Spectral properties of Google Matrix (towards two-dimensional search engines)

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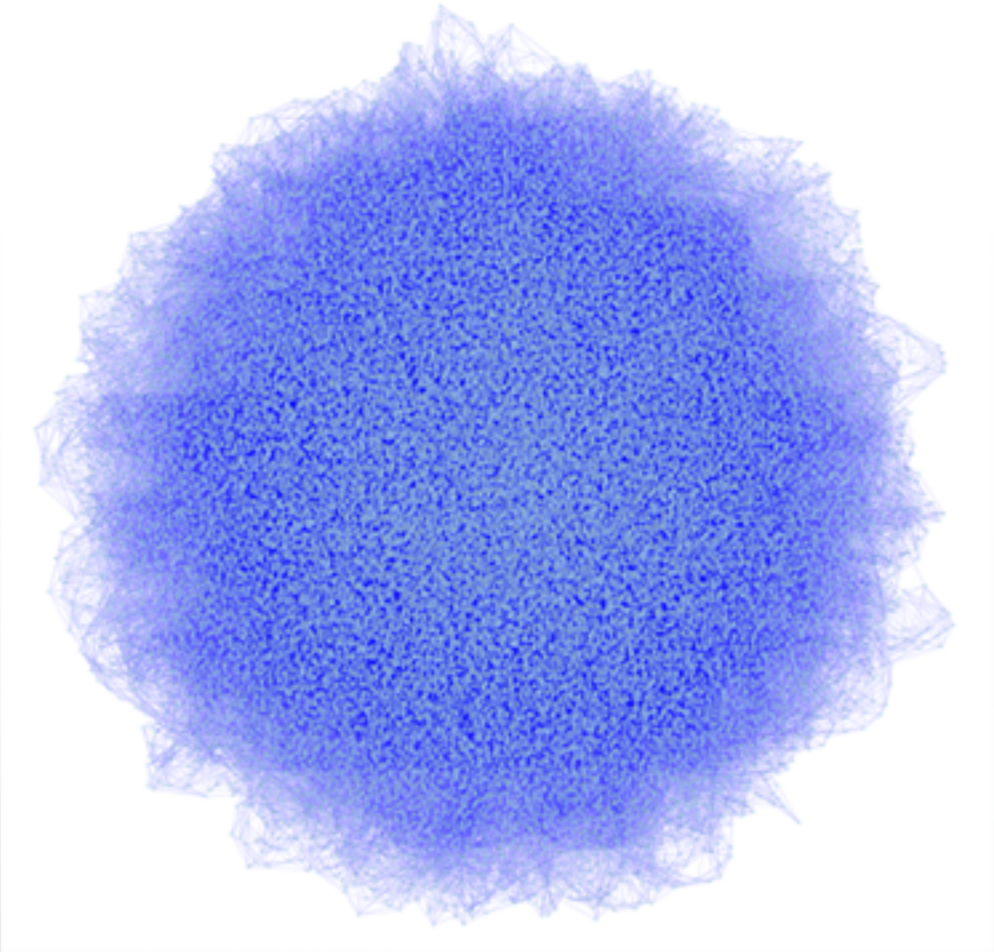
Motivations in complex networks



Outline

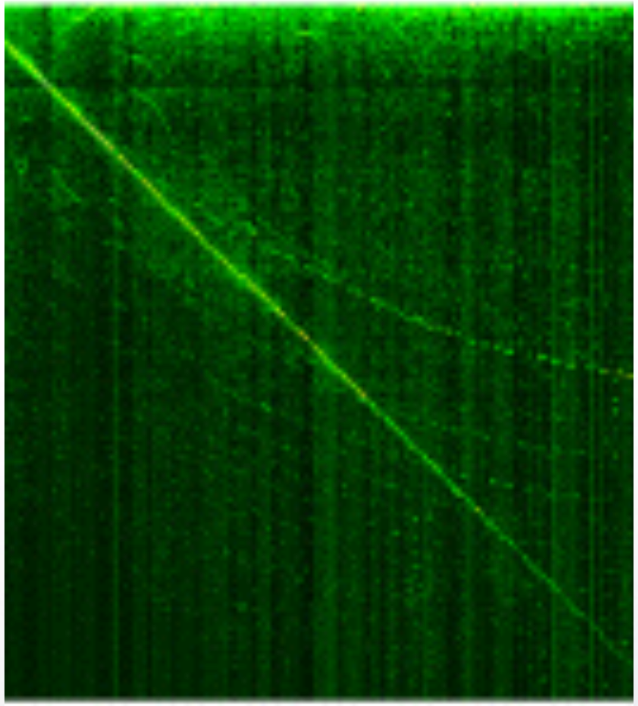
- Google matrix introduction
- PageRank and CheiRank
- Examples of 2-d rank

(Wikipedia, Univ., etc.)

- World Trade Network
- Other applications

(Ulam networks, FWL, communities)

- Conclusions



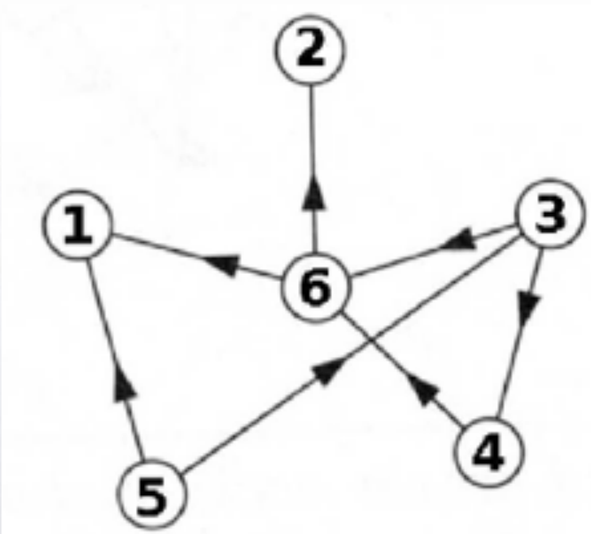
PageRank - Google Matrix

**Brin and Page (1998)*

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network



adjacency matrix

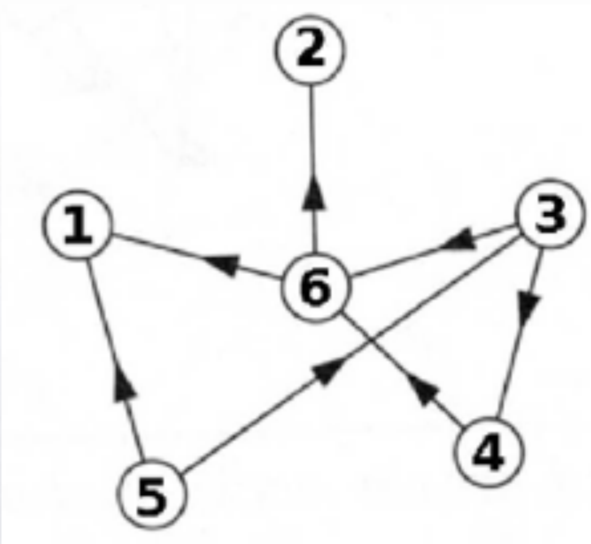
$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

PageRank - Google Matrix

**Brin and Page (1998)*

weighted adjacency matrix and dangling nodes

directed network



$$S = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 1 & 0 & 0 \end{pmatrix}$$

- $\sum_i S_{i,j} = 1$
- Perron-Frobenius (non-negative)
- $\lambda_1 = 1$ (degeneracy)

PageRank - Google Matrix

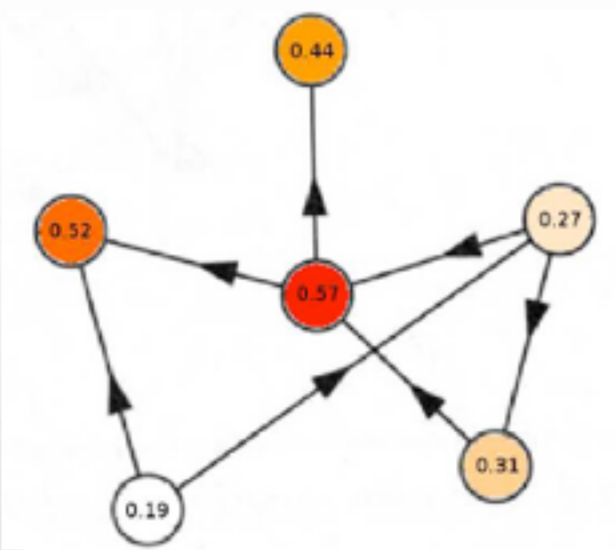
$\mathbf{G} = \alpha \mathbf{S} + (1-\alpha)\mathbf{E}/N \ (\alpha = 0.85)$

