Characteristics of the Dynamic of Mobile Networks

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Workshop: Dynamical Complex Systems
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http://cnet.fi.uba.ar/wdcs/

Outline

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- **MOSAR Project**
  - Project overview
- Dynamic Network Characterization
  - Motivation
  - Statistical analysis of snapshots of graphs
  - Towards a global analysis of the dynamics
  - Modeling of the dynamics
- Conclusion

Deployment of a large-scale dynamic networks

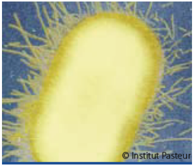
Control of antimicrobial resistance of bacteria responsible for major and emerging nosocomial infections.

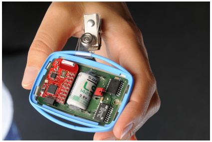
MOSAR Experiment

- Medical / staff / Patients (500 people)
- Individual antibiotic use;
- Characterization of the isolates bacteria and their epidemicity;
- 7/24 during **6 month** long period

Document contact frequencies

- Associate 1 sensor with each actor
- monitor the dynamic (inter & intra contact)





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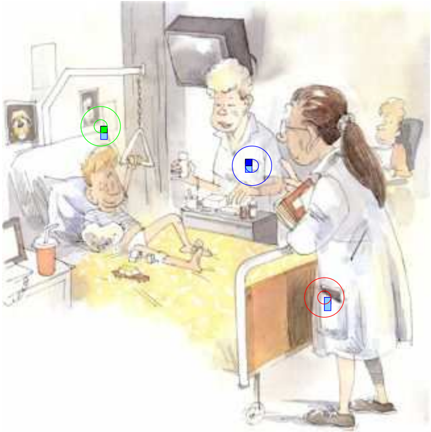

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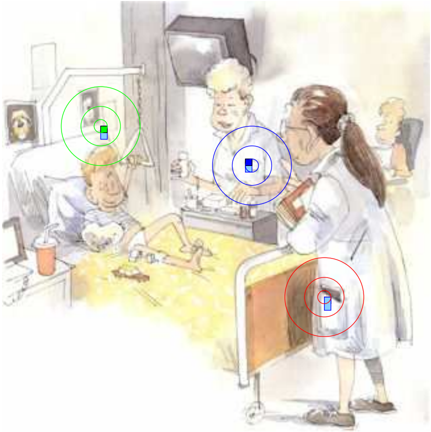

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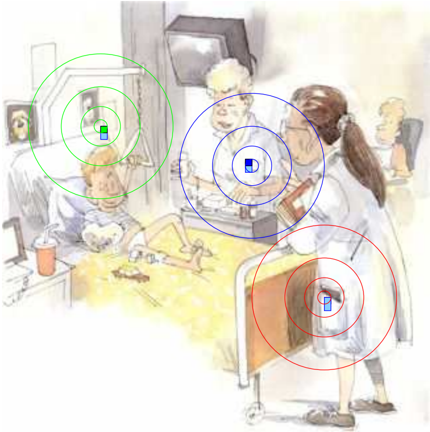

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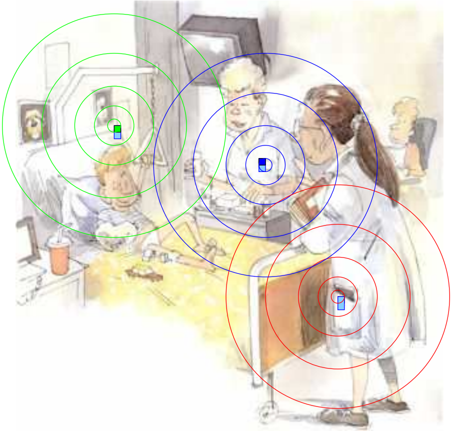

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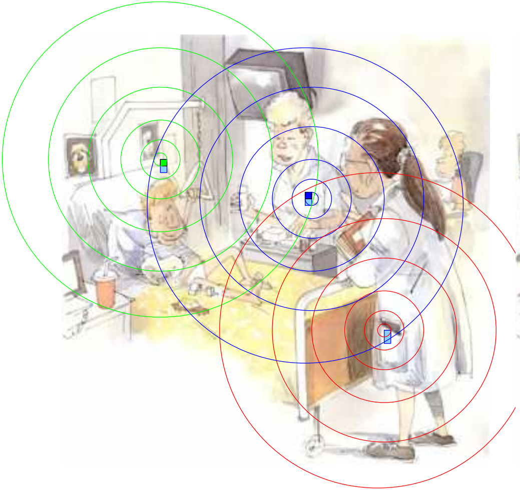

