The Role of Topology in collective behavior of excitable networks driven by noise

Pablo Balenzuela

Departamento de Física, FCEyN, UBA

IFIBA – CONICET

universidad de buenos aires - exactas
departamento de física

Workshop on Dynamic Networks 2013

Co-authors

- Jordi García–Ojalvo

  Department of Experimental & Health Science,

  Universitat Pompeu Fabra, Barcelona, Spain.
- Pau Rué

  Department of Genetics, Cambridge University, UK.
- Stefano Boccaletti

  CNR-Istituto dei Sistemi Complessi, Italy

Warning

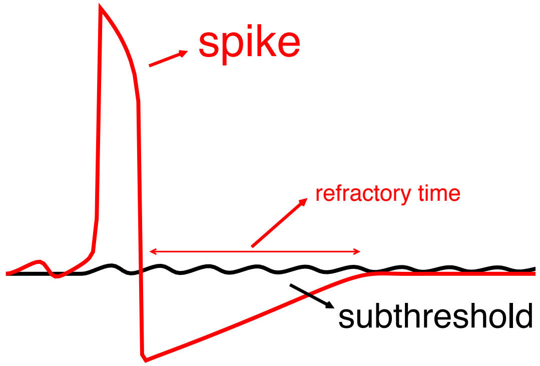
- ✓ In this work we don't analyze networks.
- ✓ We analyze what happen when certain dynamical units are connected with different networks topologies
- ✓ Input: Excitable dynamics + noise + complex networks
- ✓ Output: Collective behaviors such as Synchronization & Array Enhance Coherence Resonance.

Summary

- ✓ Stochastic Coherence & Synchronization in complex networks
- ✓ The model: Dynamical units & connectivity networks
- ✓ Results
- ✓ Conclusions

Stochastic Coherence

Excitable Unit



✚

Noise

Coefficient of Variation

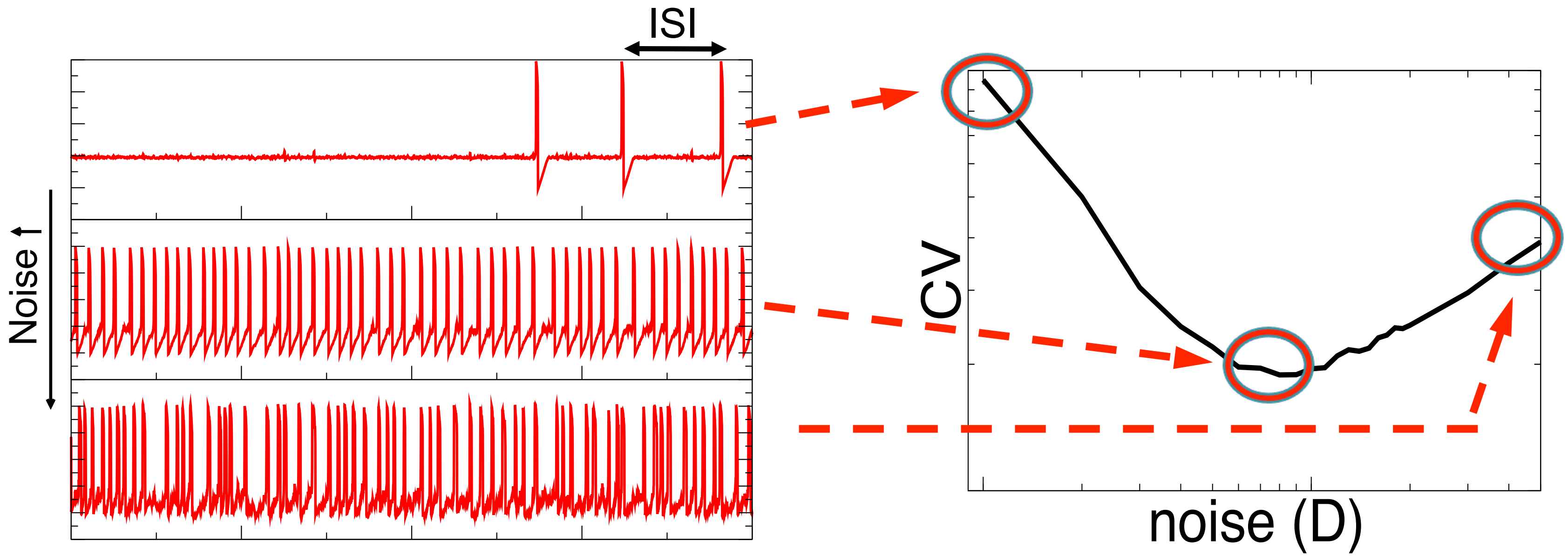
Quantifier of regular spiking

$$\langle CV \rangle = \frac{\langle \sigma_{t_{ISI}} \rangle}{\langle \overline{t_{ISI}} \rangle}$$

Low

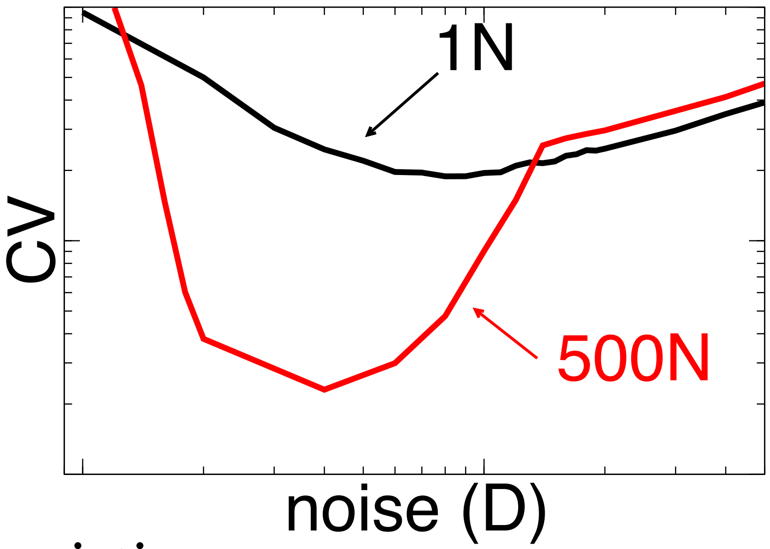
Optimal

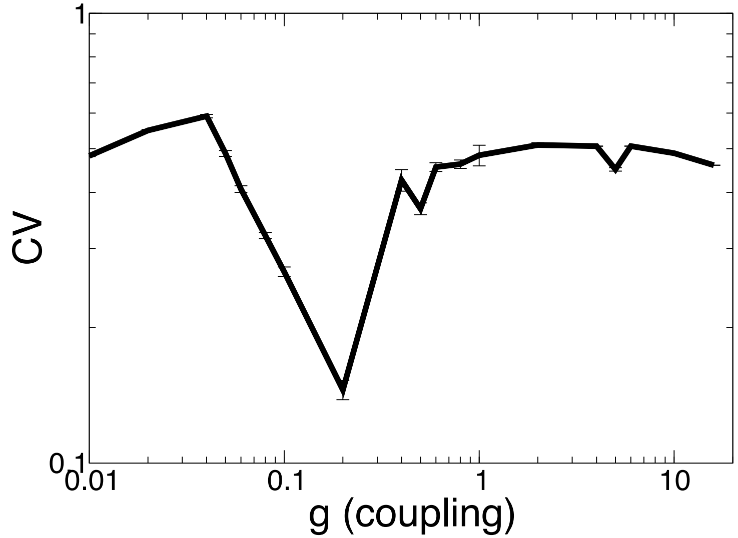
High



A.Pikovsky & J.Kurths, *PRL* **78**, 775 (1997).