Google Matrix

PageRank

$\mathbf{G}P = P$

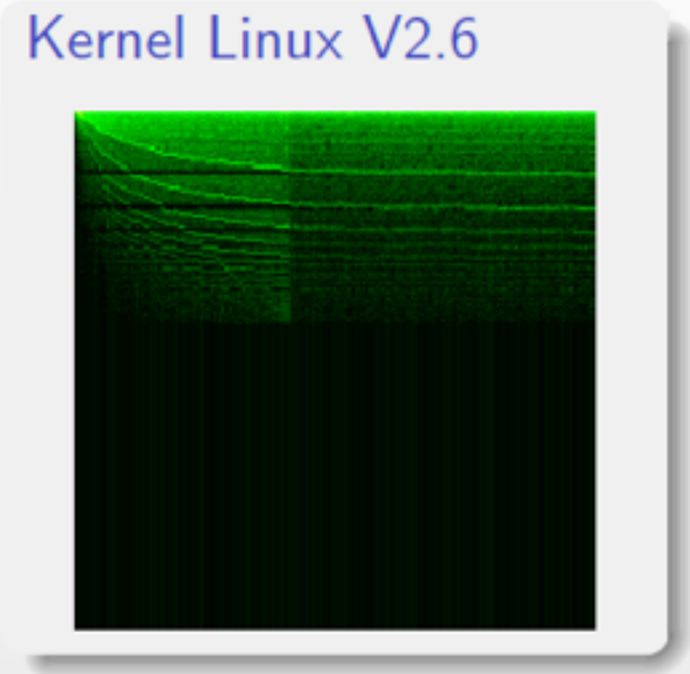
directed network

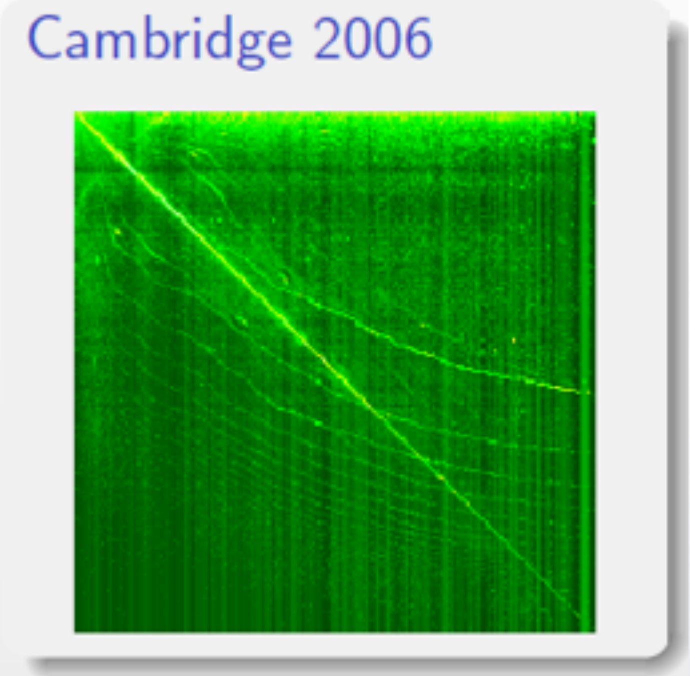


$$G = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{9}{20} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{9}{20} & \frac{7}{8} & \frac{1}{40} & \frac{1}{40} \end{pmatrix}$$

- $\alpha \to S, (1-\alpha) \to$ random node
- Perron-Frobenius (positive) $\lambda_1 = 1$
- $\Delta \geq (1-\alpha)$ (global convergence)

Google Matrix examples





L.E, Chepelianskii and Shepelyansky, Jour. Phys. A 45, 275101 (2012).

G*, CheiRank and spectrum

PageRank

Brin and Page

$\mathbf{G}P = P$

$P \sim 1/j^{\beta}, \quad (\beta = 0.9)$

CheiRank

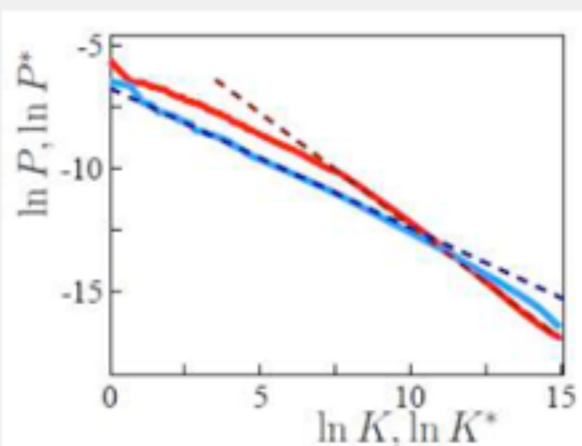
Chepelianskii (Kernel), Zhirov–Shepelyansky (Wiki)

$\mathbf{G}^{*}P^{*} = P^{*}$

(from inverted network: $\mathbf{A} = \mathbf{A}^{T}$)

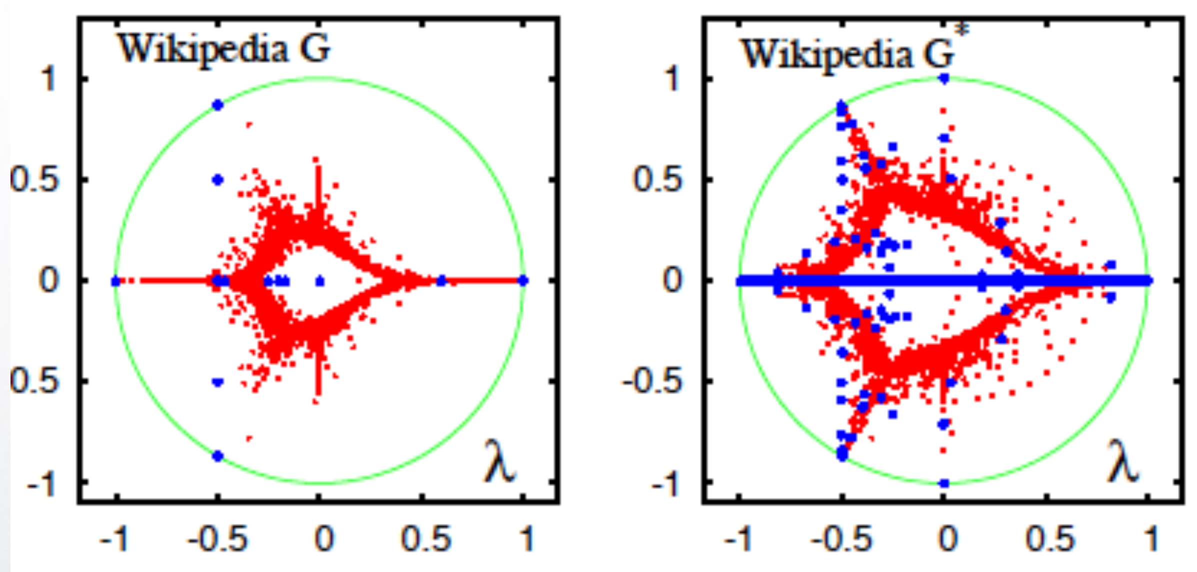
Example: Wikipedia

(Zhirov-Shepelyansky)



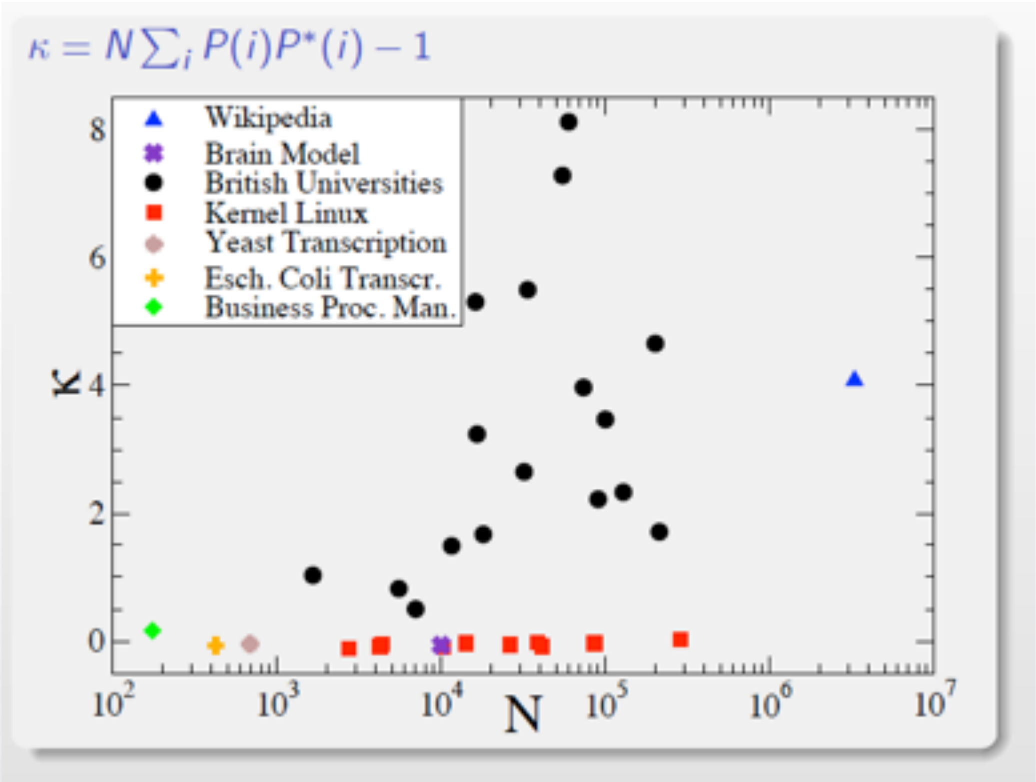
$N \sim 3.3 \times 10^{6}, \ \beta = 1/(\mu_{i,o} - 1)$

$P, \beta = 0.92$ and $P^{*}, \beta = 0.57$

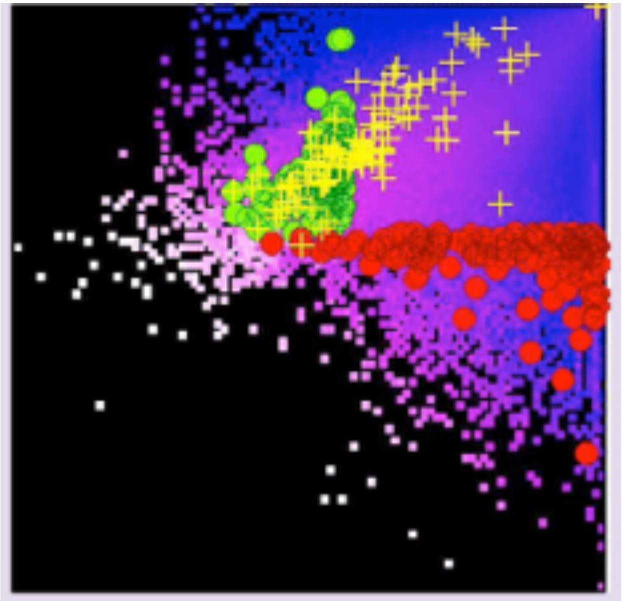


PageRank - CheiRank analysis

P-P* correlator



K-K* plane



PageRank: 1. Napoleon I of France, 2. George W. Bush, 3. Elizabeth II of the United Kingdom, 4. William Shakespeare, 5. Carl Linnaeus, 6. Adolf Hitler, 7. Aristotle, 8. Bill Clinton, 9. Franklin D. Roosevelt, 10. Ronald Reagan.