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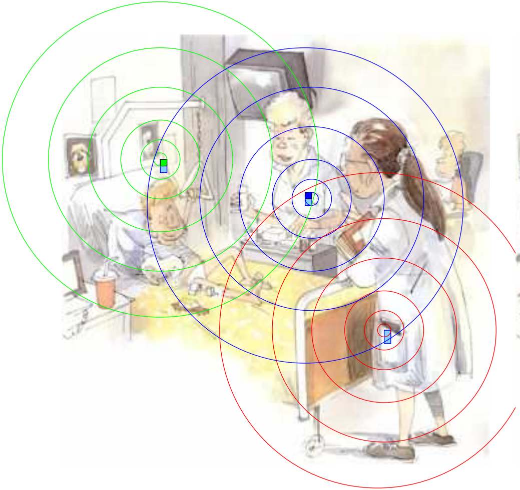

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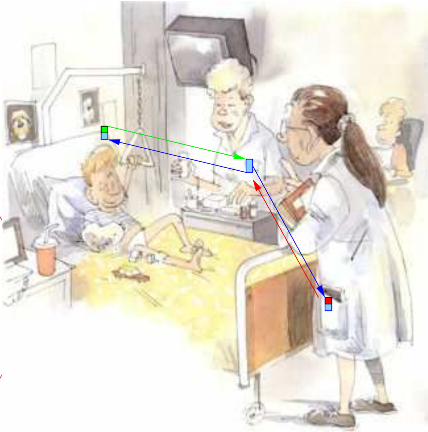

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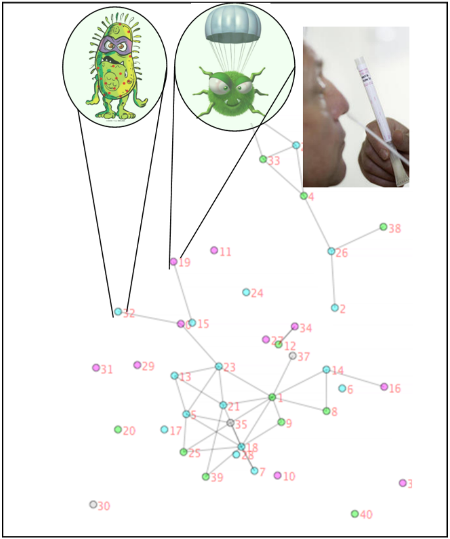
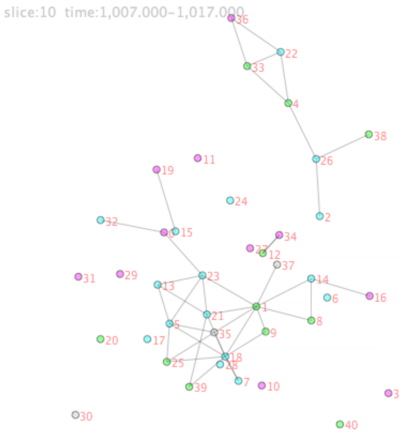
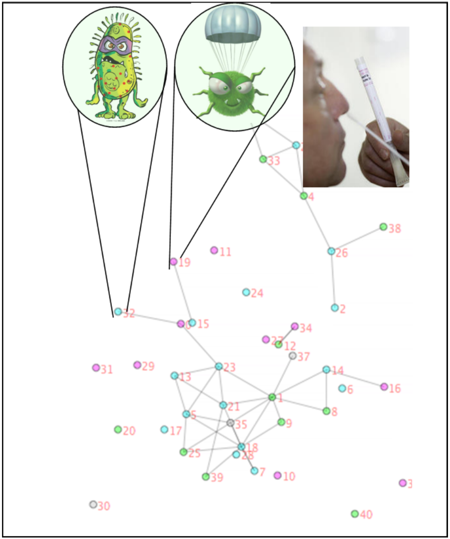
Multi modal / multi time scale







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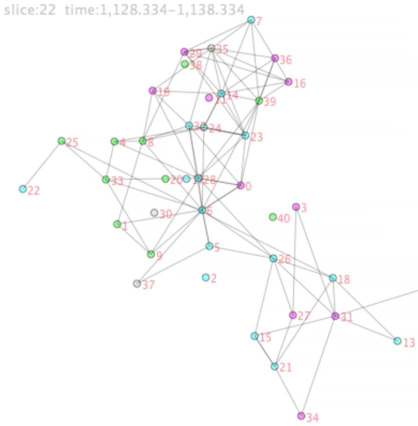

