A new model for Time-Varying Graphs

Klaus Wehmuth, Artur Ziviani, and Eric Fleury

Workshop on Dynamic Networks 2013

Buenos Aires, Argentina

Dynamic Networks

Social networks

Face to face contacts networks

Brain activity networks

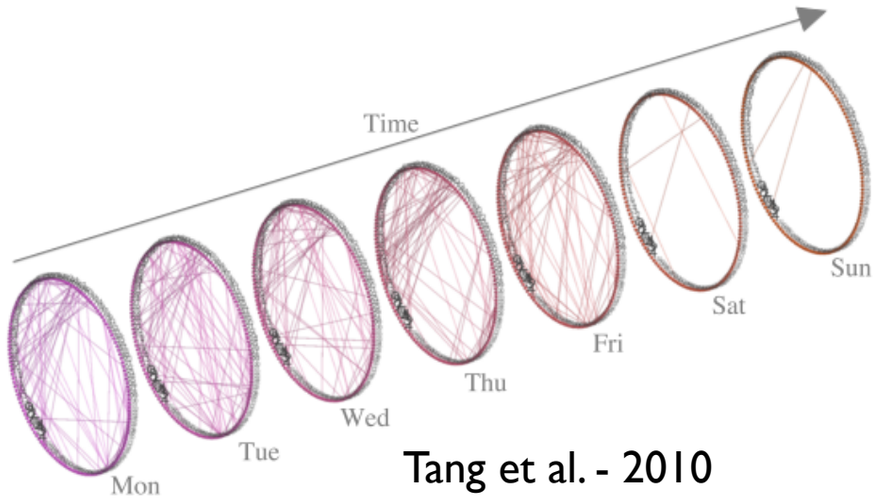
Distributed systems

...

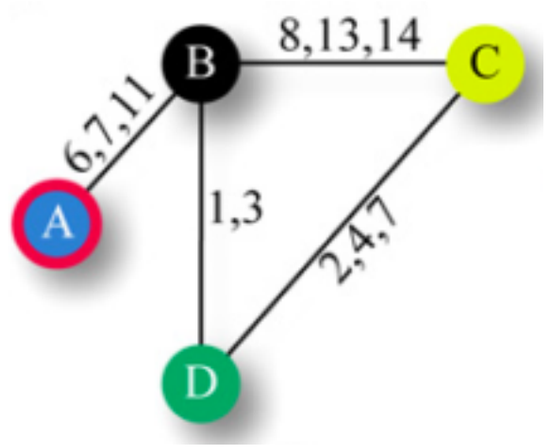
i.e., any environment where relations between objects vary in time

Dynamic Networks

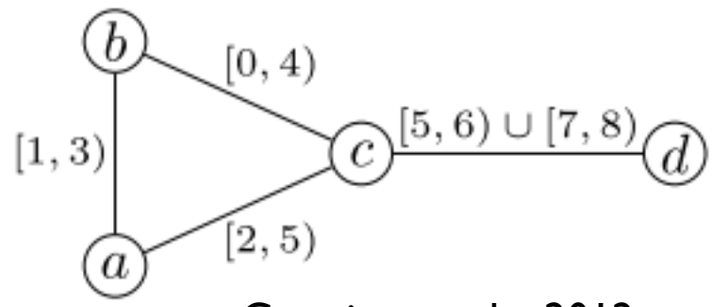
Represented by Time-Varying Graphs (TVGs)