CheiRank: 1. Kasey S. Pipes, 2. Roger Calmel, 3. Yury G. Chernavsky, 4. Josh Billings (pitcher), 5. George Lyell, 6. Landon Donovan, 7. Marilyn C. Solvay, 8. Matt Kelley, 9. Johann Georg Hagen, 10. Chikage Oogi.

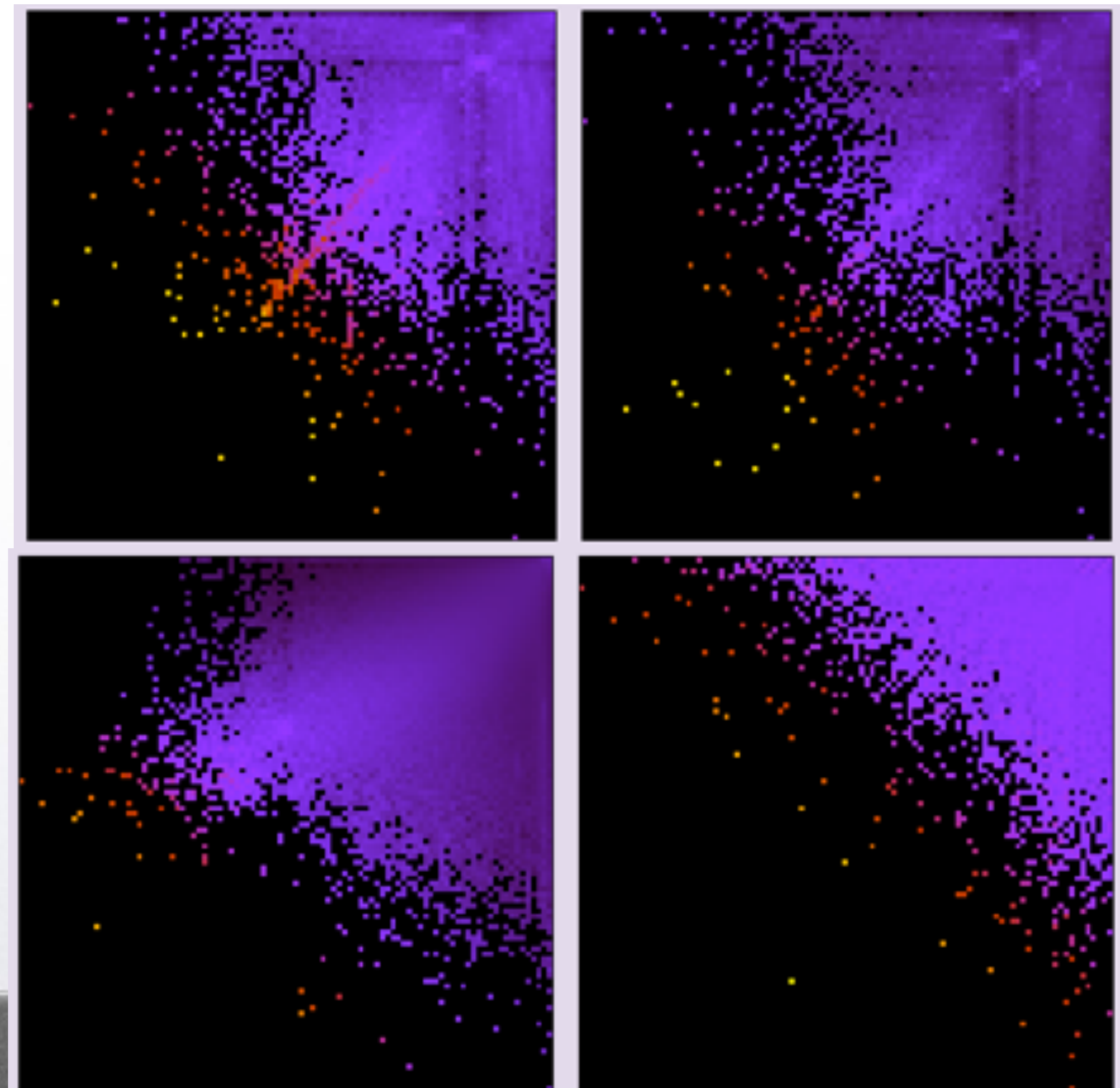
2DRank: 1. Michael Jackson, 2. Frank Lloyd Wright, 3. David Bowie, 4. Hillary Rodham Clinton, 5. Charles Darwin, 6. Stephen King, 7. Richard Nixon, 8. Isaac Asimov, 9. Frank Sinatra, 10. Elvis Presley.