Coupling enhances Stochastic Coherence





Heuristic

✓ Resonance occurs when noise makes the neuron fires after each refractory period.
✓ Coupling enhances SC:
✓ Firing of a neighbor reminds a given element to fire at the "correct time"

Previous Results

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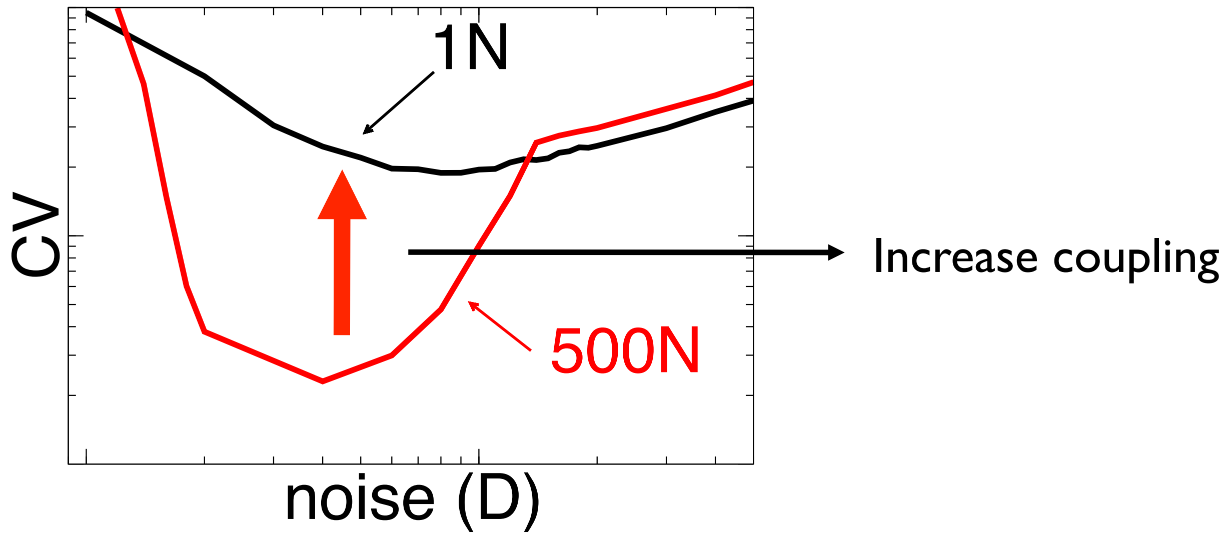
  (NN Ring of FN Neurons with Diffusive Coupling and Gaussian noise)
- C. Zhou, J. Kurths and B. Hu, PRL (2001) → Heterogeneity & Independent Noise Favours AESC (NN Ring of FN Neurons with Diffusive Coupling and Gausian Noise)
- O. Kwon and H.T Moon, Phys.Lett A (2002) → SC persist in SW Networks
  (HH neurons on WS networks)

Synchronization affects AESC

✓ Coupling favours Stochastic Coherence because dynamical units are not fully synchronized

✓ If $x_i(t) = x_s(t), \forall i = 1, N, \forall t$ → CV(N exc. elem) = CV (1 exc. elem)

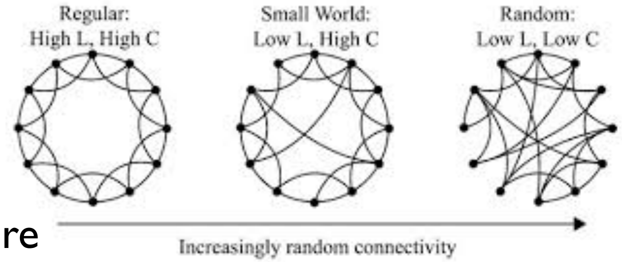


<u>Synchronization also affects other collective properties driven by noise</u>

✓ Full synchronization has a negative effect in Global Amplification (Acebron et al, 2007).
✓ Diffusive coupling (DC) vs Chemical Synapse (ChS, only couples during spiking): ChS shows better AECR than DC, but synchro is worst (Balenzuela & Ojalvo, 2005).
✓ Stochastic Resonance (SR) in weakly paced SFN is weaker when coupling increases (M. Perc 2008).

Synchronization on Complex Networks

• M. Barahona & L.M. Pecora, PRL (2002)

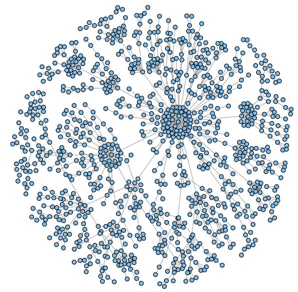


Synchronization in Watts-Strogatz Small World networks via Master Stability Function Analysis.

✓ The addition of shortcuts produces synchronizability more efficiently than deterministic & random graph

✓ However, the SW property does not guaranty synchronization

• T. Nishikawa, A.E.Motter et al, PRL (2003)



They study the role of heterogeneity in synchronization via Master Stability Function Analysis in a semirandom model of SF networks [$P(k) \sim k^{-\gamma}$].

✓ The more heterogeneus → the less synchronizable,

Paradox of Heterogeneity

✓ Heuristic:

Hubs tend to be overloaded by the traffic of communications.

Networks with few links concentrating lot of traffic → Less Synchronizable

Improving synchronization on Complex Networks

Synchronization in Networks with heterogeneous degree distribution could be improved if links are properly weighted

Binary undirected network

$$\frac{dx_i}{dt} = f(x_i) - \sigma \sum_{j=1}^{N} A_{ij}[h(x_i) - h(x_j)]$$

Weighted directed networks

$$\frac{dx_i}{dt} = f(x_i) - \sigma \sum_{j=1}^{N} G_{ij}[h(x_i) - h(x_j)]$$

(Local) Weighted by its degree

•A. E.Motter, C. Zou & J. Kurths, PRE (2005)

$$G_{ij} = \frac{1}{k_i} A_{ij}$$

(Global) Weighted by its load (l_{ij})

•M.Chavez, S. Boccaletti et al, PRL (2005)

$$G_{ij} = \frac{l_{ij}^{\ \alpha}}{\sum_{j \in K_i} l_{ij}^{\ \alpha}} A_{ij}$$

A_{ij} = Adjancecy Matrix

Load=betweenes: measure the traffic of each link

Array Enhanced Stochastic Coherence & Synchronization

- ✓ Are they really incompatible phenomena?
- ✓ Does it exist a *network architecture* showing *strong synchronizability* and high levels of *stochastic coherence*?

We analyze excitable dynamical units on
Weighted (by its load)
Scale Free Networks (WSFN).

Dynamical Model

FitzHugh-Nagumo Model

$$\varepsilon \frac{dx_i}{dt} = x_i - \frac{x_i^3}{3} - y_i + I_i,$$

$$\frac{dy_i}{dt} = x_i + a + D\xi_i(t).$$

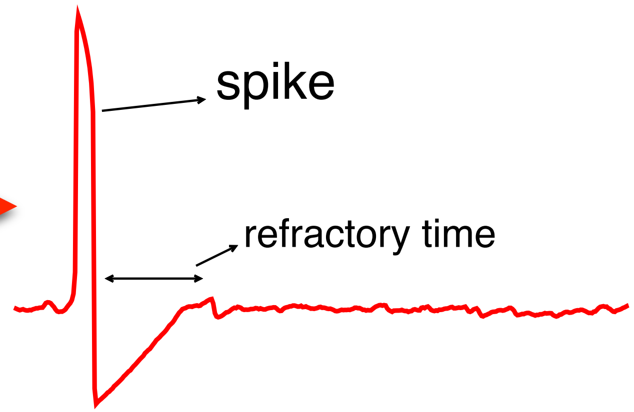
If |a|> solution has a only fixed point.
If |a|< solution is a limit cycle.
When |*a*| slightly greater than 1 → excitable.