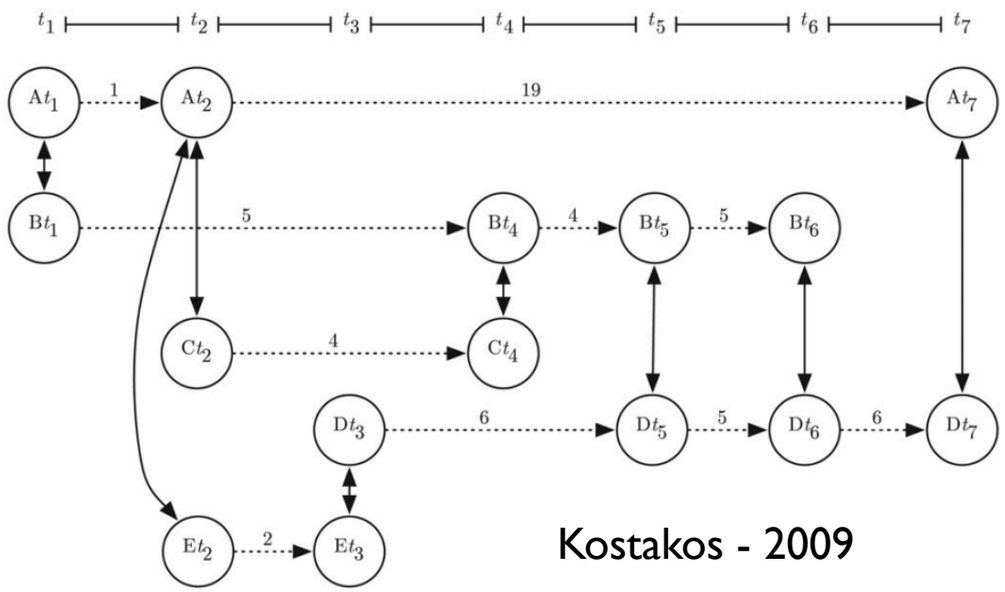
Tang et al. - 2010



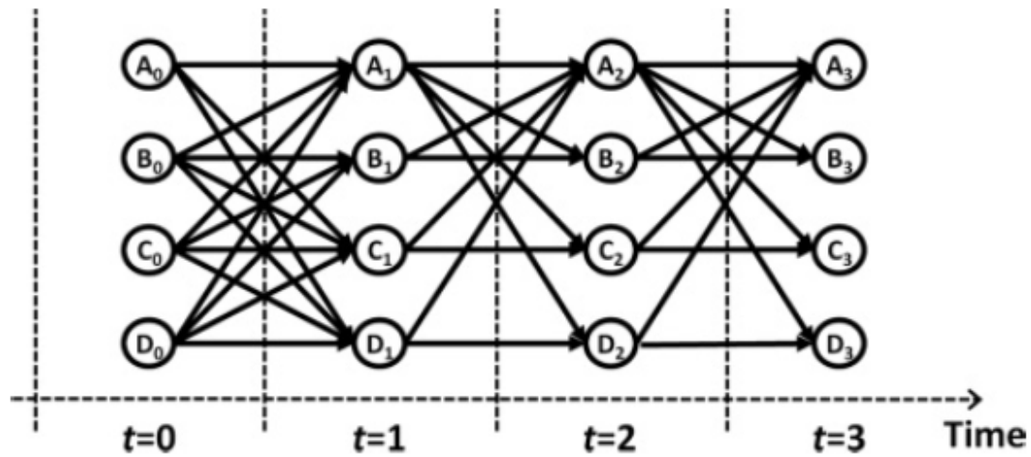
Saramäki & Holme - 2012



Casteigts et al. - 2012



Kostakos - 2009



Kim & Anderson - 2012

Time-Varying Graphs

Representation issues

Don’t look and feel like graphs

Some are not totally discrete

- use continuous time edges

Auxiliary assumptions needed

- node memory
- time interval intersections

Time-Varying Graphs

Our proposed representation model

G = (V, E, T)

