When Schelling met Michael Jackson

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Segregation Models: Schelling

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DYNAMIC MODELS OF SEGREGATION†

THOMAS C. SCHELLING

Harvard University

If blacks exclude whites from their church, or whites exclude blacks, the segregation is organized, and it may be reciprocal or one-sided. If blacks just happen to be Baptists and whites Methodists, the two colors will be segregated Sunday morning whether they intend to be choot. If blacks join a black church because they are more comfortable among their ow, color, and whites a white church for the same reason, undirected individual choice can lead to segregation. And if the church bulletin board is where people advertise cooms for rent, blacks will rent rooms from blacks and whites from whites because of a communication system that is correlated with churches that are correlated with color.

discriminatory behavior \rightarrow awareness, conscious or unconscious, of sex or age or color that influences decisions: • where to live; • whom to sit by; • what occupation to join or to avoid; • whom to play with; • whom to talk to. individual residential preferences \rightarrow segregated neighborhood patterns. modest levels of racial oreferences \rightarrow high levels of foldal segregation. $\langle \Box \rangle \langle \Box \rangle \langle \Xi \rangle \langle \Xi \rangle \langle \Xi \rangle \langle \Xi \rangle$

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original model

- Agents are black or white, and they are located in a square lattice with a proportion of empty sites.
- An agent is unhappy when the proportion of agents of the other color among its neighbors is greater than his tolerance (T).
- Unhappy agents will move to an acceptable empty place if they can find one.



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Opinion and Language Dynamics Models

Voter model

- Holley et al. (1975): two neighboring agents are selected at random, and one of them changes its opinion as a result of the interaction.
- interaction: agent follows the opinion of the majority.
- For regular lattices it is well known that one of the opinions prevails and the other disappears.
- extensively studied by researchers from complex systems and statistical mechanics.

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Language model

- Nettle (Lingua, 1999) considered several agents interacting in a network, and changing their languages according to certain rules.
- Abrams and Strogatz (Nature, 2003): approach based on ordinary or partial differential equations.
- Pinasco and Romanelli (Phy.A 2006): possibility of coexistence of two languages by including resources for each population as in Lotka-Volterra models.

Schelling-Voter model:

- An agent is unhappy when the ratio between the number of agents of the opposite language and the number of agents in his neighborhood is greater than *T*. An unhappy agent can play one of two different games:
- (Schelling) The agent tries to change his location (with probability *p*).
- (Voter) The agent tries to change his color (with probability 1 − p).



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Model parameters:

- $1 \rho \in (0, 1)$ is the *density* of empty sites.
- $T \in [0, 1]$ is the *tolerance*.
- $p \in [0, 1]$ is the possibility to migrate
- $s \in (0, 1)$ is the prestige of b, which defines $q_{b \to w} = s \frac{W}{(B+W)}$ and $q_{w \to b} = (1-s) \frac{B}{(B+W)}$ W = total agents w

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B = total agents b

Schelling-Voter Model: results

Model parameters:

- ρ ∈ (0, 1) is the *density* of empty sites.
- T ∈ [0, 1] is the tolerance.
- s ∈ (0, 1) is the prestige of b.
- p ∈ [0, 1] is the possibility to migrate.

Simulations:

- \bullet start with two similar populations in a square lattice of side L=50.
- free boundary conditions.
- the same prestige s = 0.5 to both languages.

Size of the population speaking the minority language as a function of $1 - \rho$ and as a function of p and T = 0.3



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Schelling-Voter Model: empty spaces



Available empty spaces for the populations.

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Schelling-Voter Model: Phase space



Below red symbols $\langle S \rangle < 0,1$; between red and green symbols: $0,1 < \langle S \rangle < 0,45$; beyond green symbols: $\langle S \rangle > 0,45$ *p* is high when the difficulty to learn the other language is high or when it is easy to move to other sites http://www.unesco.org/new/en/culture/themes/endangered-languages.

 $\label{eq:chicago} Chicago, data from Census 2010. Base map © OpenStreetMap, CC-BY-SA Eric Fischer, Race and ethnicity (2010) http://www.flickr.com/photos/walkingsf/sets/$

Red is White, Blue is Black, Green is Asian, Orange is Hispanic, Yellow is Other, and each dot is 25 residents.



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US Census Data: US Census Bureau (2010). http://www.census.gov/hhes/socdemo/language/data/acs/appendix.htm

...almost 30 percent of the population older than 5 years who speak Spanish, Chinese, or Korean languages at home, speak English *not well* or *not at all*..

Segregation by ethnicity observed in the maps is also segregation by language.



Lewis, M. Paul, *Ethnologue: Languages of the world, sixteenth edition*, Dallas, TX, USA: SiL International. Online version: http://www. ethnologue. com (2009).

Nigeria has more than 500 languages belonging to at least three main families (Nilo-Saharan, Afro-Asiatic, and Niger-Congo), with around 169 million people, and 950.000 km^2 ; its density of inhabitants is higher than the one of the European Union.





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Ghana has several languages, two main families (Gur, Kwa). 30 different languages coexist in each zone around the bifurcation of the Black and White Volta Rivers.



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Lewis, M. Paul, *Ethnologue: Languages of the world, sixteenth edition*, Dallas, TX, USA: SiL International. Online version: http://www. ethnologue. com (2009).

Niger has a very low population density (around 12 persons per km²), several different languages coexist



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Related work

• (2006) D. Zanette and S. Gil, *Opinion spreading and agent* segregation on evolving networks (Physica D) stochastic model where the distribution of opinions in a population of agents coevolves with their interaction network. The system evolves towards a state where the network's structure and the opinion distribution is frozen, and the population is divided into disconnected communities.

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- (2008) F. Vazquez, V. M. Eguíluz, M. San Miguel, Generic Absorbing Transition in Coevolution Dynamics (PRL): coevolution voter model on a complex network. A mean-field approximation.

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- (2012) Durrett, Gleeson, Lloyd, Mucha, Shi, Sivakoff, Socolar, Varghese, *Graph fission in an evolving voter model* (PNAS): model of a social network in which opinions of the individuals and the network connections coevolve.

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- *Future work:* variations in the tolerance parameter (heterogeneos), more than two languages, and the dependence with the topology of the underlying network.

...I Said If You're Thinkin' Of Being My Brother It Don't Matter If You're Black Or White...

Michael Jackson

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bilingual agents

poblacion final - cambios de idioma



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nomads

poblacion final - mudanzas



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agents who never change



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Schelling-Voter Model: agents playing ...

Start movie

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