We use ε=0.01 y *a*=1.05

White Noise

$$\langle \xi(t) \rangle = 0,$$

$$\langle \xi(t)\xi(t') \rangle = \delta(t - t')$$



Coupling

$$I_i = g \sum_{j=1}^{N} n_{ij}(x_j - x_i)$$

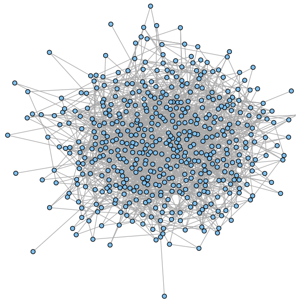
n_{ij}>0 if i and j connected
n_{ij}=0 otherwise

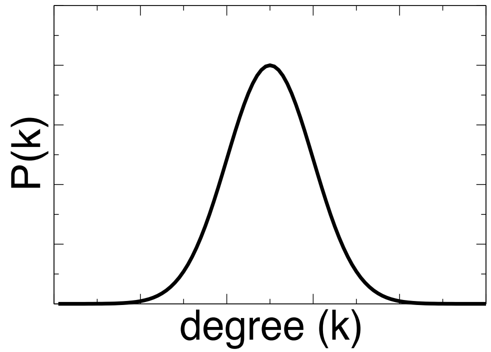
$\sum_{i,j} n_{ij} = 2M$ always

M= # of links

Connectivity Networks

Random Erdos-Renyi Networks





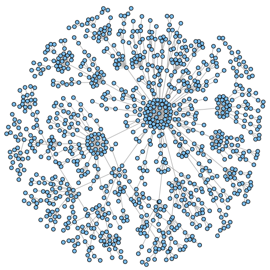
$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

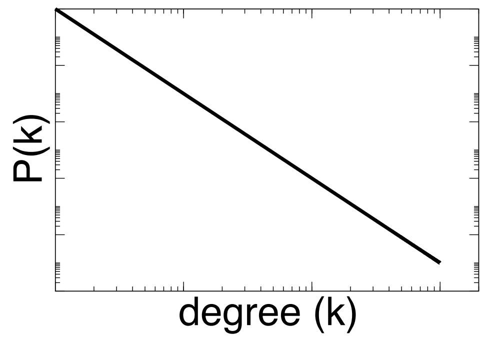
n_{ij}= 1 for all connected pairs

Scale Free Networks (Barabasi-Albert Model)

UnWeighted

n_{ij}= 1 for all connected pairs



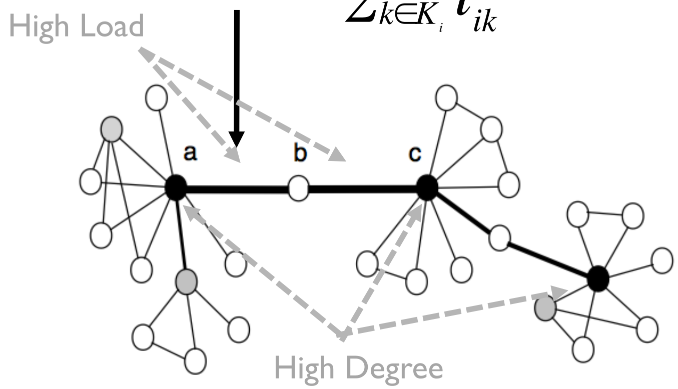


$$P(k) = Ak^{-\gamma}$$

Weighted

$$n_{ij} = \frac{l_{ij}}{\sum_{k \in K_i} l_{ik}}$$

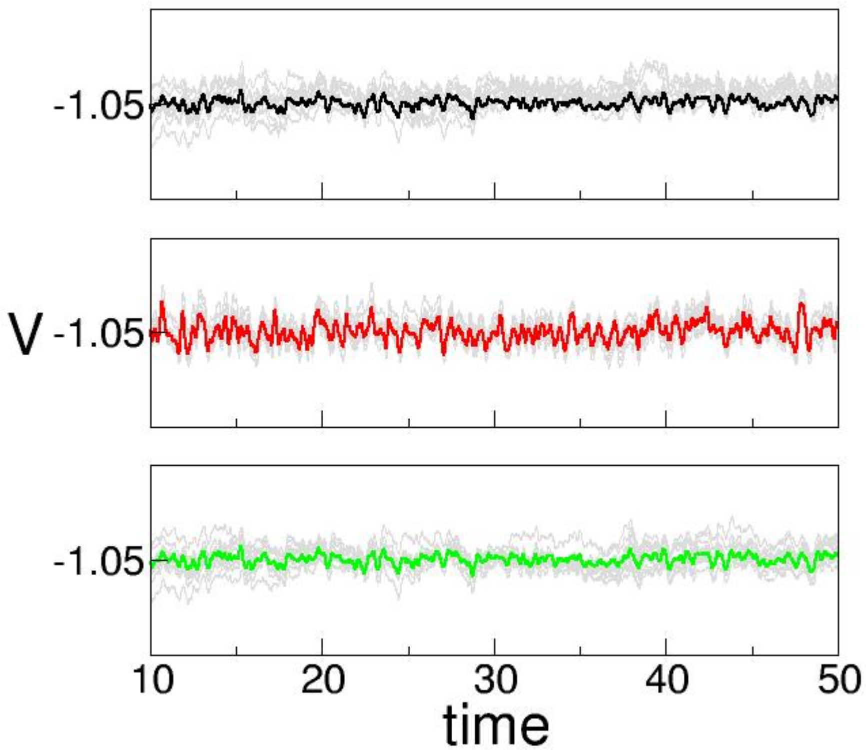
→ load



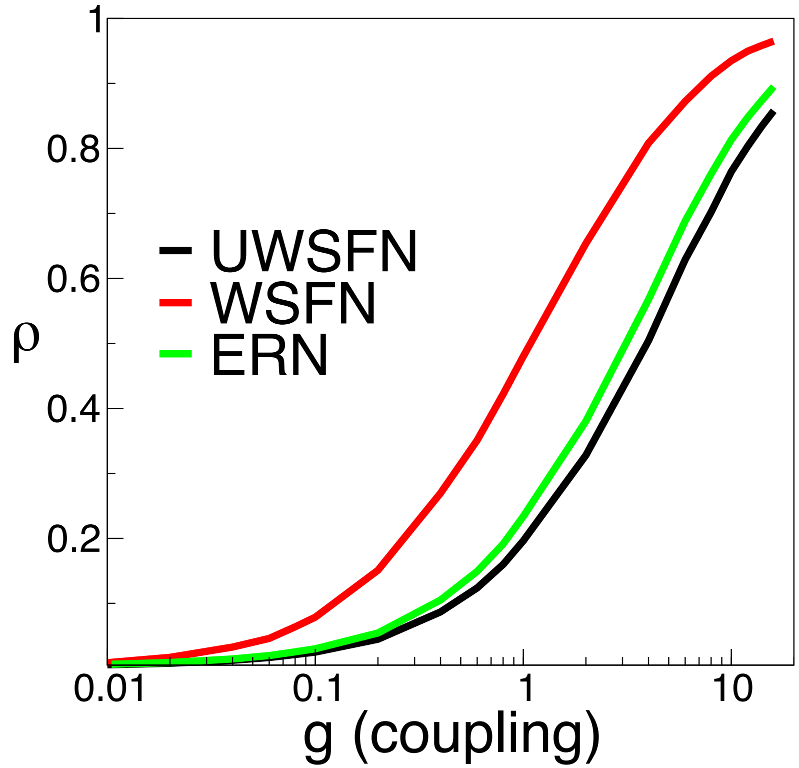
Results

Synchronization in the subthreshold regime

Time series (g=1)