Global 2-d examples

$W(K, K^*) = dN_i/dKdK^*$

Cambridge
(2006)
N~2.10^5

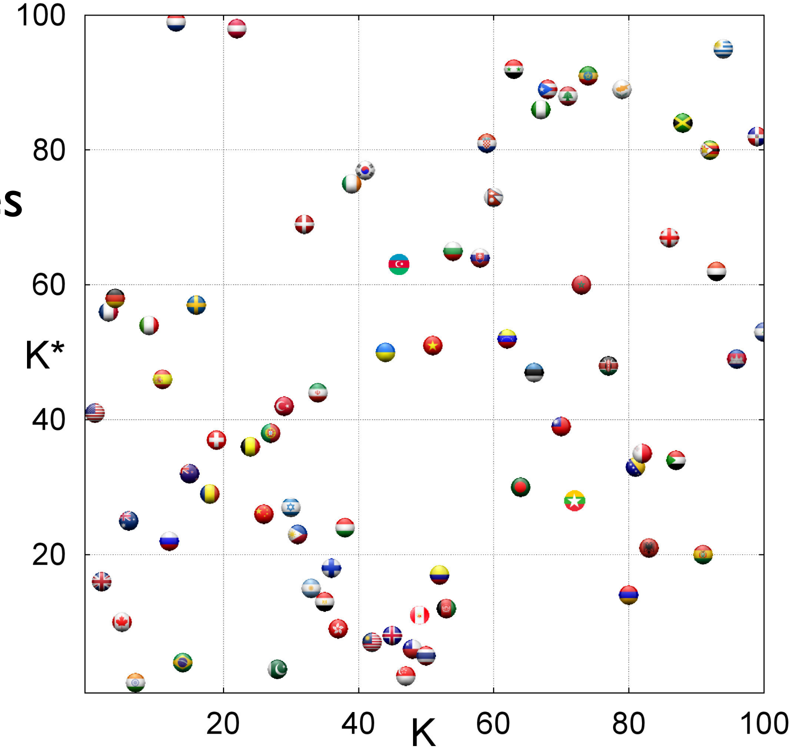
Oxford
(2006)
N~2.10^5



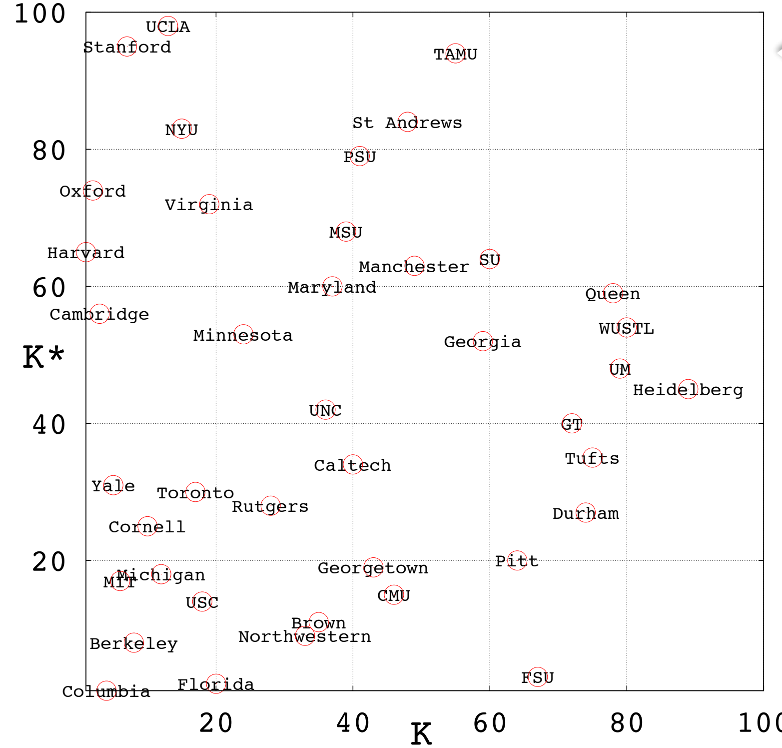
wikipedia
N~2.10^6

Linux Kernel
V2.4
N~3.10^5

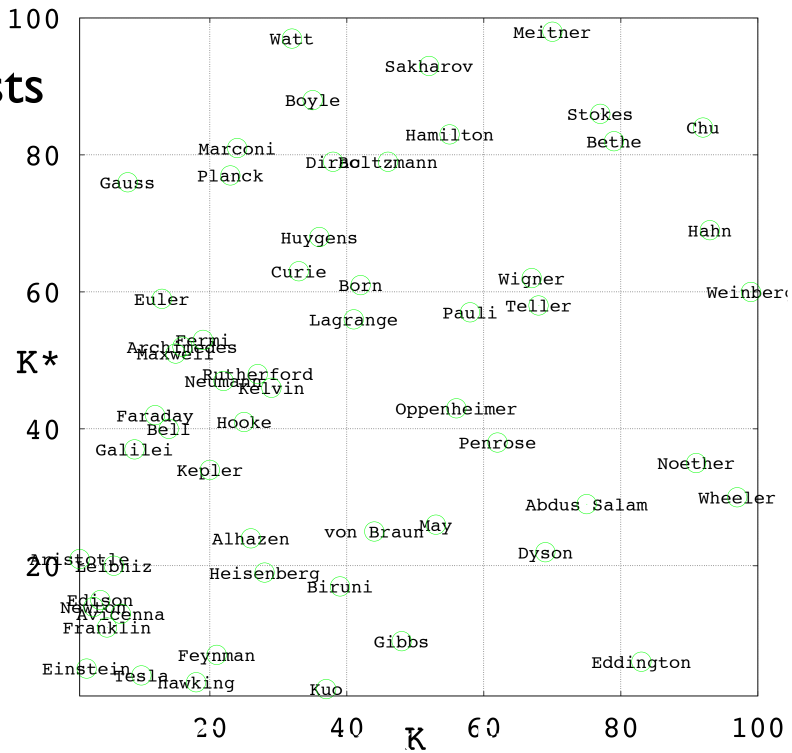
countries



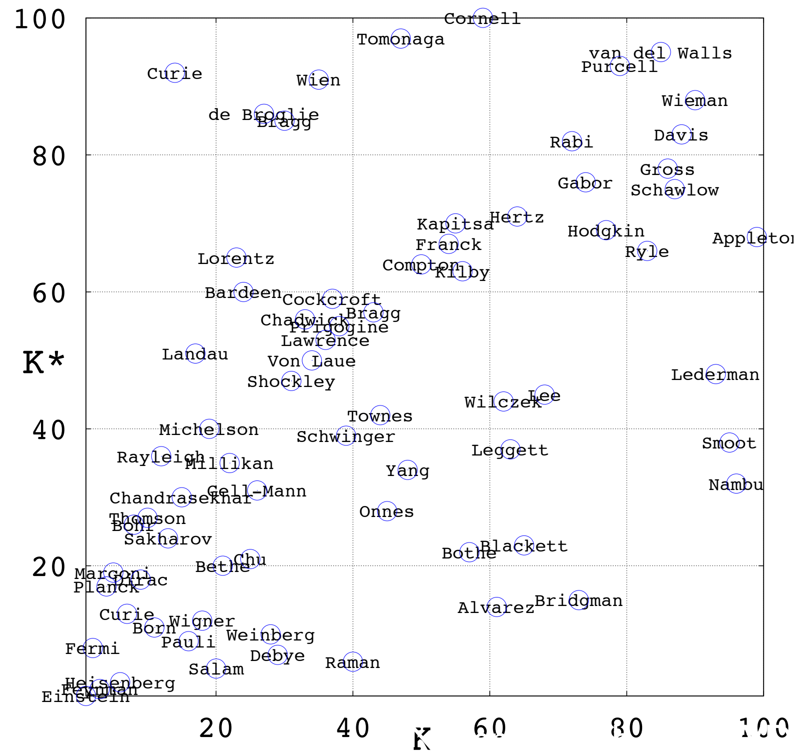
universities



physicists



Nobel laureates in physics



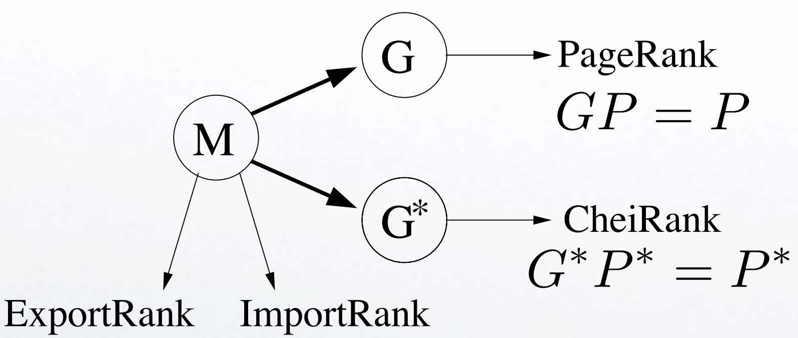
Google Matrix of the WTN

United Nation Commodities Trade Network

all countries of UN, from 1962 to 2011, all commodities or some specific products

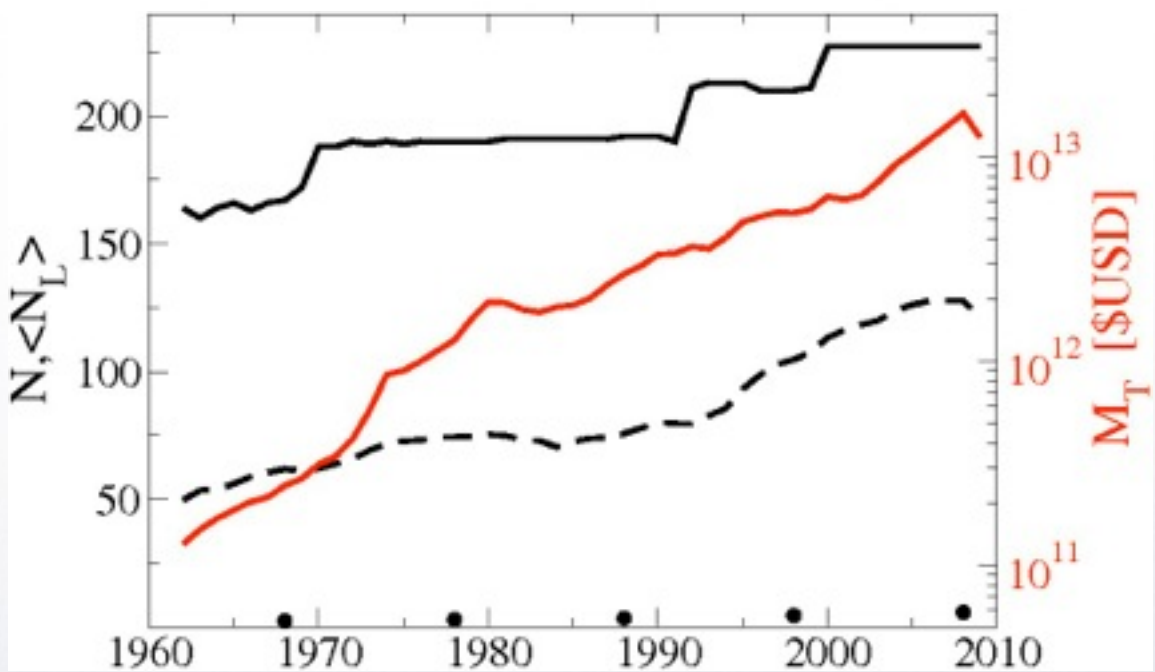
Money Matrix

$$M_{i,j} = U\$S(j \to i)$$



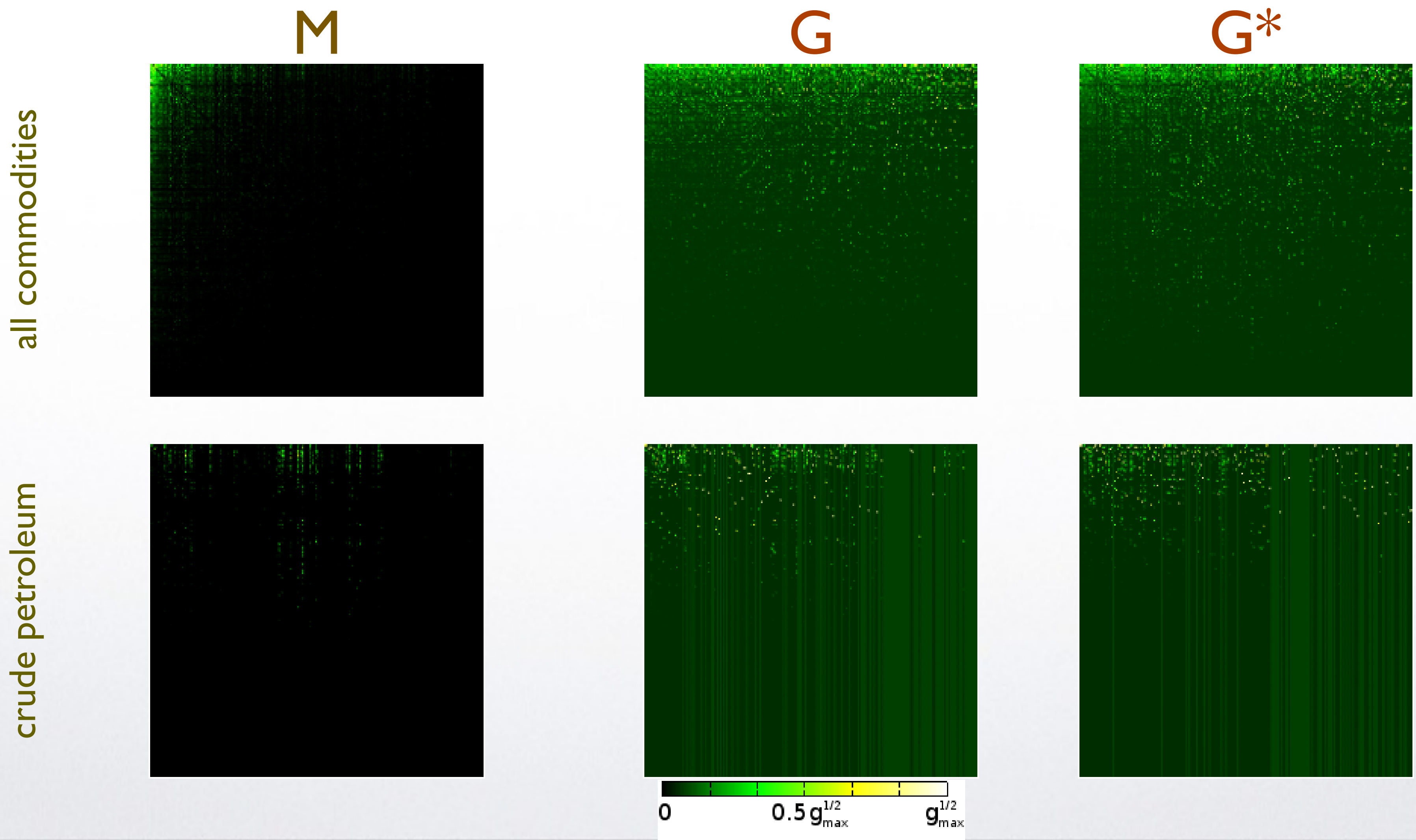
$$(\tilde{K}, \tilde{K}^*) \qquad (K, K*)$$