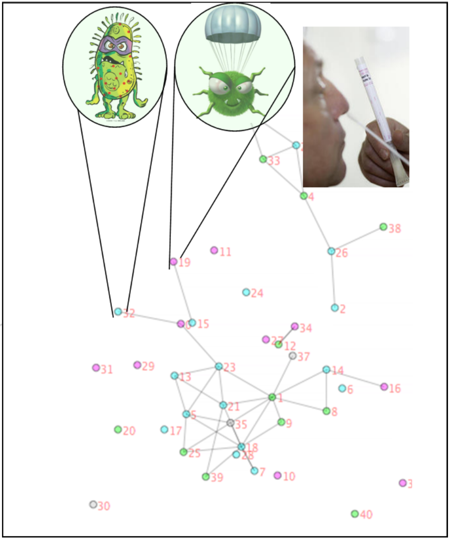

Multi modal / multi time scale

Outline

Objectives

MOSAR project

- Better understand the intrinsic characteristics / **properties** of dynamic networks
- Model / analyze interaction between node/users
- **Describe accurately the dynamics**

Two central questions:

- Obtaining random models that reproduce "*these*" properties
- How do their functionalities constrain the structures of real network?

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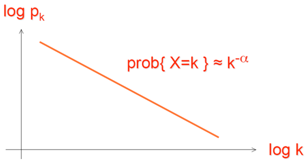
Preliminary data[1]

Mosar experiment: May 2009 – Nov 2009.

"Toy" traces are now available

- 41 nodes, 3 days (254 151 sec), every 120sec
- 820 possible links,
- *"[...] inter contact time distribution can be compared to the one of power law [...]"*



Power law...

- What do power law really signify?
- Is it the ultimate argument?

[1] A. Chaintreau and J. Crowcroft and C. Diot and R. Gass and P. Hui and J. Scott, *Impact of Human Mobility on the Design of Opportunistic Forwarding Algorithms*, INFOCOM 2006

Methodology

Descriptive: Standard graph properties

1. as a function of time to to provide an empirical statistical characterization of the dynamics.
2. temporal evolution of the snapshots
3. statistical signal processing

Analysis: global indicators

- connected components, triangles, and communities

Model

- We propose models to perform random dynamic networks simulations.

Standard graph properties

Snapshots $G_t = (V^0, E_t)$

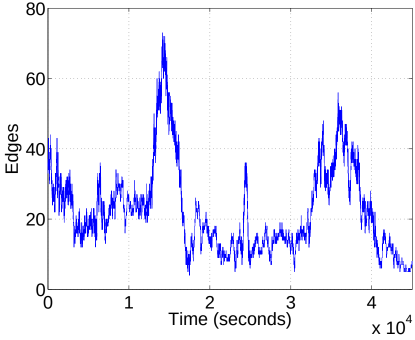
- Active links: $E(t) = |E_t|$
- Connected vertcices: $V(t) = |\{u \in V^0, d_{G_t}(u) > 0\}|$
- Average degree of connected vertices is $D(t) = \sum_{u \in V^0} d_{G_t}(u) / V(t)$
- Number of connected components (*maximal subgraph such as every node of the subgraph is connected to each another node*): $N_c(t) = |C_{G_t}|$
- Number of triangles: $T(t) = |T_{G_t}|$

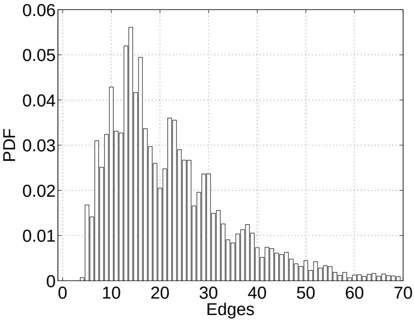
Property		IMOTE Mean	IMOTE Std. Dev.	IMOTE Corr. Time (s)
#Active links	$E(t)$	21.9	12.4	5200
#Connected vertices	$V(t)$	19.9	4.7	7400
Avg degree	$D(t)$	2.1	0.8	3600
#CC	$N_c(t)$	4.8	2.1	5600
#Triangles	$T(t)$	6.9	8.30	4700

Standard graph properties (cont)