Synchronization (ρ)



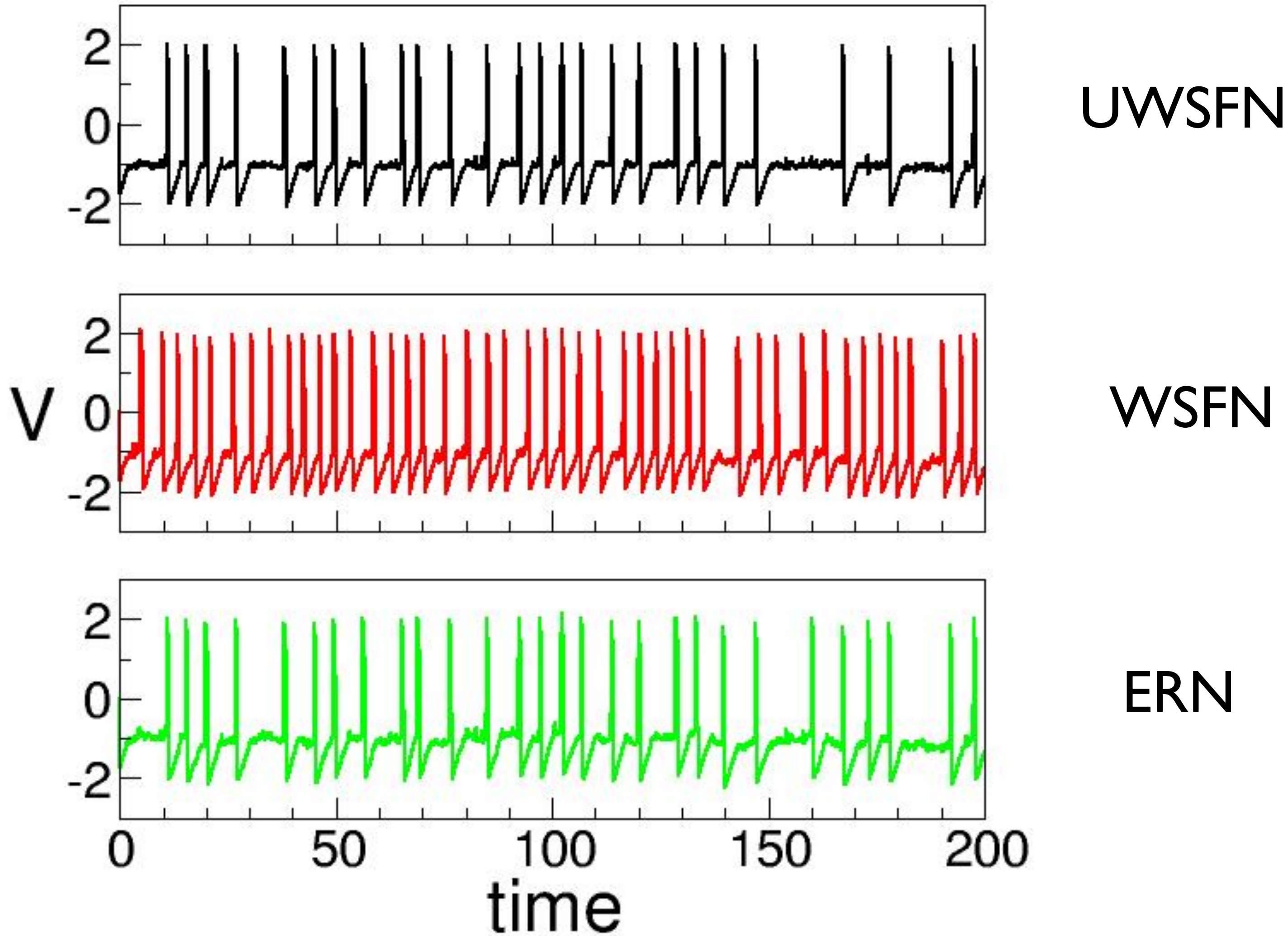
Synchronization at a glance:
Amplitude of Averaged signal is Maximal at WSFN

$$\rho = \frac{\langle \overline{v_i}^2 \rangle - \langle \overline{v_i} \rangle^2}{\overline{\langle v_i^2 \rangle - \langle v_i \rangle^2}}$$

Weighted Networks Synchronize better!

Stochastic Coherence

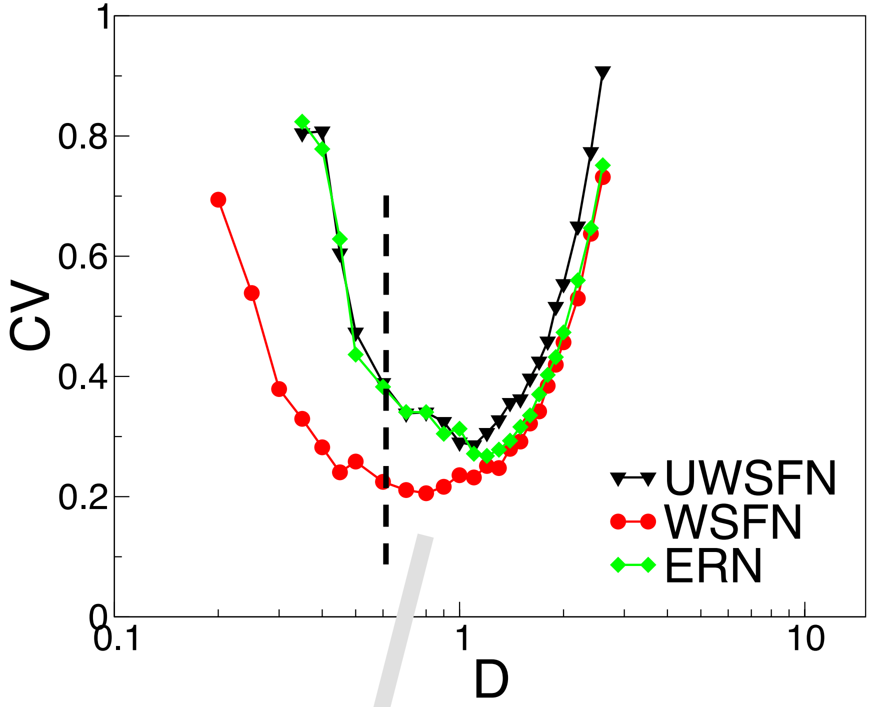
Time series (g=1, D=0.5)

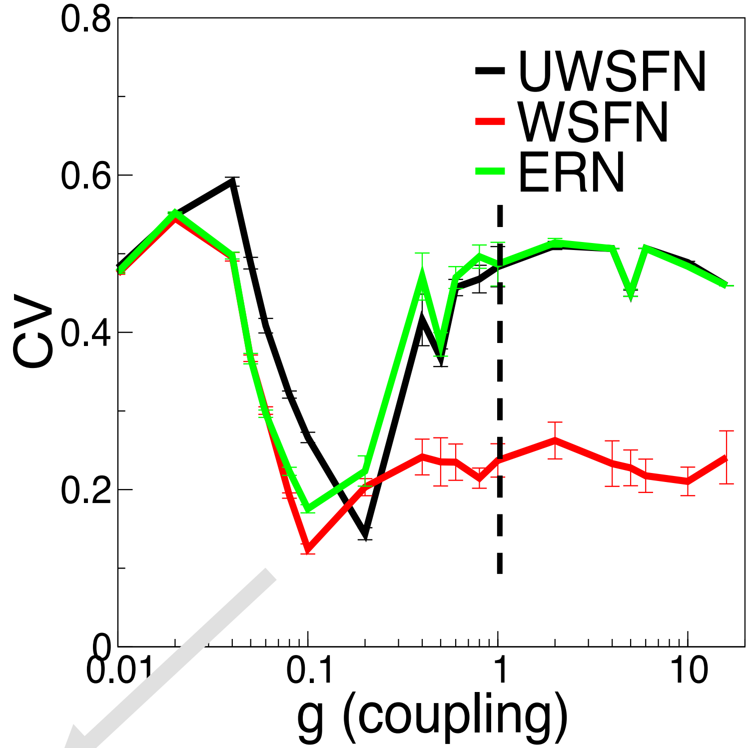


Weighted Networks spike more regularly!