ImportRank, ExportRank   PageRank, CheiRank



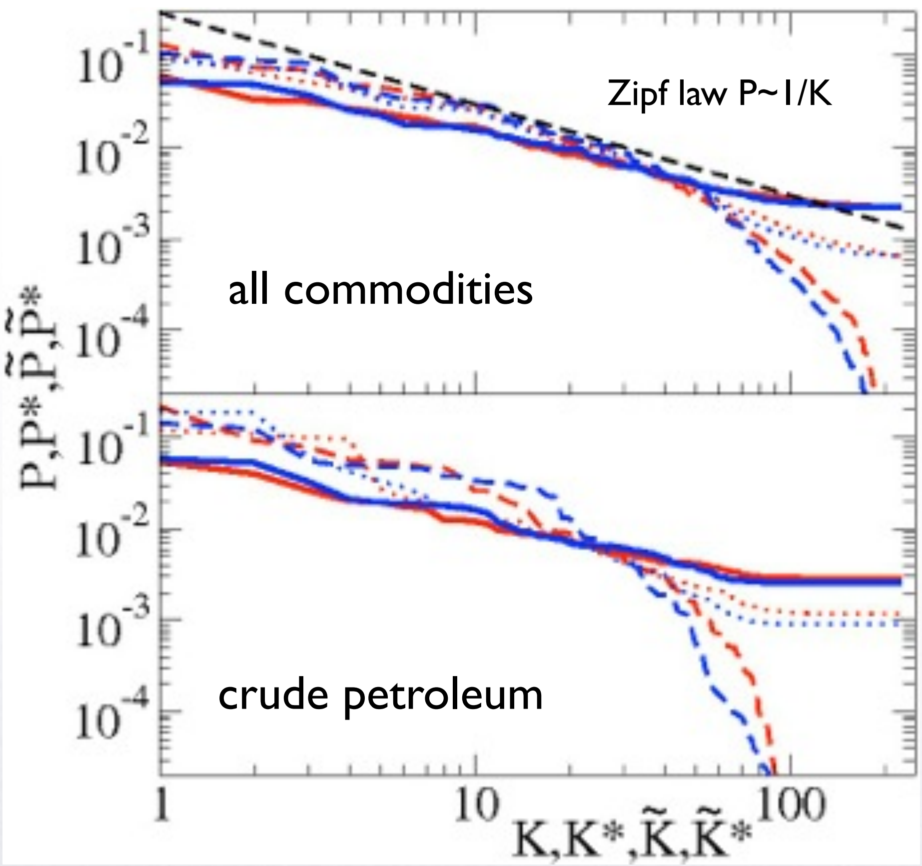
L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011), arXiv:1103.5027, http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank

G and G* matrices of the WTN (2008)

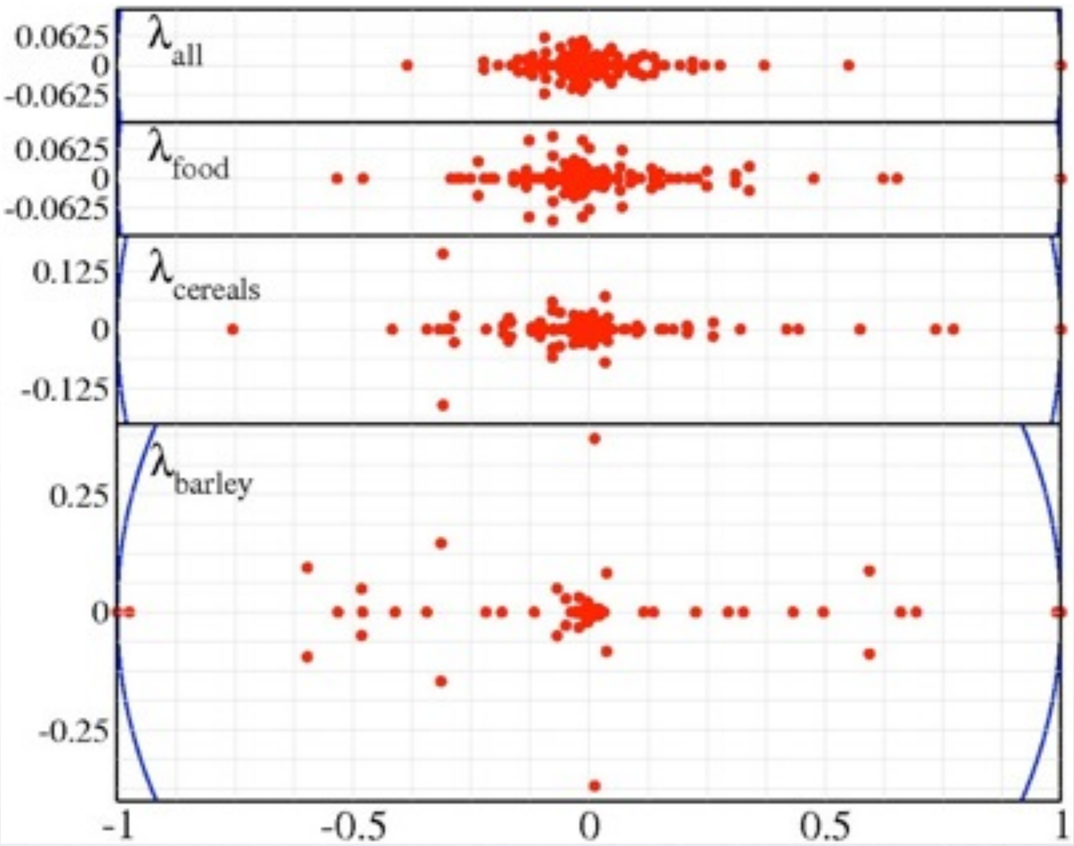


PageRank, CheiRank and Spectrum

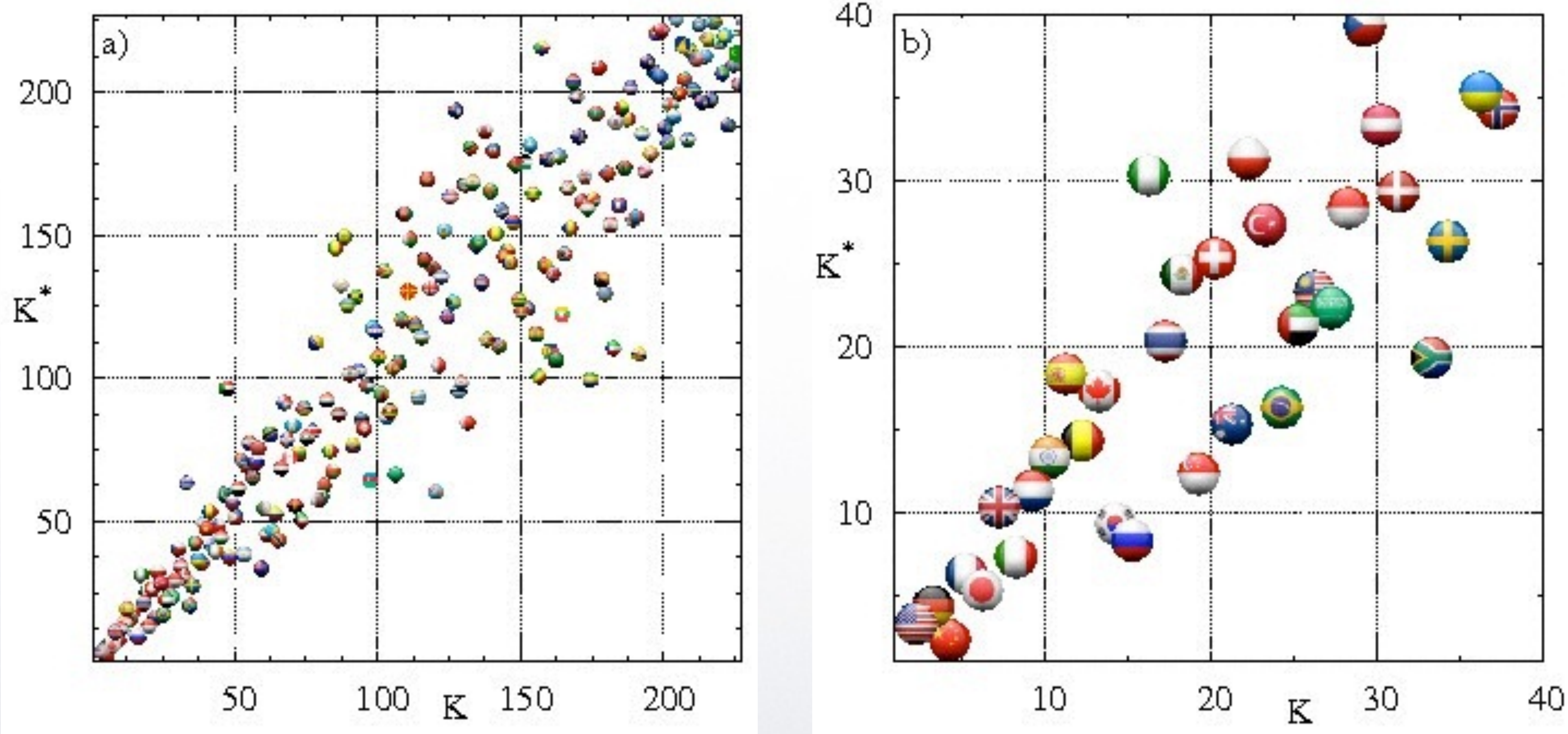
PageRank, CheiRank, ImportRank, ExportRank $\alpha = 0.85$ $\alpha = 0.5$