V - Node set

E - Edge set

T - Time instants set

Dynamic Edges

$E \subseteq V \times T \times V \times T$

$e \in E,\ e = (n_1, t_a, n_2, t_b)$

$n_1, n_2 \in V$ - nodes $\quad$ $t_a, t_b \in T$ - time instants

A dynamic edge expresses a relation between two nodes at two time instants

Dynamic Edges

$E \subseteq V \times T \times V \times T$

$e \in E,\ e = (n_1, t_a, n_2, t_b)$

Are represented by an ordered quadruple

Can be represented by an entry on a 4th order tensor

Temporal Nodes

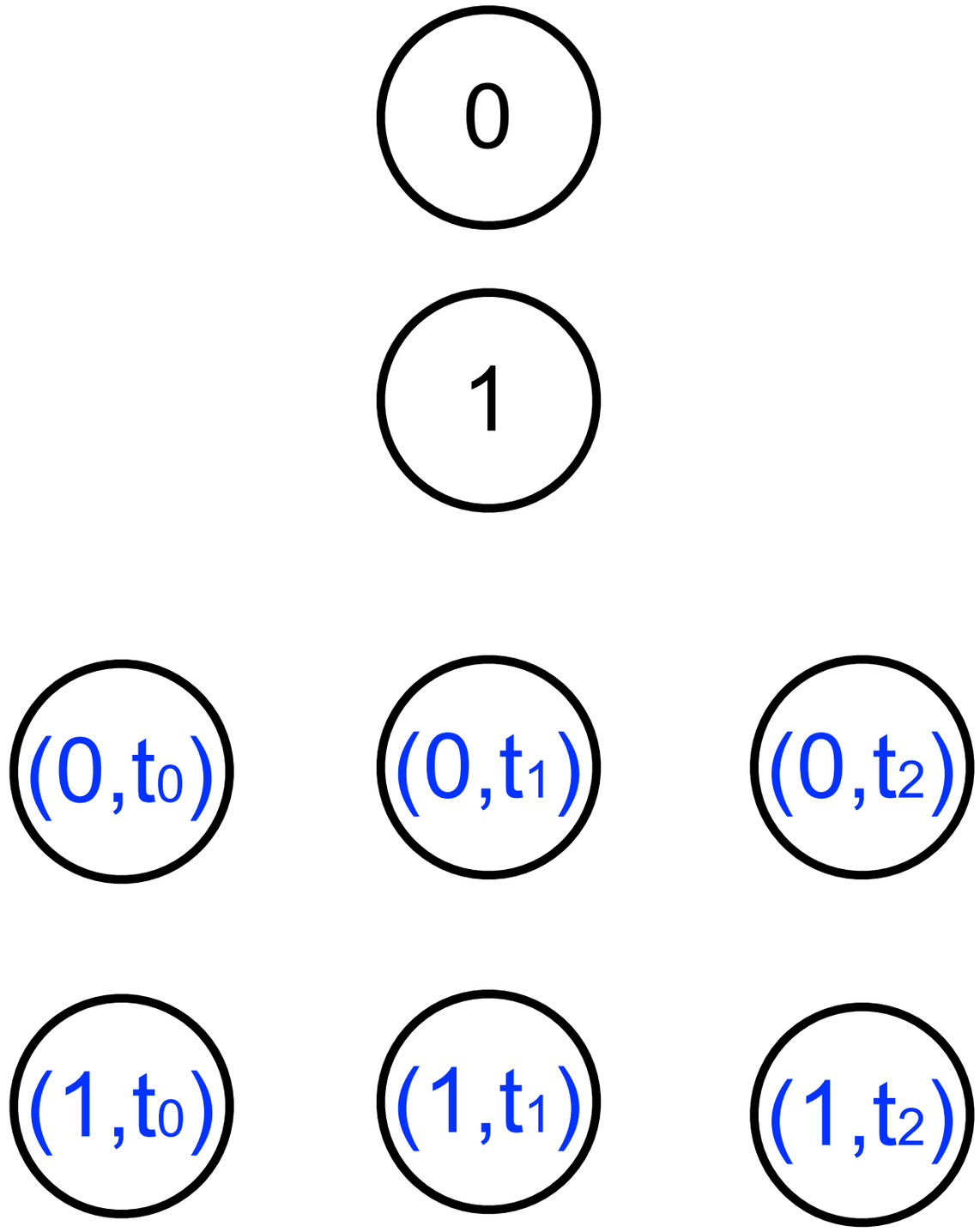
$G = (V,E,T)$

$V = \{0, 1\}$

$T = \{t_0, t_1, t_2\}$

$e = (\underbrace{n_1, t_a}_{\mathbf{u}}, \underbrace{n_2, t_b}_{\mathbf{v}})$

$\mathbf{u},\mathbf{v} \in V_s = V \times T$



Temporal Node Representation

$G = (V,E,T)$

$e = (\underbrace{n_1, t_a}_{\mathbf{u}}, \underbrace{n_2, t_b}_{\mathbf{v}})$

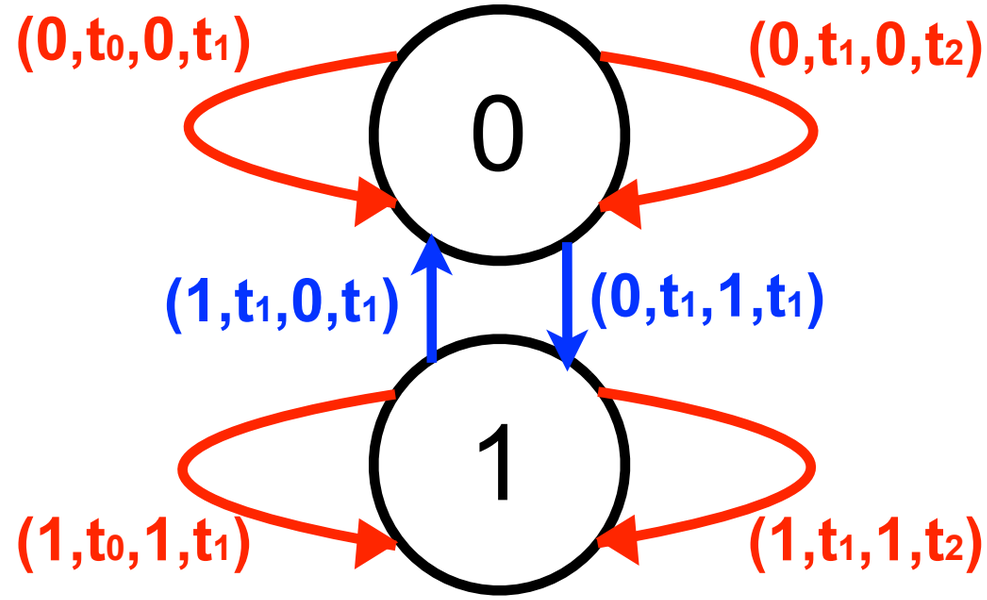
$\mathbf{u},\mathbf{v} \in Vs = V \times T$

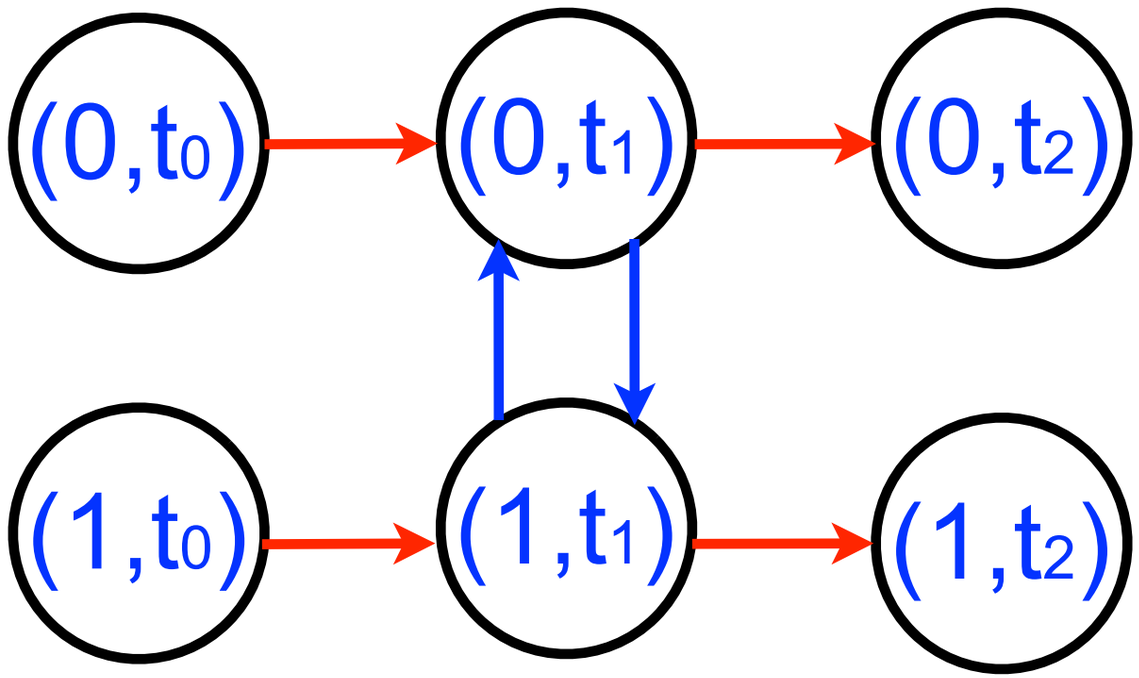
$e_s = (\mathbf{u},\mathbf{v}) \in Vs \times Vs = Es$