Stochastic Coherence

Spiking Regularity: Minimun of CV



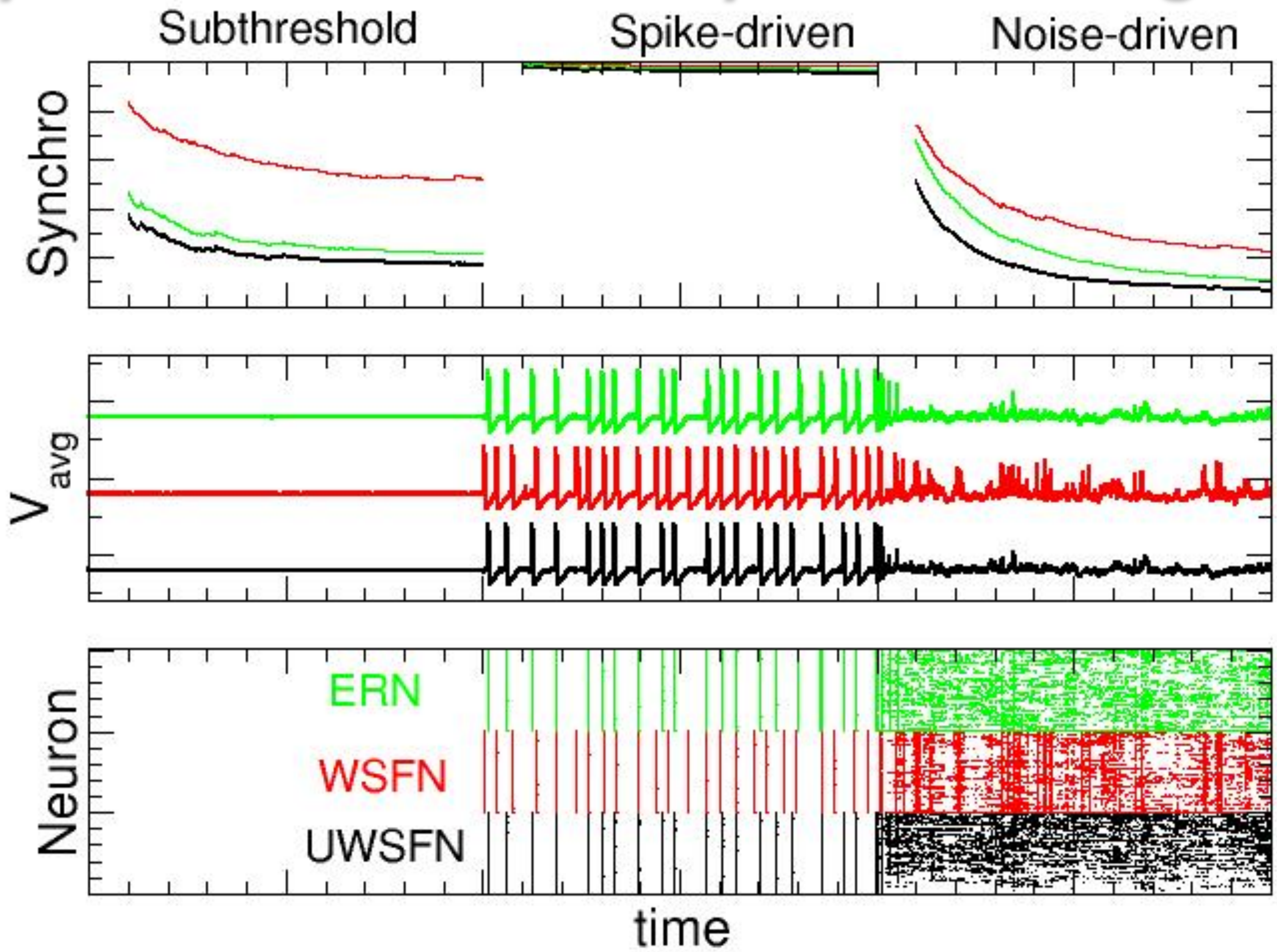


Minimun of CV at intermediate coupling
and noise strenght:
Array Enhanced Coherence Resonance
AECR

$$\langle CV \rangle = \frac{\langle \sigma_{t_{ISI}} \rangle}{\langle \overline{t_{ISI}} \rangle}$$

Weighted Networks show better SC !