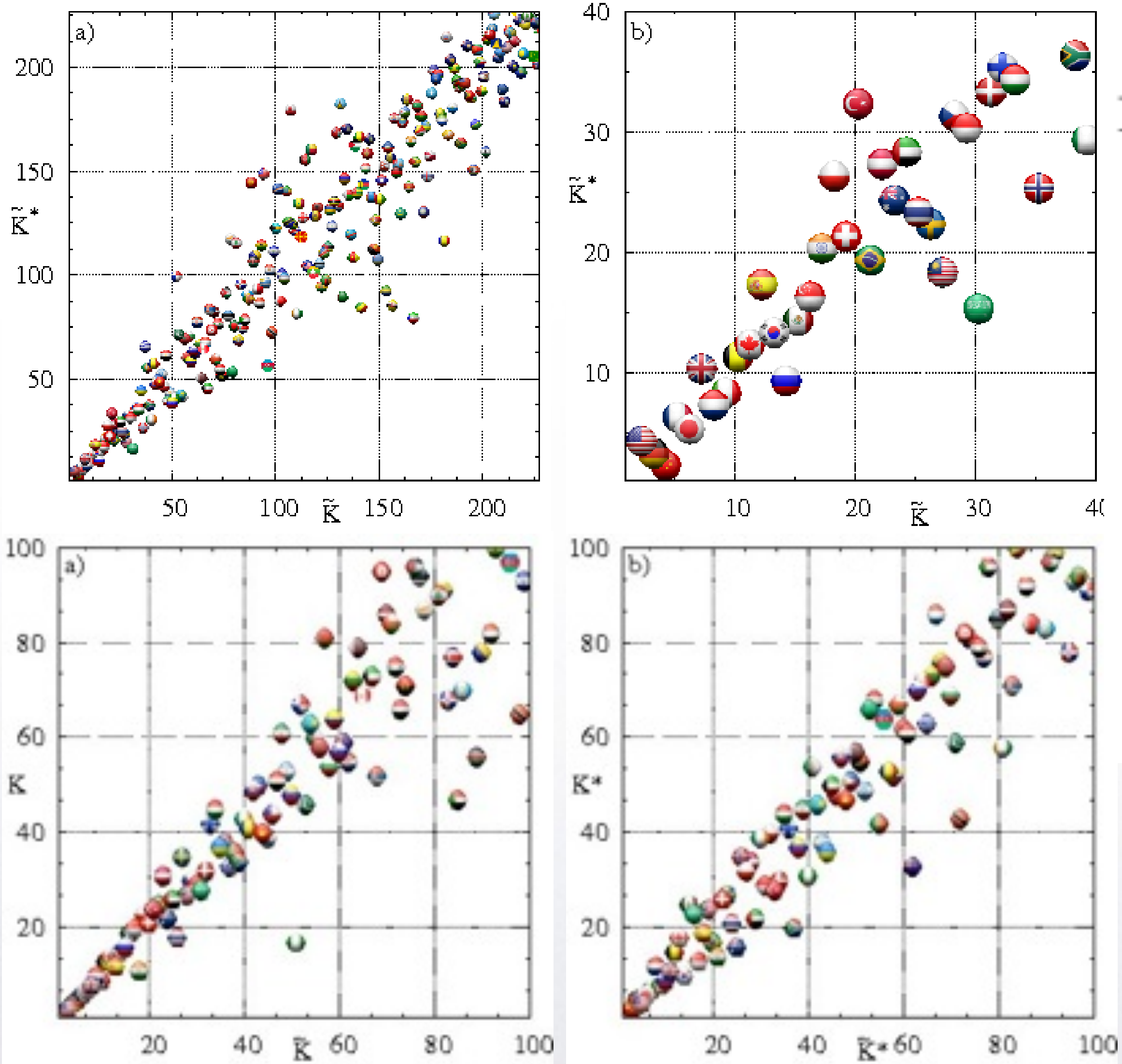
Spectra $\alpha = 1$



2d ranking “all commodities”



PageRank, CheiRank vs. ImportRank, ExportRank



countries are treated on equal democratic ground

G-20 ~ 74%

Ran	K	K^*	K_2	$\tilde{K}$	$\tilde{K}^*$
1	USA	China	USA	USA	China
2	Germany	USA	China	Germany	Germany
3	China	Germany	Germany	China	USA
4	France	Japan	Japan	France	Japan
5	Japan	France	France	Japan	France
6	UK	Italy	Italy	UK	Netherlands
7	Italy	Russian Fed.	UK	Netherlands	Italy
8	Netherlands	Rep. of Korea	Netherlands	Italy	Russian Fed.
9	India	UK	India	Belgium	UK
10	Spain	Netherlands	Rep. of Korea	Canada	Belgium
11	Belgium	Singapore	Belgium	Spain	Canada
12	Canada	India	Russian Fed.	Rep. of Korea	Rep. of Korea
13	Rep. of Korea	Belgium	Canada	Russian Fed.	Mexico
14	Russian Fed.	Australia	Spain	Mexico	Saudi Arabia
15	Nigeria	Brazil	Singapore	Singapore	Singapore
16	Thailand	Canada	Thailand	India	Spain
17	Mexico	Spain	Australia	Poland	Malaysia
18	Singapore	South Africa	Brazil	Switzerland	Brazil
19	Switzerland	Thailand	Mexico	Turkey	India
20	Australia	U. Arab Emir.	U. Arab Emir.	Brazil	Switzerland

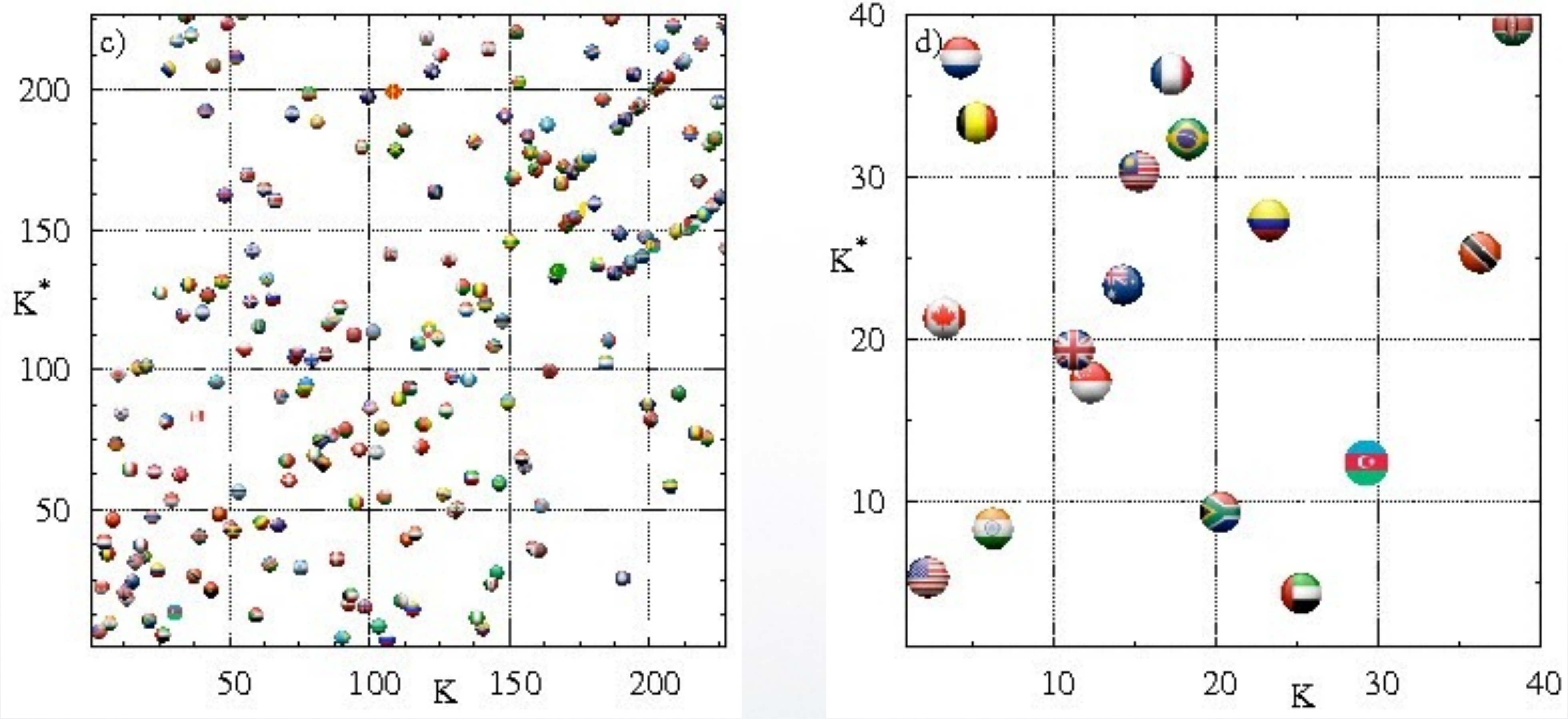
$$\tilde{K}^* = 11 \longrightarrow K^* = 16$$