$Gs = (Vs, Es)$

$\Omega: V \times T \rightarrow Vs$

$e_s \in Es \longleftrightarrow e \in E$



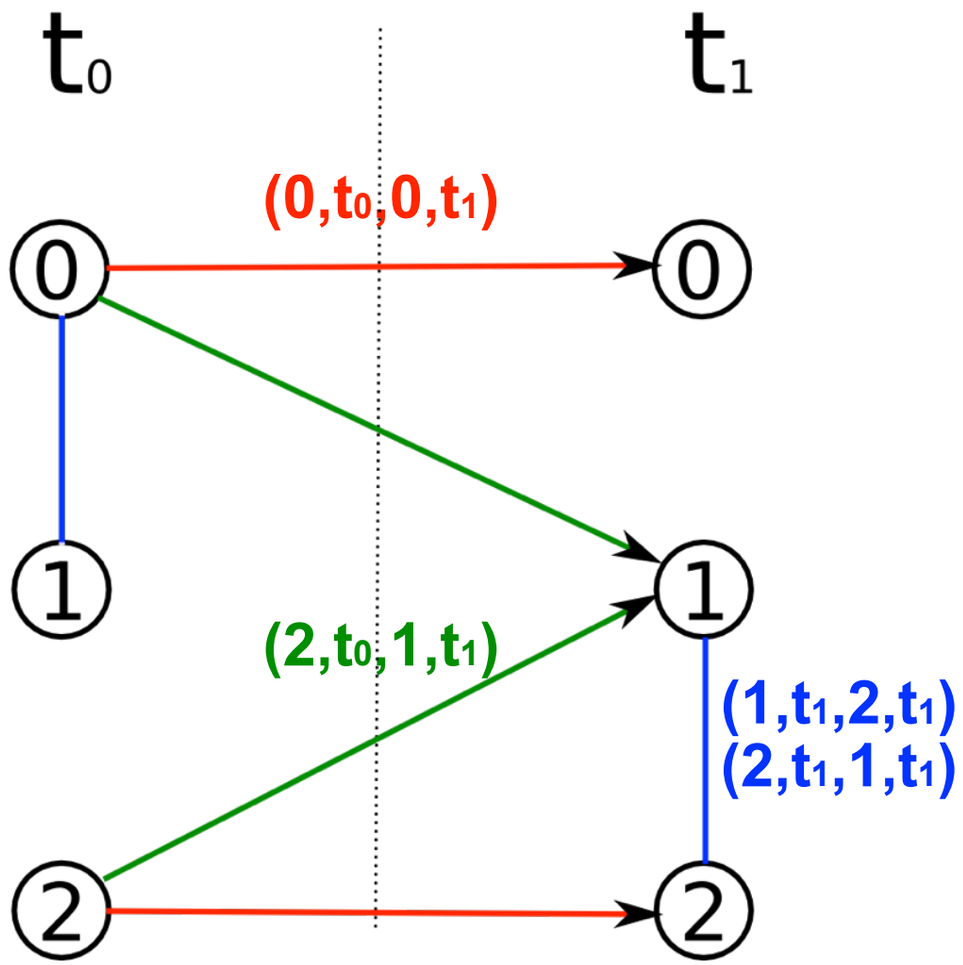


Dynamic Edges

Temporal Edges

Spatial Edges

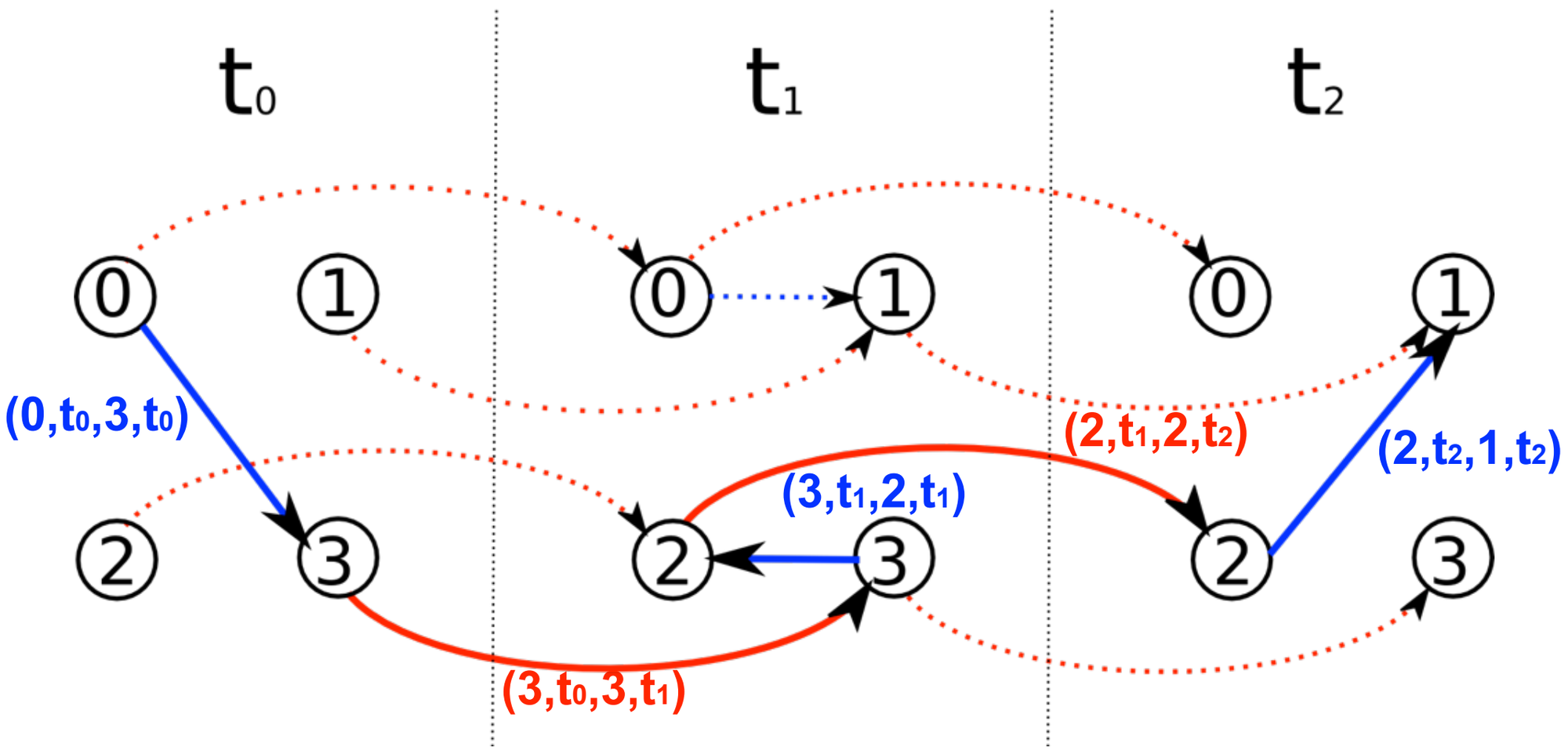