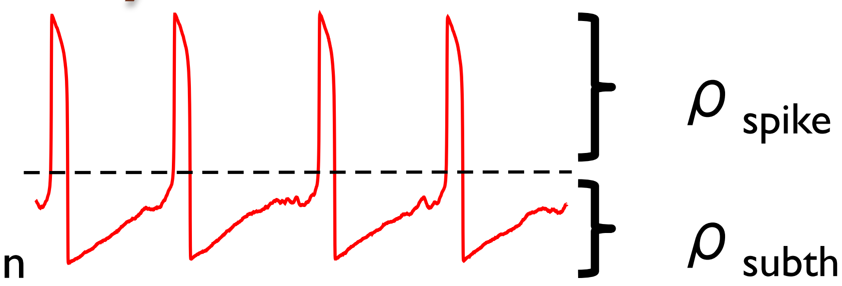
Synchro & SC in 3 Dynamical Regimes

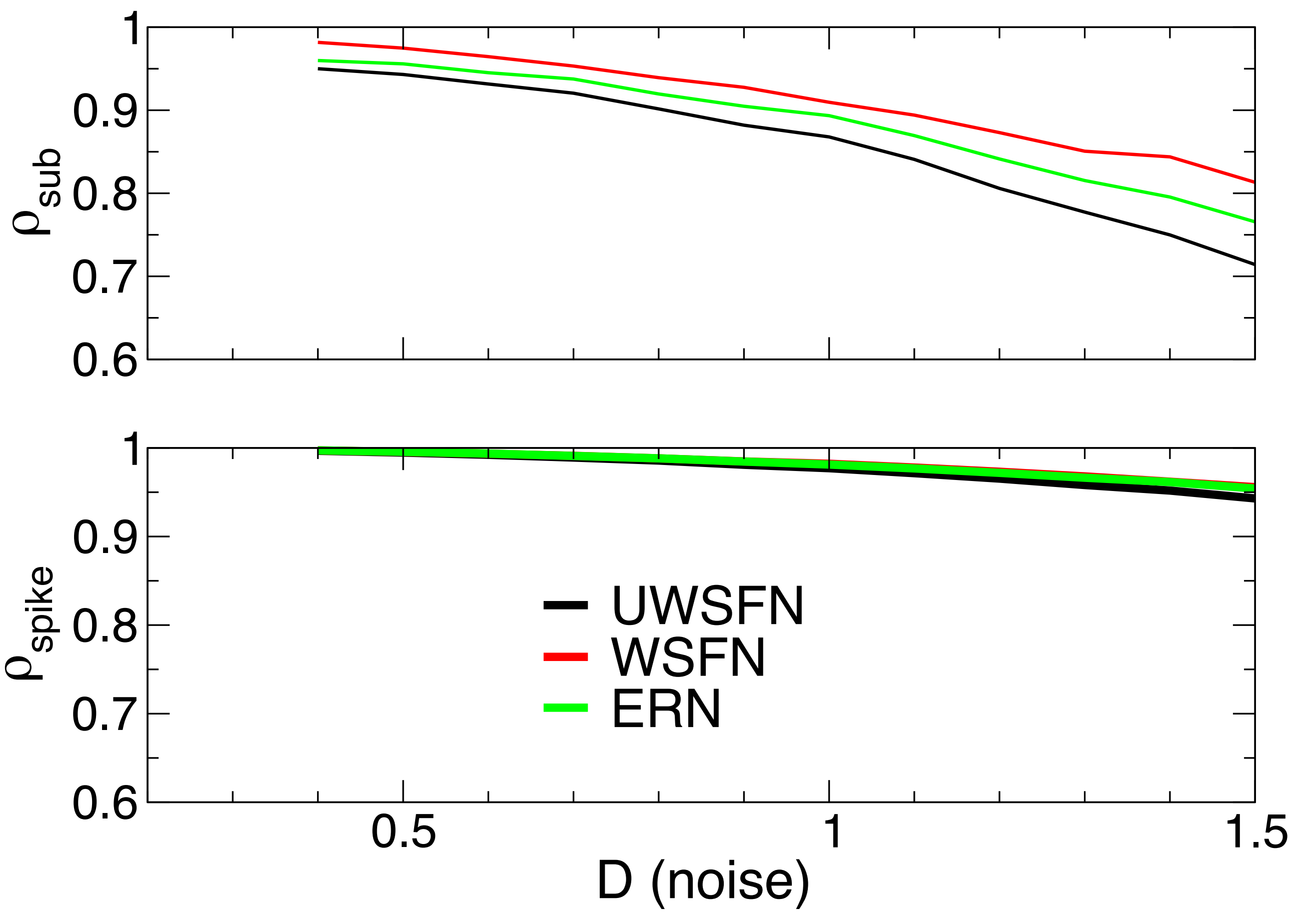


What happen with Synchronization in spiking regime?

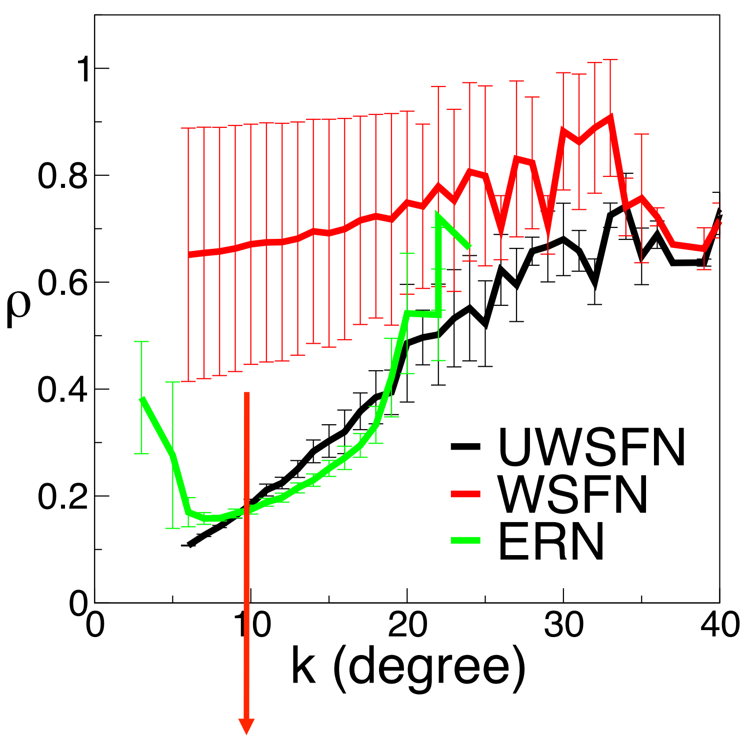
Spike and Subthreshold Synchronization

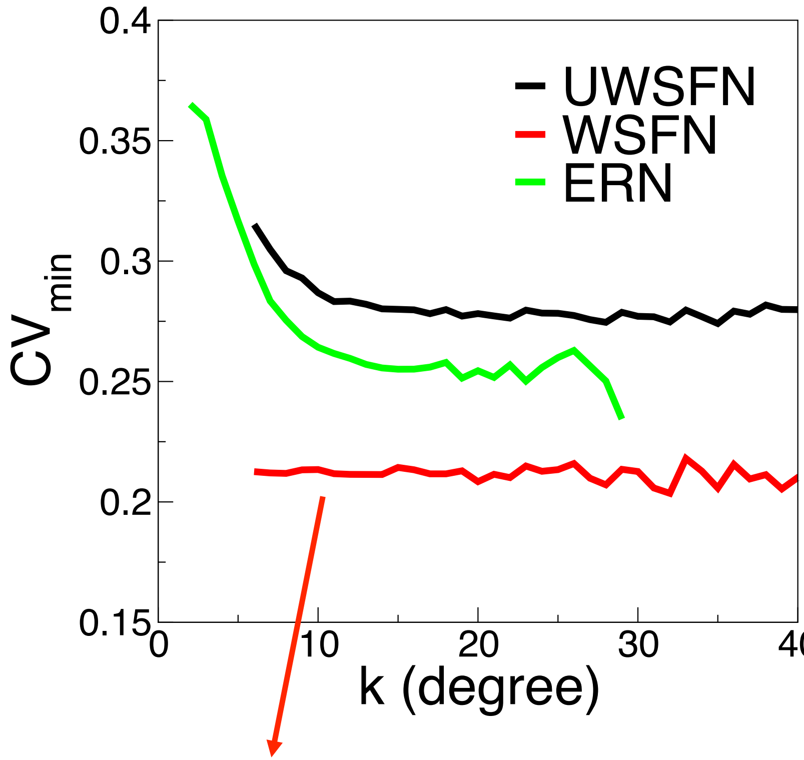
- ✓Time series: Spike signal + subthreshold signal
- ✓ We separate spike and subthreshold dynamics contributions to synchronization