$$\tilde{K}^* = 13 \longrightarrow K^* > 20$$

$$\tilde{K}^* = 15 \longrightarrow K^* = 11$$

$$\tilde{K}^* = 19 \longrightarrow K^* = 12$$

2d ranking: crude petroleum

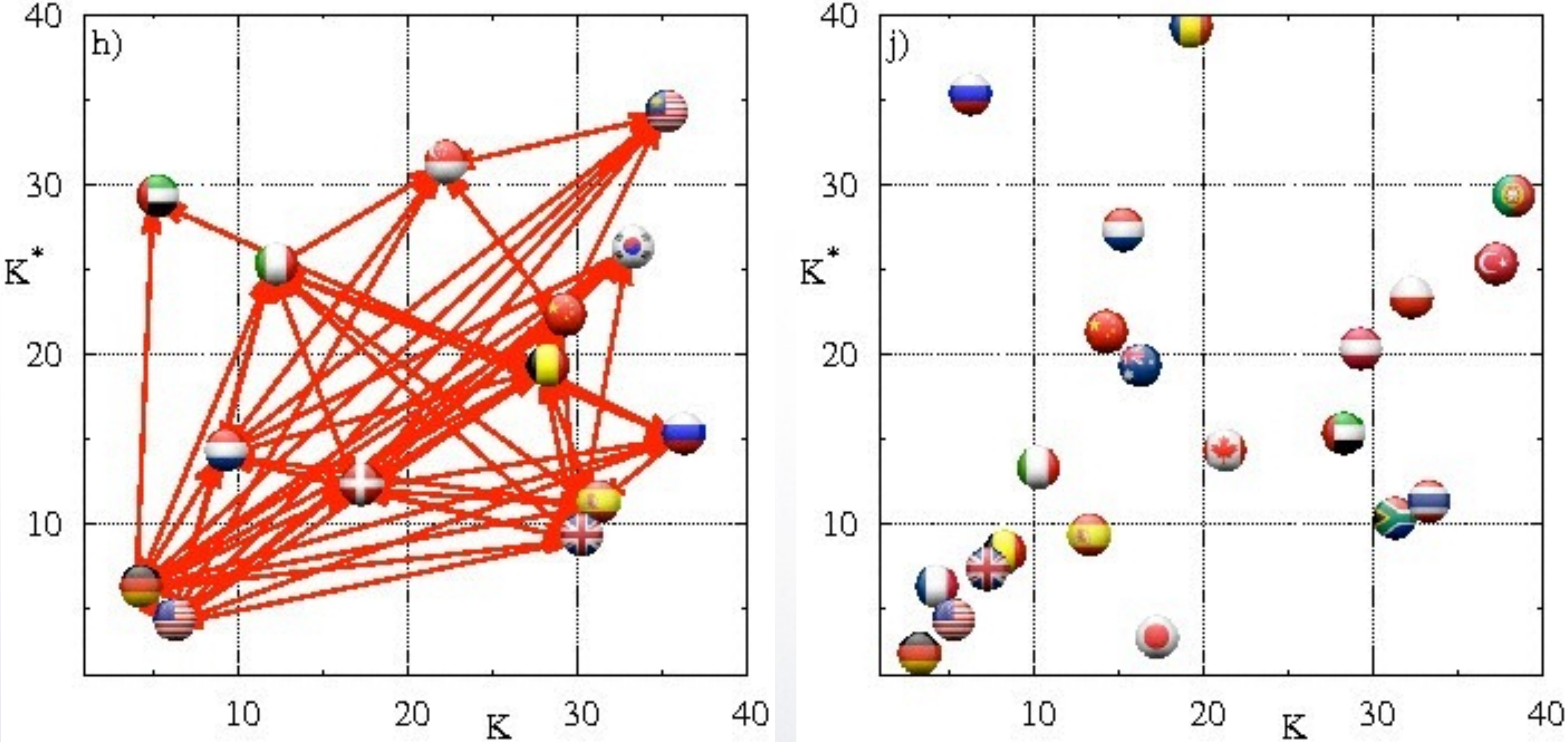


Russia $\tilde{K}^* = 2 \rightarrow K^* = 1$ its trade network in this product is better and broader than the one of Saudi Arabia (1st)

Iran $\tilde{K}^* = 5 \rightarrow K^* = 14$ its trade network is restricted to a small number of nearby countries.

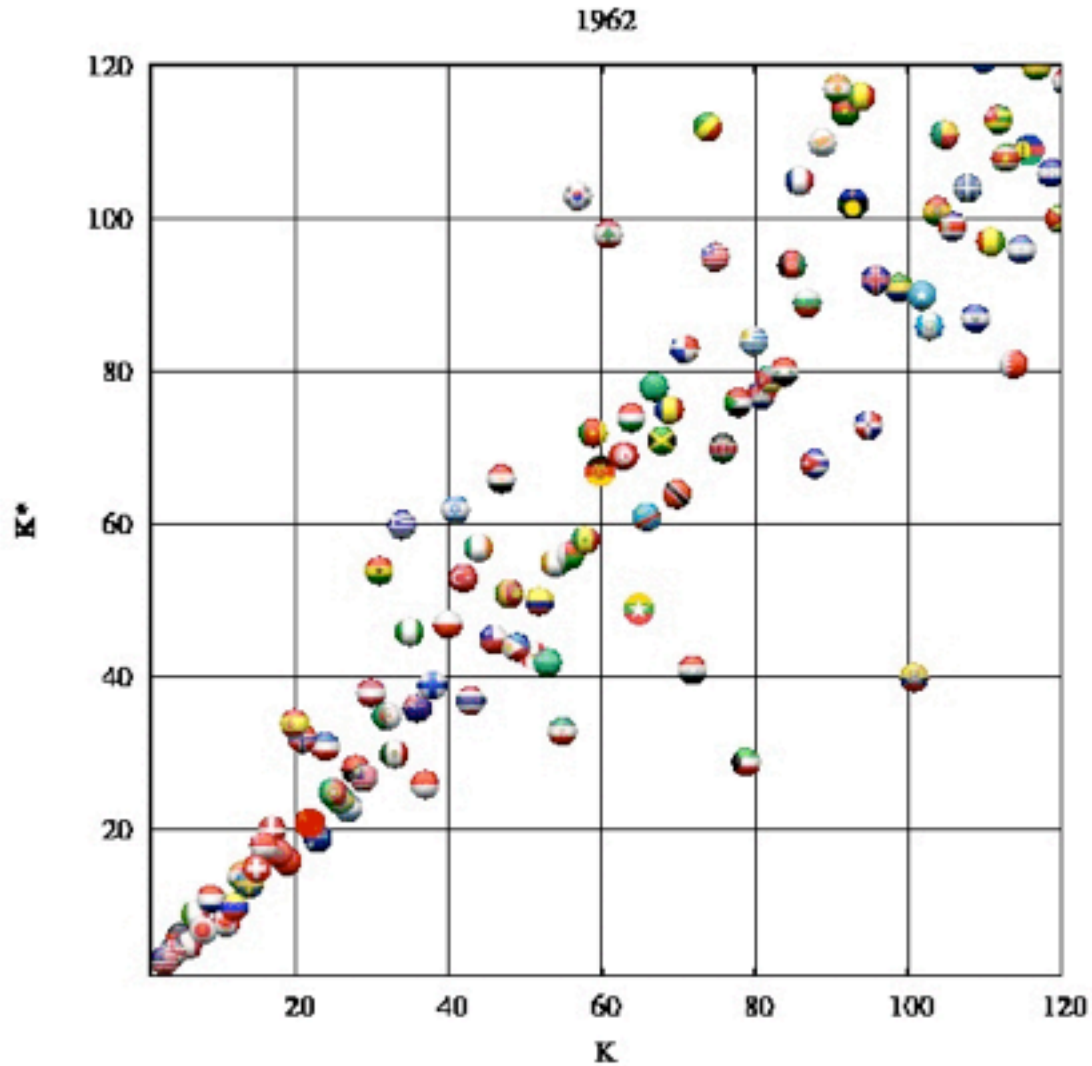
Kazakhstan $\tilde{K}^* = 12 \rightarrow K^* = 2$ is practically the only country which sells crude petroleum to the CheiRank leader in this product Russia.

2d ranking: barley and cars



Ukraine (K* 1st to 6th) USA (K* 8th to 3rd) France (K* 7th to 3rd) Thailand (K* 19th to 10th)

2d ranking evolution

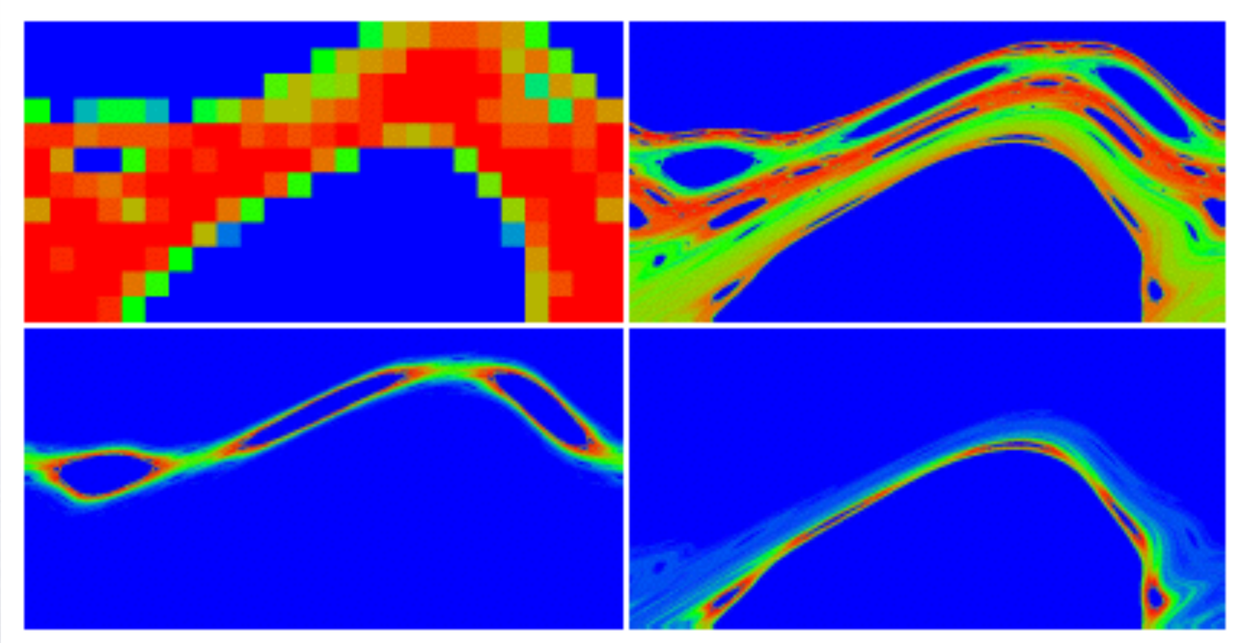


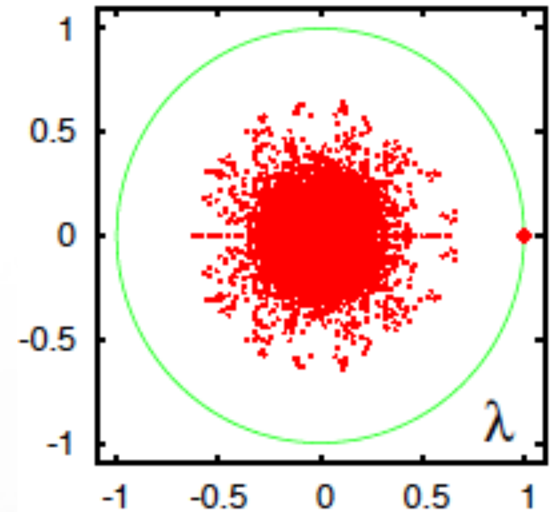
Other applications of Google Matrix

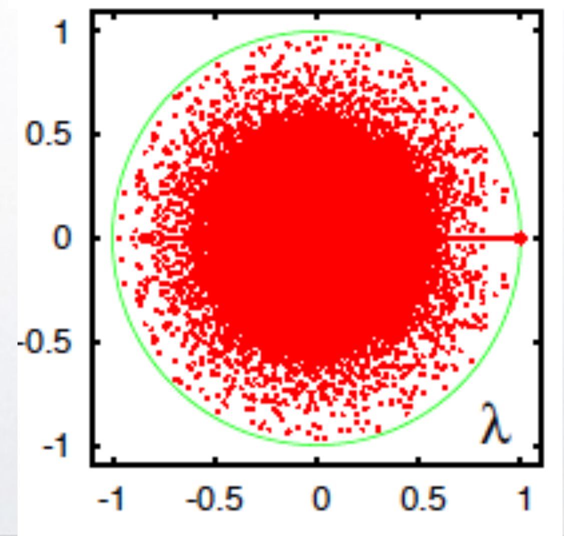
Ulam Networks

Chirikov standard map

$$\boxed{\begin{aligned}\bar{p} &= p + K \sin x \\ \bar{x} &= x + \bar{p}\end{aligned}}$$