Mixed Edges



TVG Paths

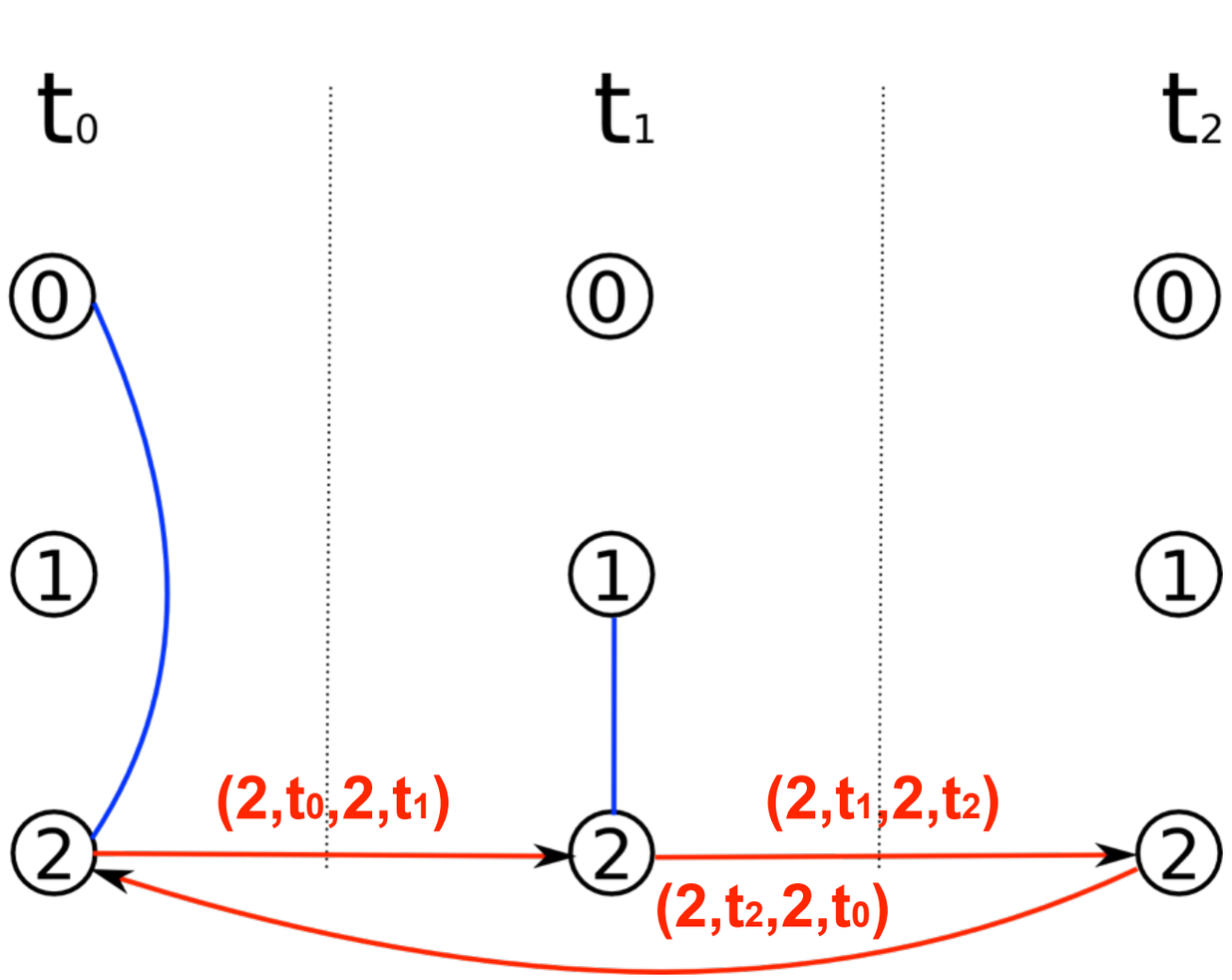
Paths are established by dynamic edge adjacency

