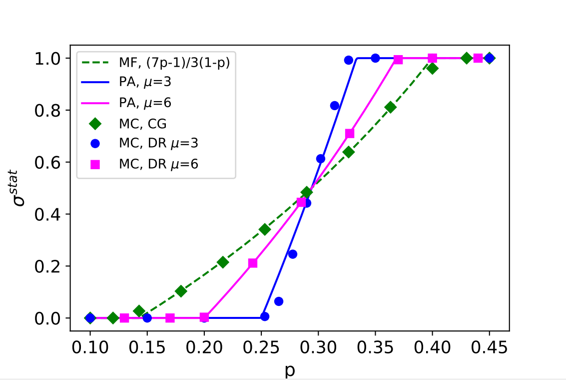
The evolution in time of σ is given by:

$$\frac{d\sigma}{dt} = \left[p(P_2 - P_1) + (1 - p)(\tilde{P}_2 - \tilde{P}_1)\right]\sigma(1 - \sigma), \qquad (3)$$

Coincides with:

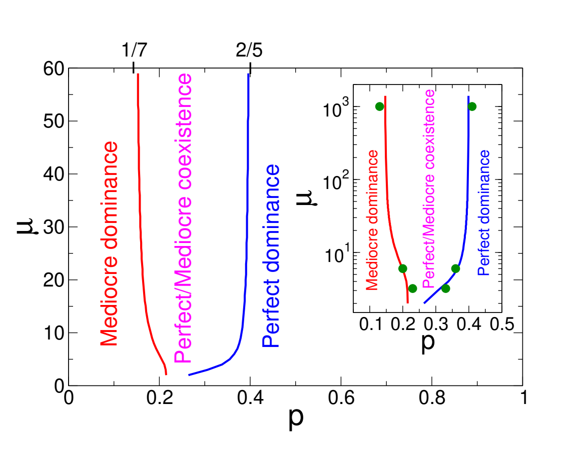
$$\frac{d\sigma}{dt} = \left[\frac{1}{4}p - \left(\frac{1}{24} + \frac{\sigma}{8}\right)(1 - p)\right]\sigma(1 - \sigma) \qquad (4)$$

The interval of p coexistence occurs depends on the mean degree of the network μ.



DR: Degree Regular Random Graph.

The coexistence region depends on *p* and increases with μ.



Conclusions

- We found games that have mixed equilibria that are ESS.

Conclusions

- We found games that have mixed equilibria that are ESS.
- We introduced the replicator dynamics over complex networks.

Conclusions

- We found games that have mixed equilibria that are ESS.
- We introduced the replicator dynamics over complex networks.
- We found that the interval of p for which there is coexistence depends on the mean degree of the network.

Conclusions

- We found games that have mixed equilibria that are ESS.
- We introduced the replicator dynamics over complex networks.
- We found that the interval of p for which there is coexistence depends on the mean degree of the network.
- Starting at the pure equilibria fase for a given p, stable coexistence may be induced by inceasing the amount of neighbors.

¡Thank you for your attention!

Chaos 32, 033128 (2022); https://doi.org/10.1063/5.0080137