Synchronization & SC vs degree





✓ Weighting behavior balance the heterogeneity of the SF networks

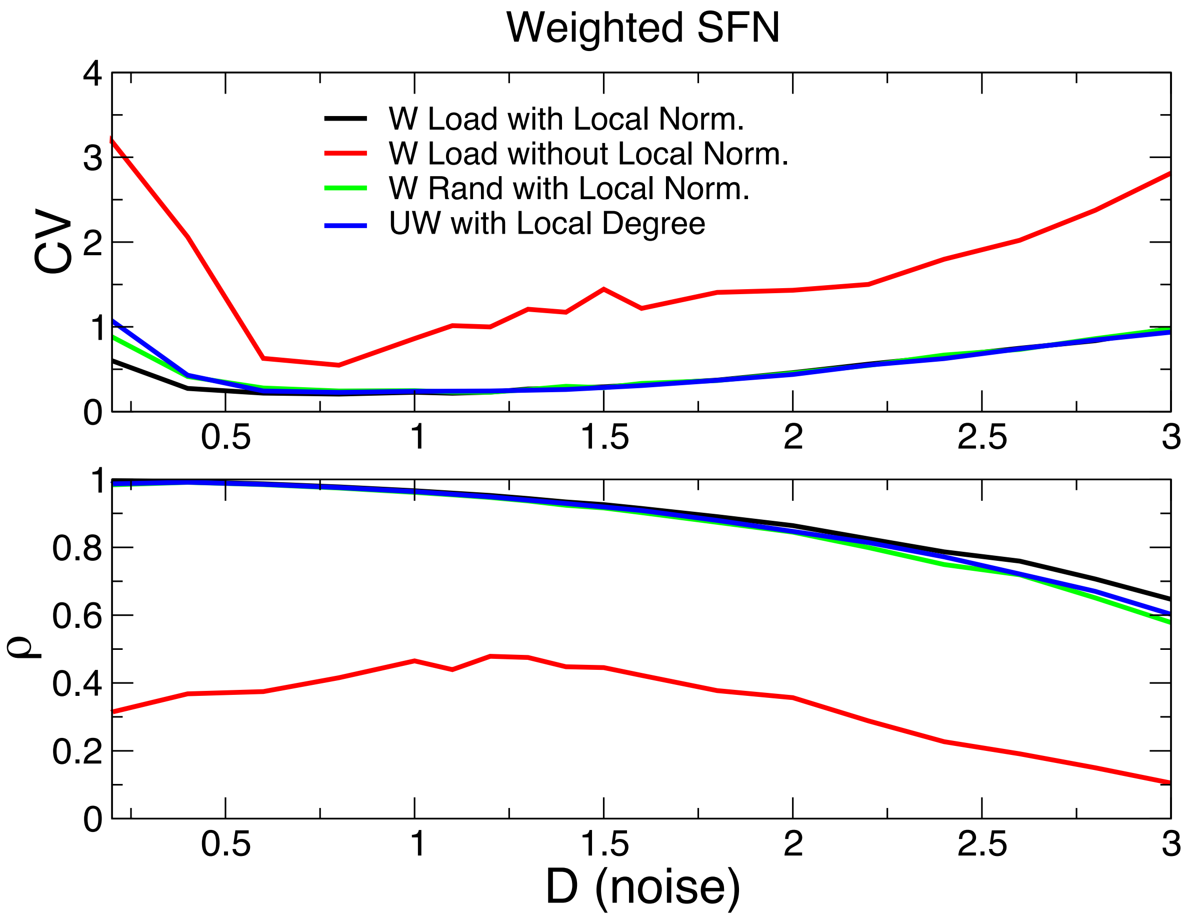
Conclusions

- ✓ Synchronization and Stochastic Coherence seems not to be compatible phenomena.
- ✓ We show that both collective behaviors can be found together in Weighted Scale Free Networks
- ✓ The weighting procedure balance the intrinsic heterogeneity of the SFN giving optimal conditions for synchronization as well as Stochastic Coherence

Balenzuela, Rue, Boccaletti & Garcia-Ojalvo, ArXiv:1309.2184v1 (2013)

Thank you!

The key-role of normalization



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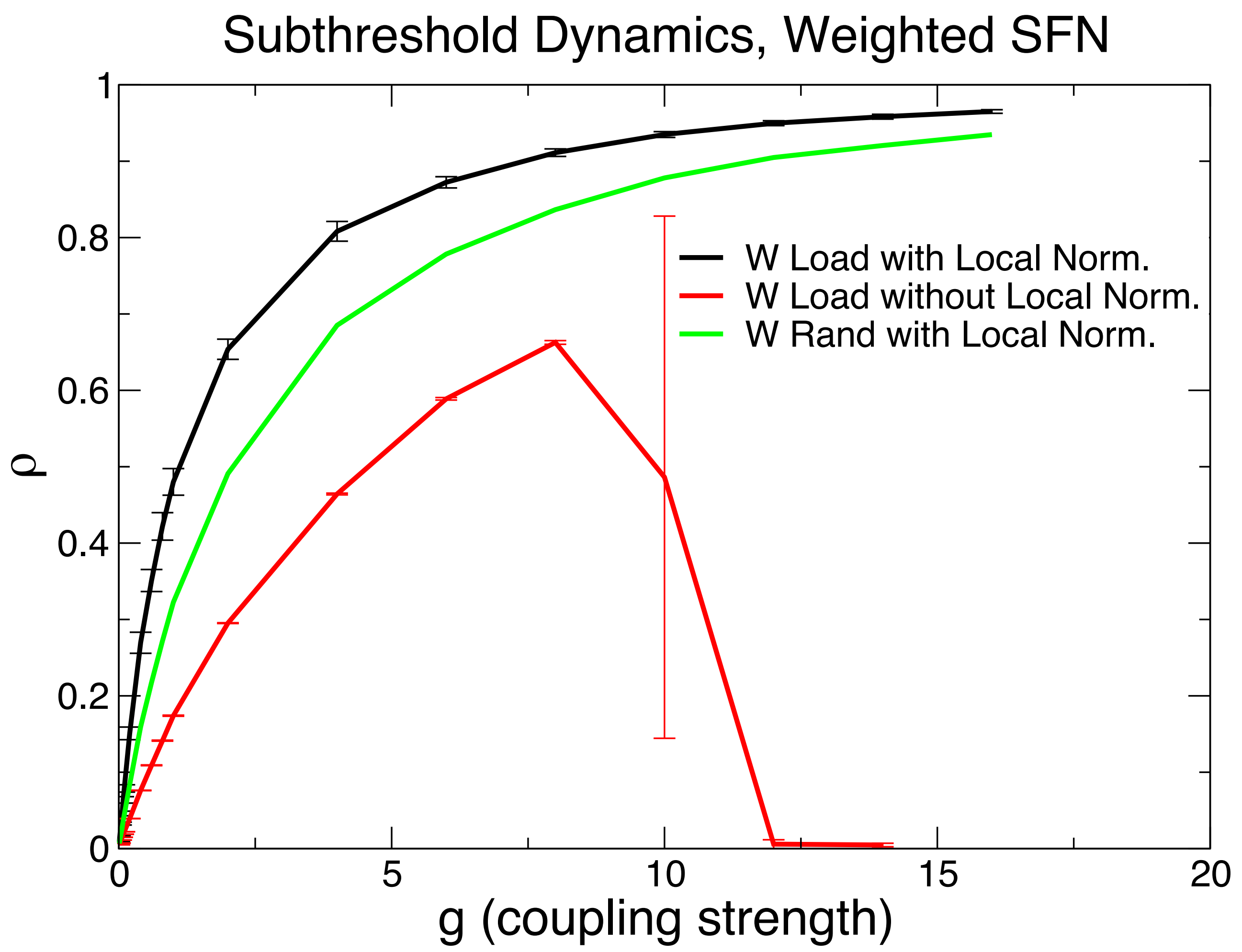