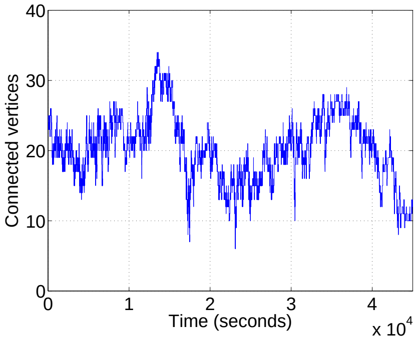
Probability distribution

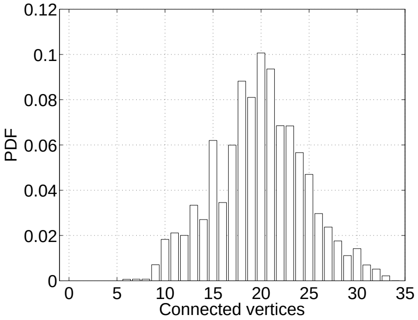
- time bin of $1s << $ period.
- PDF obtained are not heavy tailed
- variability is not very large (stdv is a good measurement of the variability)





Standard graph properties (cont)

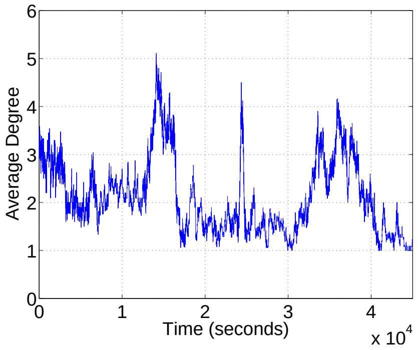


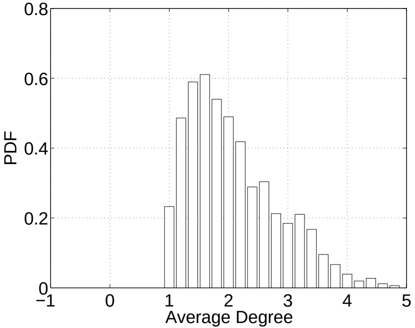


Network is sparse

- less than 10% of active links among the 820 possible links
- at no time the network is a single connected component.
- many nodes remain isolated during long times (around 50% on average for daytime and more than 90% for nighttime).

Standard graph properties (cont)



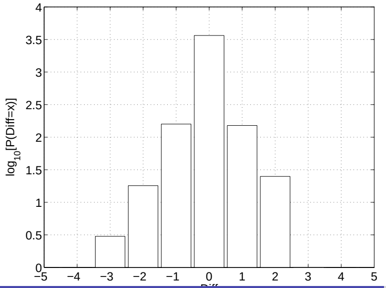


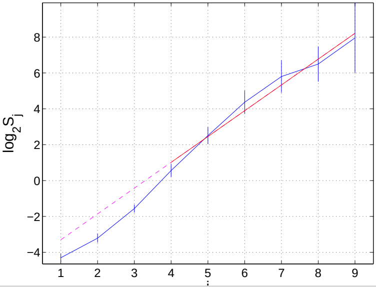
Standard graph properties (cont)

differential sequence: $DS[k] = D[k+1] - D[k]$

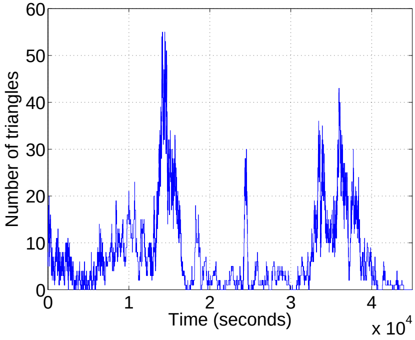
- log-log representation of the covariance in the wavelet domain[a]
- S_j is roughly the average of the wavelet coef. at scale j
- Hurts exponent is close to the special value 0.5.
- no long range $\longrightarrow$ Independent Identically Distributed (IID)

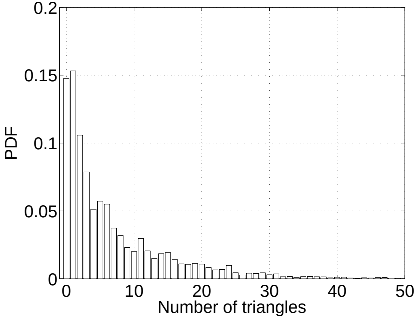
[a] P. Abry and D. Veitch, Wavelet analysis of long-range dependent traffic, TIT, 1998





Standard graph properties (cont)