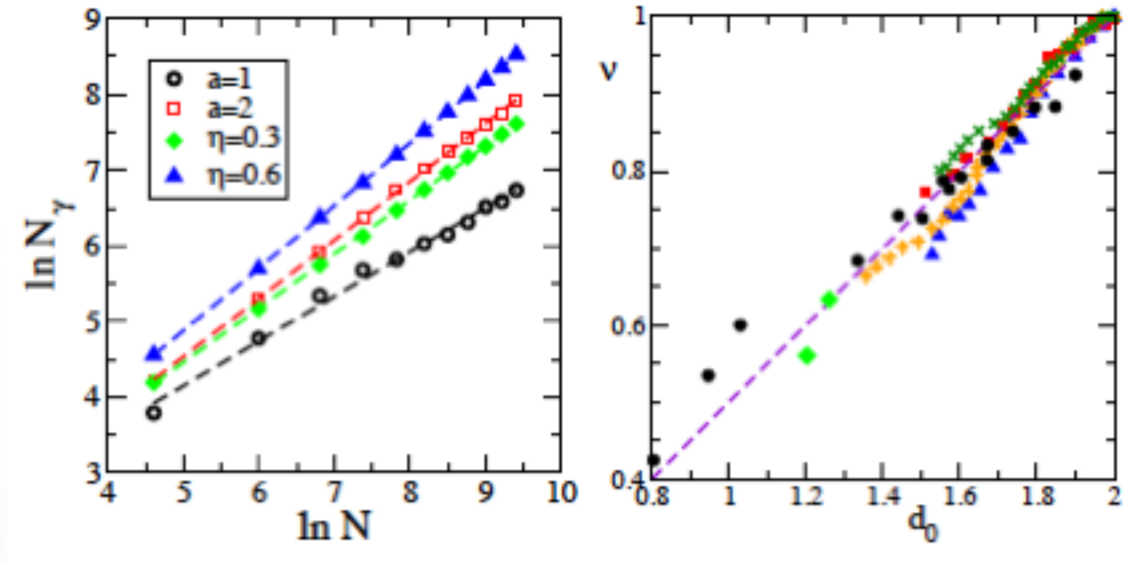




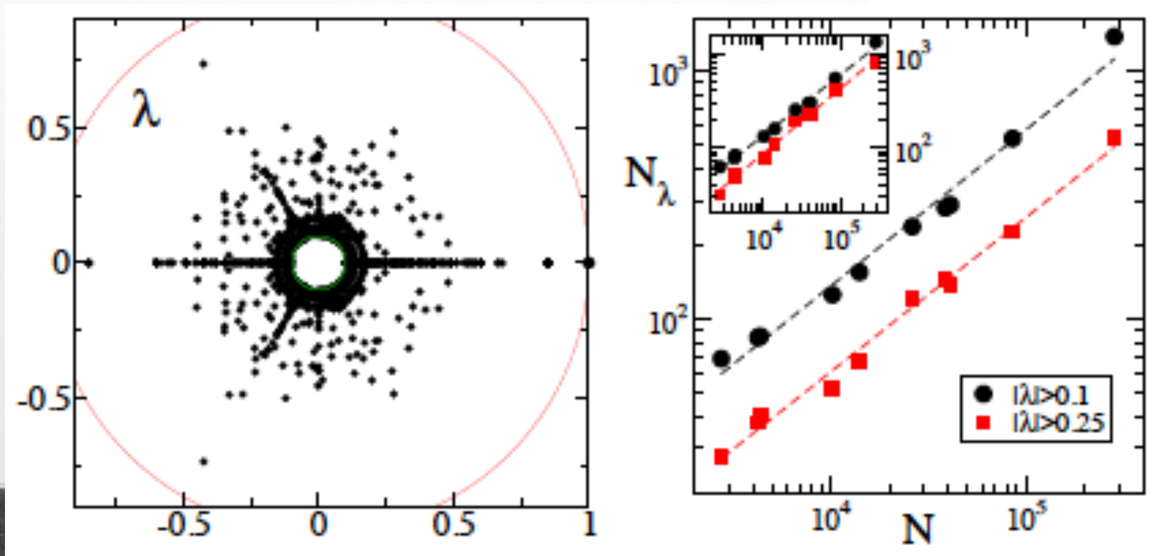


Fractal Weyl law

Standard map



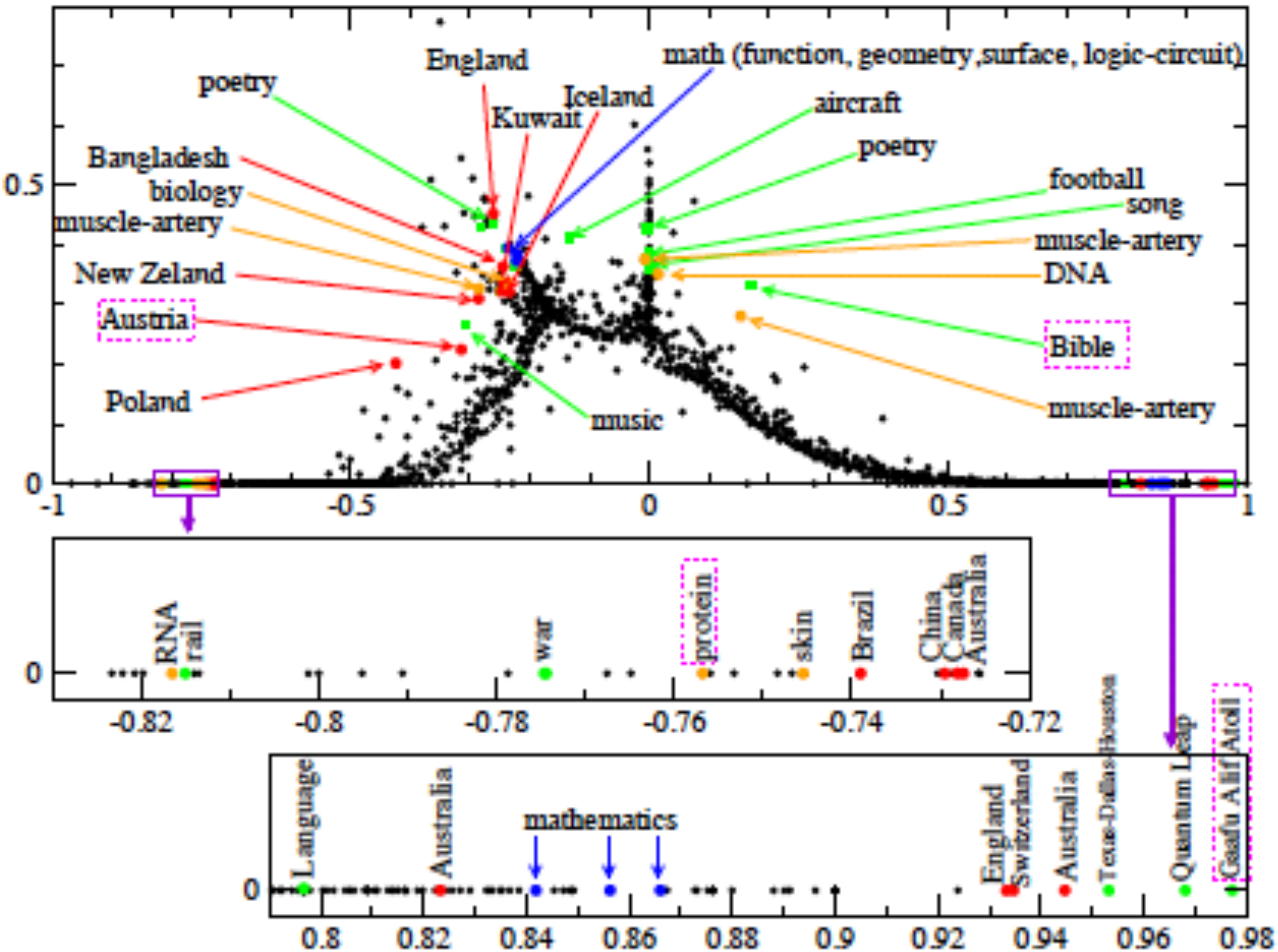
L.E and Shepelyansky, Eur. Phys. J. B 75, 299 (2010)



Linux Kernel architecture network

L.E, Chepelianskii and Shepelyansky, Eur. Phys. J. B 79, 115 (2011)

Wikipedia communities in G eigenstates



L.E, Frahm and Shepelyansky, Eur. Phys. J. B 86, 193 (2013)

Conclusions

- Google Matrix analysis of different networks (www: wikipedia, universities, softwares, etc.)
- CheiRank and 2-d rankings
- WTN example
- Other applications:
  - Ulam Networks
  - Fractal Weyl law in maps and Linux Kernel
  - Communities in Wikipedia

Thank You