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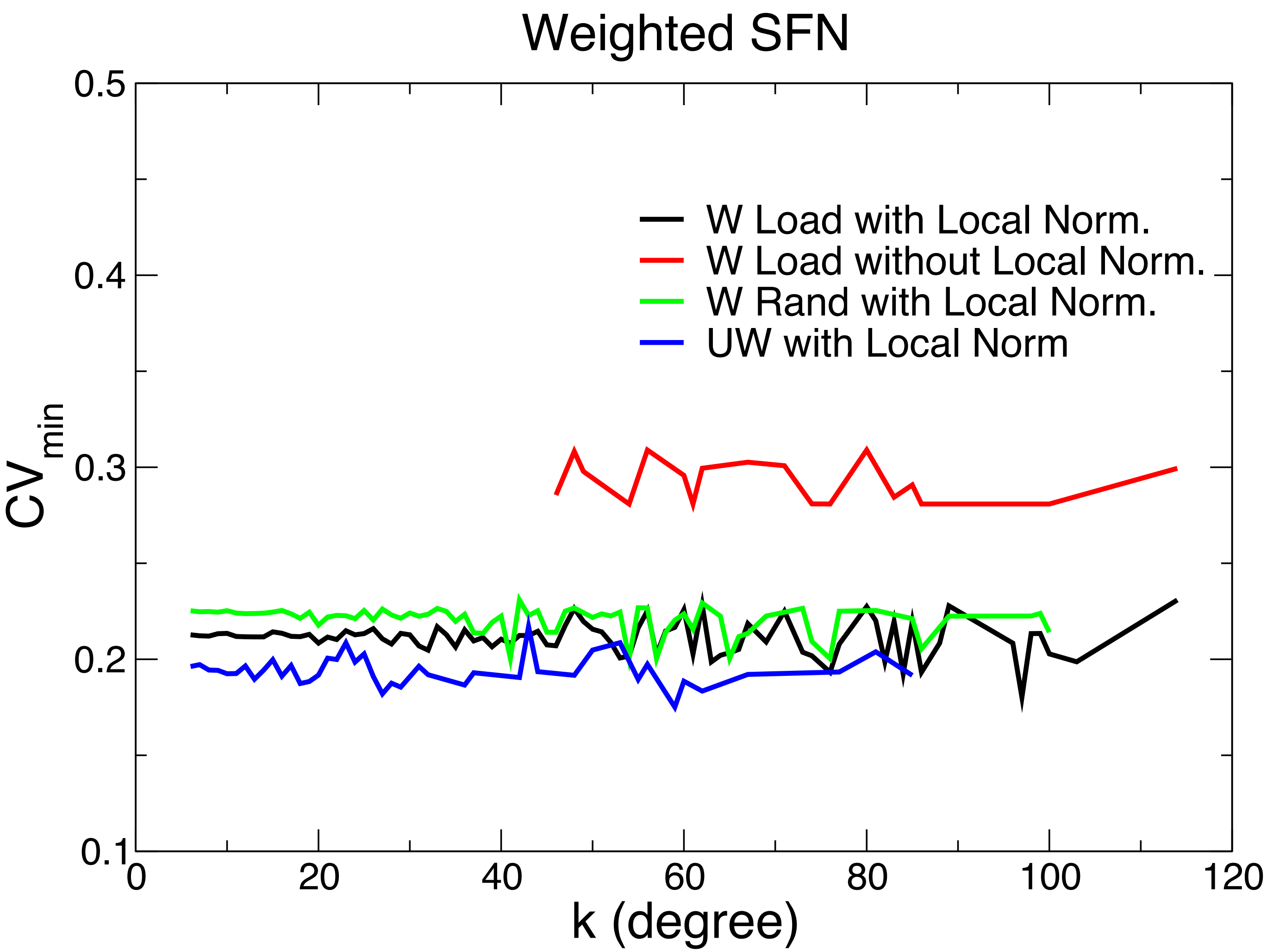
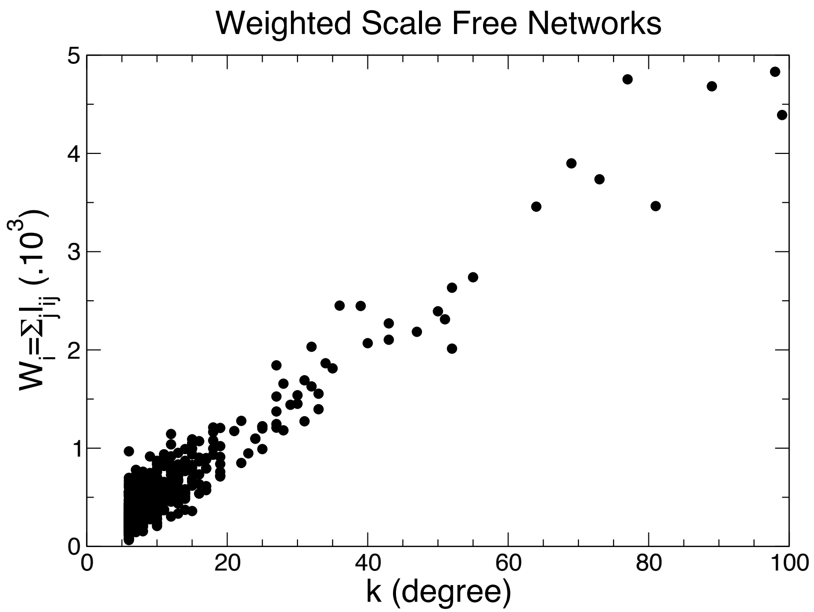
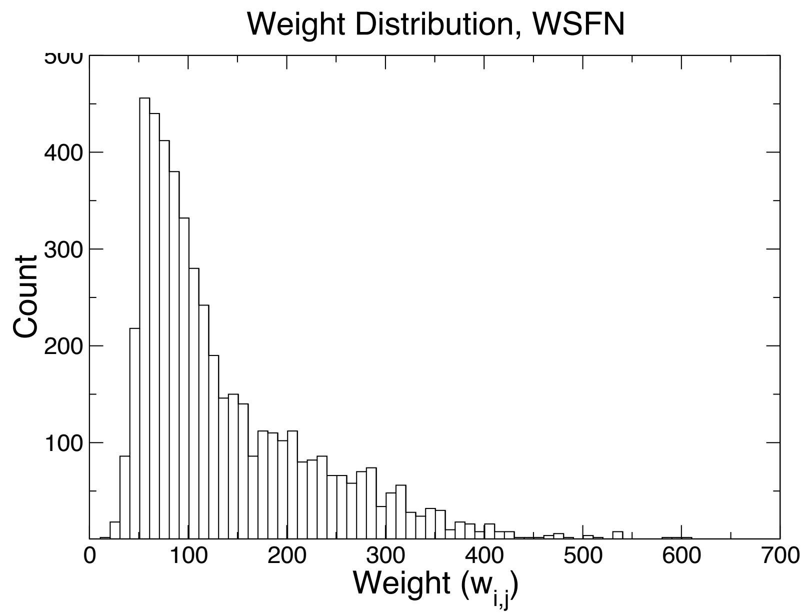


Weight Distributions





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•M.Chavez, S. Boccaletti et al, PRL (2005)

Weighting by its load (l_{ij})

$$\frac{dx_i}{dt} = f(x_i) - \sigma \sum_{j=1}^{N} \frac{l_{ij}^{\alpha}}{\sum_{j \in K_i} l_{ij}^{\alpha}} A_{ij} [h(x_i) - h(x_j)]$$

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Coupling enhances Stochastic Coherence

Heuristic

✓ Resonance occurs when noise makes the neuron fires after each refractory period.
✓ Coupling enhances SC:
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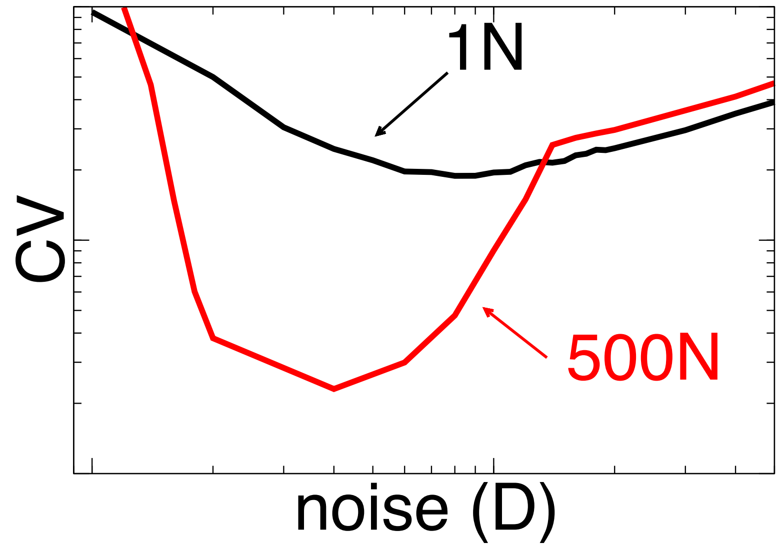

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