Large number of triangles

- $\mathbb{E}(T(G(p,n))) = \binom{N}{3}3!p^3$ & $\mathbb{E}(E(G(p,n))) = p\frac{N(N-1)}{2}$
- When there is k links, $\mathbb{E}(T(G(n,k))) \sim \frac{8k^3(N-2)}{N^2(N-1)^2}$
- 70 links (max) $\longrightarrow$ 40(60)
- 22 links (avg) $\longrightarrow$ 1(7)

Dynamical characteristics

Correlation times

- temporal evolution ($X(t)$: univariate time-series)
- The autocorrelation function of $X(t)$:

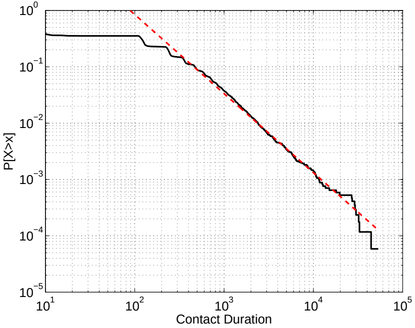
$$C_X(\tau) =< X(t+\tau)X(t) >_t - (< X(t) >_t)^2$$

- correlation time: **first time where the function $C_X(\tau)$ goes to zero**

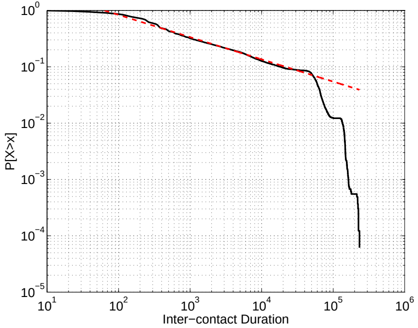
notes

- correlation times of E, V and N_c are rather large: $\sim$ 1h15.
- D and T have comparable correlation times.
- **This suggests that these properties evolve under a common cause.**

Dynamical characteristics (cont)



Mean : 140; $\alpha = 1.66$

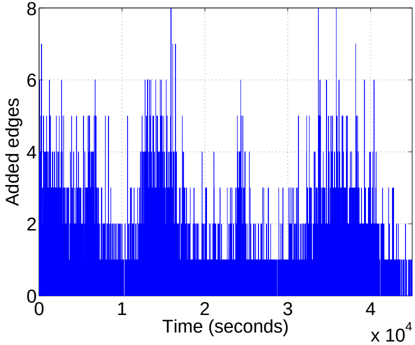


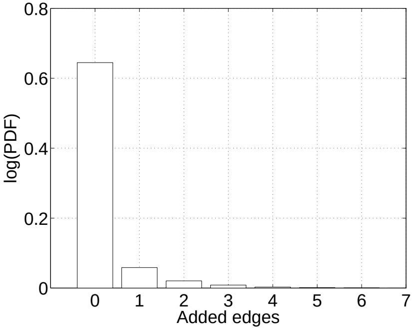
Mean : 3680; $\alpha = 0.60$

Contact and inter-contact durations

- $P[X > x] \underset{x \to \infty}{\sim} cx^{-\alpha}$.
- $\alpha > 2$: finite mean/variance; $\alpha < 2$, infinite variance (*heavy tailed*).
- $\alpha < 1$, infinite mean/variance.

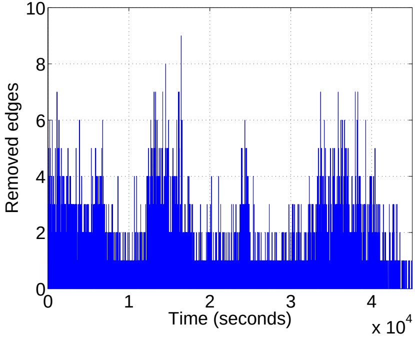
Dynamics of links creation and deletion

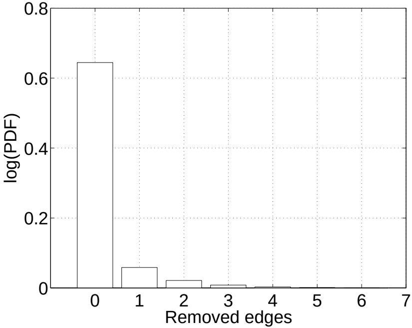




- $E_{\oplus}(t) = |\{e \in E_t, e \notin E_{t-1}\}|$, the number of links added at time t

Dynamics of links creation and deletion (cont)





- $E_{\ominus}(t) = |\{e \in E_{t-1}, e \notin E_t\}|$, the number of links removed at time t

		IMOTE		
Property		Mean	Std. Dev.	Corr. Time (s)
Edge creation	$E_{\oplus}(t)$	0.15	0.55	680 $\sim$ 12min
Edge delation	$E_{\ominus}(t)$	0.15	0.55	680$\sim$ 12min

Multivariate statistics of graph properties

Cross-correlations

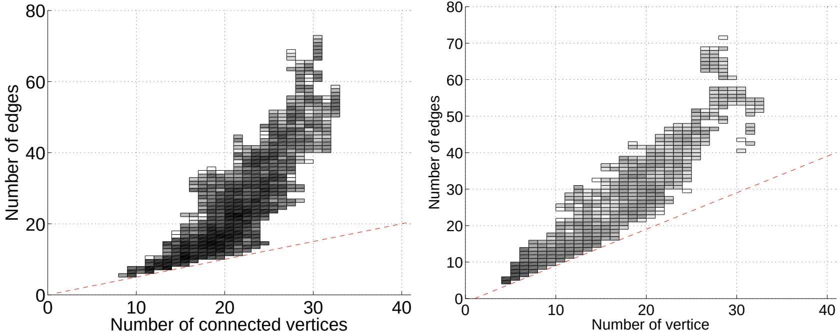
- Strong influence $E(t)$ over $V(t)$;
- $N_c(t)$ related to $E(t)$
- Less related: $N_c(t)$ and $V(t)$
- $E_\oplus(t)$ and $E_\ominus(t)$: mostly uncorrelated

	$E(t)$	$V(t)$	$N_c(t)$	$D(t)$	$T(t)$	$E_\oplus(t)$	$E_\ominus(t)$
$E(t)$	1	0.85	-0.56	0.95	0.90	**0.19**	**0.15**
$V(t)$	0.85	1	-0.20	0.70	0.66	**0.15**	**0.11**
$N_c(t)$	-0.56	-0.20	1	-0.70	-0.41	**-0.16**	**-0.15**
$D(t)$	0.95	0.69	-0.69	1	0.86	**0.19**	**0.15**
$T(t)$	0.90	0.66	-0.41	0.86	1	**0.15**	**0.11**
$E_\oplus(t)$	**0.19**	**0.15**	**-0.16**	**0.20**	**0.15**	1	**0.03**
$E_\ominus(t)$	**0.15**	**0.11**	**-0.15**	**0.16**	**0.10**	**0.03**	1

Multivariate statistics of graph properties

Joint distributions

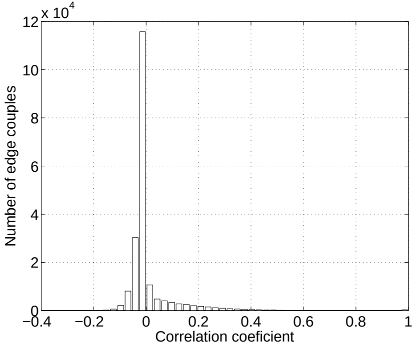
- $P_{XY}(x, y) = P[X = x \text{ and } Y = y] = P[X = x / Y = y]P[X = x]$
- variation of the # links is not constant over the # vertices



Multivariate statistics of graph properties

Link correlations

- Most pairs of links have a very low correlation coefficient.



Markovian evolution

1. Correlation time link creation/deletion is small
2. Independent from the evolution of other graph properties
3. Links are independents

Multivariate statistics of graph properties

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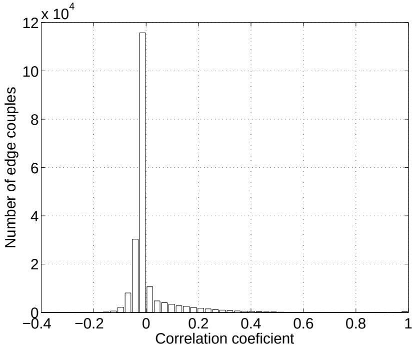

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Towards a global analysis of the dynamics

global properties

- not directly interpretable in the sequence of static graphs
- stability of connected components
- communities embedded in the network
- proportion of creation of triangles

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global properties

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- stability of connected components
- communities embedded in the network
- **proportion of creation of triangles**

Triangles in the graphs

	$P_{+/tri+}$	$P_{+/tri=}$	$f_{+/tri+}$	$f_{+/tri=}$
IMOTE	44 %	56 %	6 %	94 %
RANDOM	10 %	90 %	5 %	95 %

links / triangles

- $P_{+/tri+}$: link creation → triangle
- $f_{+/tri+}$: innactive link → triangle
- 40% of link creations increase the number of triangles
- proportion of inactive links that would create a triangle is very low
- More potential links doesn not imply higher $P_{+/tri+}$

Modeling of the dynamics

Simulation algorithm

- transition model with Markovian property
- links e are independent
- state of the network
- links e changes with $P_{tr}(e, G_t)$
- duration $\tau(e)$ since the link e has last changed its status

Ingredients

- contact / inter contact duration distribution
- elaborated graph properties $(E(t), V(t), N_C(t), D(t))$
- dynamical information (triangles)

Modeling of the dynamics

Input: Simulation time

Output: Random Dynamic Graph

```
foreach Simulation Time Step t do
    foreach link e do
        P_tr(e, G_t) = TransitionProbability(e) given the state G_t ;
        p_r = Uniform(0,1);
        if (p_r ≤ P_tr(e)) then
            ChangeState(e);
        end
    end
end
```

Ingredients I

Contact distribution

- heavy-tailed distributions for contact P_{ON} and inter-contact P_{OFF} durations
- $P_+(\tau)$: probability that one link that was OFF since τ ($\tau \geq 1$) is activated
- $P_{ON}(\tau) = P_-(\tau) \times \prod_{i=1}^{\tau-1}(1 - P_-(i))$

$$P_-(\tau) = \frac{P_{ON}(\tau)}{\prod_{i=1}^{\tau-1}(1 - P_-(i))}, \quad \tau \geq 2, \quad P_-(1) = P_{ON}(1) \tag{1}$$

$$P_+(\tau) = \frac{P_{OFF}(t)}{\prod_{i=1}^{\tau-1}(1 - P_+(i))}, \quad \tau \geq 2, \quad P_+(1) = P_{OFF}(1) \tag{2}$$

Ingredients II

Imposed graph property distribution

- Rejection Sampling based on a Metropolis-Hastings algorithm
- new state $G'_t = \{G_t + S_e(t) \text{ changed}\}$, is accepted with probability $P_{RS}(G_t, G'_t) = \min\left(1, \frac{F(x(G'_t))}{F(x(G_t))}\right)$
- F is the target PDF for the graph
- The total probability of transition of link e is then: $P_{tr}(e, G_t) = P_{-/+}(\tau(e)) \cdot P_{RS}(G_t, G'_t)$.

Ingredients III

Imposed dynamics of triangles

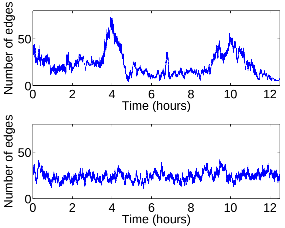
- reproduce the correct dynamical transition process concerning triangles
- do not want to change the mean probabilities of transition
- The weighted probabilities are then:

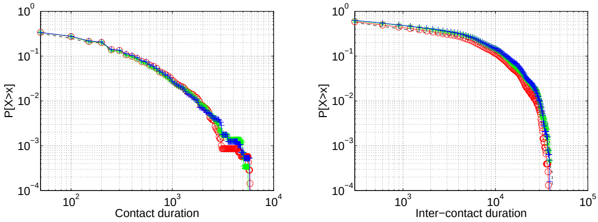
$$P_{tr}(e, G_t) = \begin{cases} P_+(\tau(e))\frac{P_{+/tri=}}{f_{+/tri=}} & \text{for link creation without new triangle,} \\ P_+(\tau(e))\frac{P_{+/tri+}}{f_{+/tri+}} & \text{for link creation with a new triangle.} \end{cases}$$

Simulation results

Investigated models

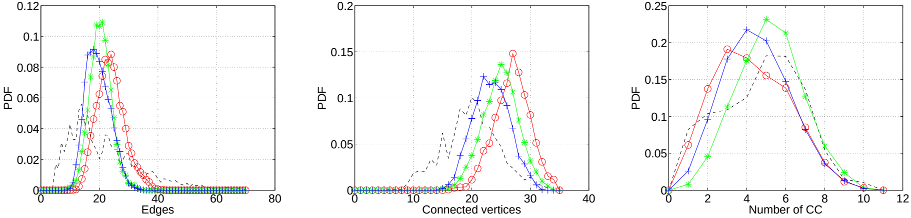
- $\mathcal{A}$: imposed empirical contact and inter-contact duration distribution only.
- $\mathcal{B}$: imposed distributions of contact / inter-contact durations , and of number of connected components.
- $\mathcal{C}$: distributions imposed contact / inter-contact durations and of number of connected vertices.





−− Imote / o Model $\mathcal{A}$ / $*$ Model $\mathcal{B}$ / + Model $\mathcal{C}$

Simulation results (cont)

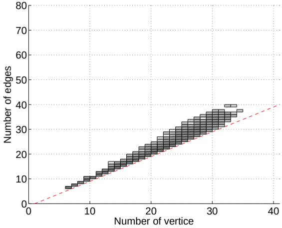


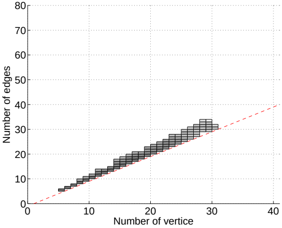
$--$ Imote / o Model $\mathcal{A}$ / $*$ Model $\mathcal{B}$ / + Model $\mathcal{C}$

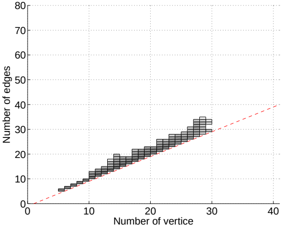
$\mathcal{A}$: sole contact and inter-contact duration fails

- the number of connected vertices is strongly over-estimated
- the number of connected components is under-estimated

Simulation results (cont)



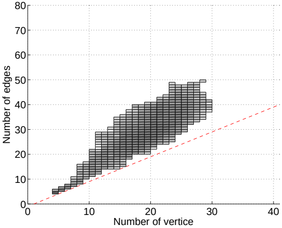


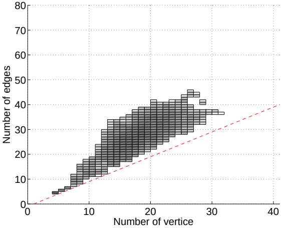


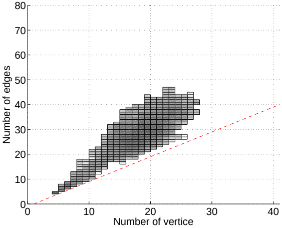
$\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ fail!

- The density of the connected components (the groups) is underestimated
- Links are spread uniformly in the graph

Simulation results (cont)



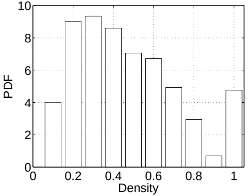




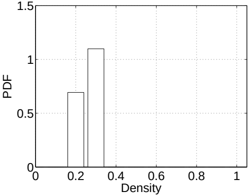
Weighted models

- does not have an impact on the contact and inter-contact duration distributions
- the density of connected components is comparable to the experimental data

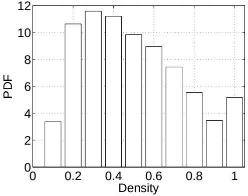
Simulation results (cont)



IMOTE$\mathcal{A}$



$\mathcal{B}_\omega$



Density of frequent connected components

- ($\tau = 7$ and $\sigma = 6$)
- classical models fail to create dense frequent connected components
- the number of frequent connected subgraphs is larger in the simulated data than in the original

Outline

- MOSAR Project
  - Project overview
- Dynamic Network Characterization
  - Motivation
  - Statistical analysis of snapshots of graphs
  - Towards a global analysis of the dynamics
  - Modeling of the dynamics
- **Conclusion**

Conclusion

contributions

- rigorous / coherent set of properties (basic / advanced)
- probability distribution of contacts and inter contacts is only one parameter
- global analyses to characterize the dynamics of the graph as a whole:
  - correlation between links
  - stability of the connected components
  - number of triangles
  - evolution of communities inside the interaction networks.
- simple / accurate models that generate random interaction graphs with satisfactory temporal properties.

Conclusion

Futur / On going works

- Introduce non-stationarity (piecewise stationary model)
- Dynamic community computation
- Overlapping community detection
- Trajectories of individuals as a *signature*
- Large in situ test beds to be deployed...

Some references

Dynamic networks

- Antoine Scherrer, Pierre Borgnat, Éric Fleury, Jean-Loup Guillaume and Céline Robardet, *Description and simulation of dynamic mobility networks*, in Computer Network 2008.
- Pierre Borgnat, Éric Fleury, Jean-Loup Guillaume, Céline Robardet and Antoine Scherrer, *Analysis of Dynamic Sensor Networks: Power Law Then What?*, in Comsware 2007.