Two dynamic edges are adjacent if they are incident to a common temporal node

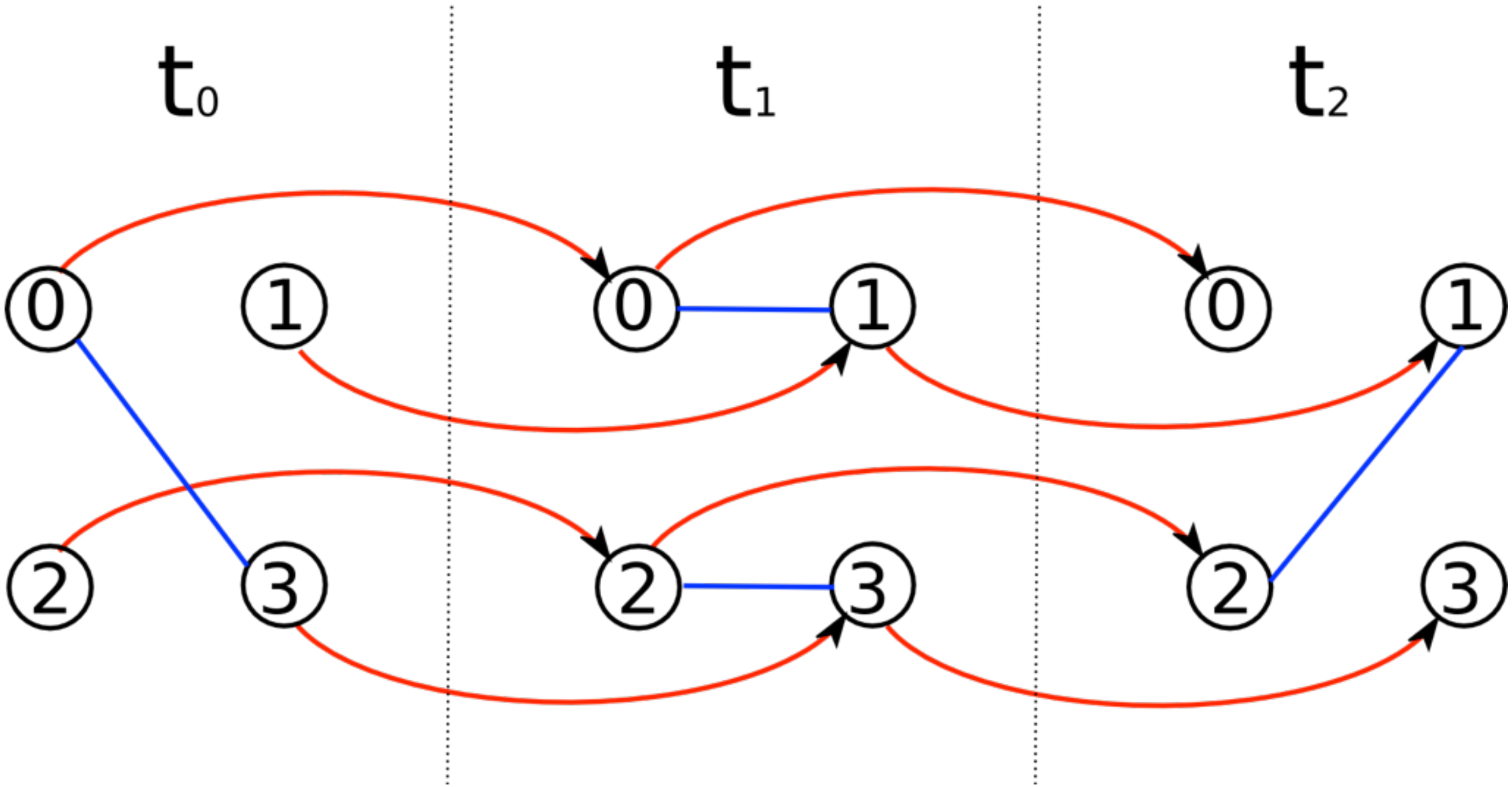


TVG Cycles

A cycle is a closed path, i.e. a path that starts and ends on the same temporal node

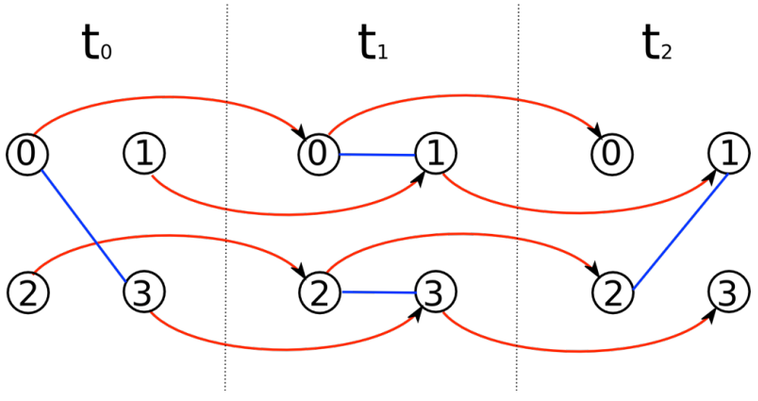


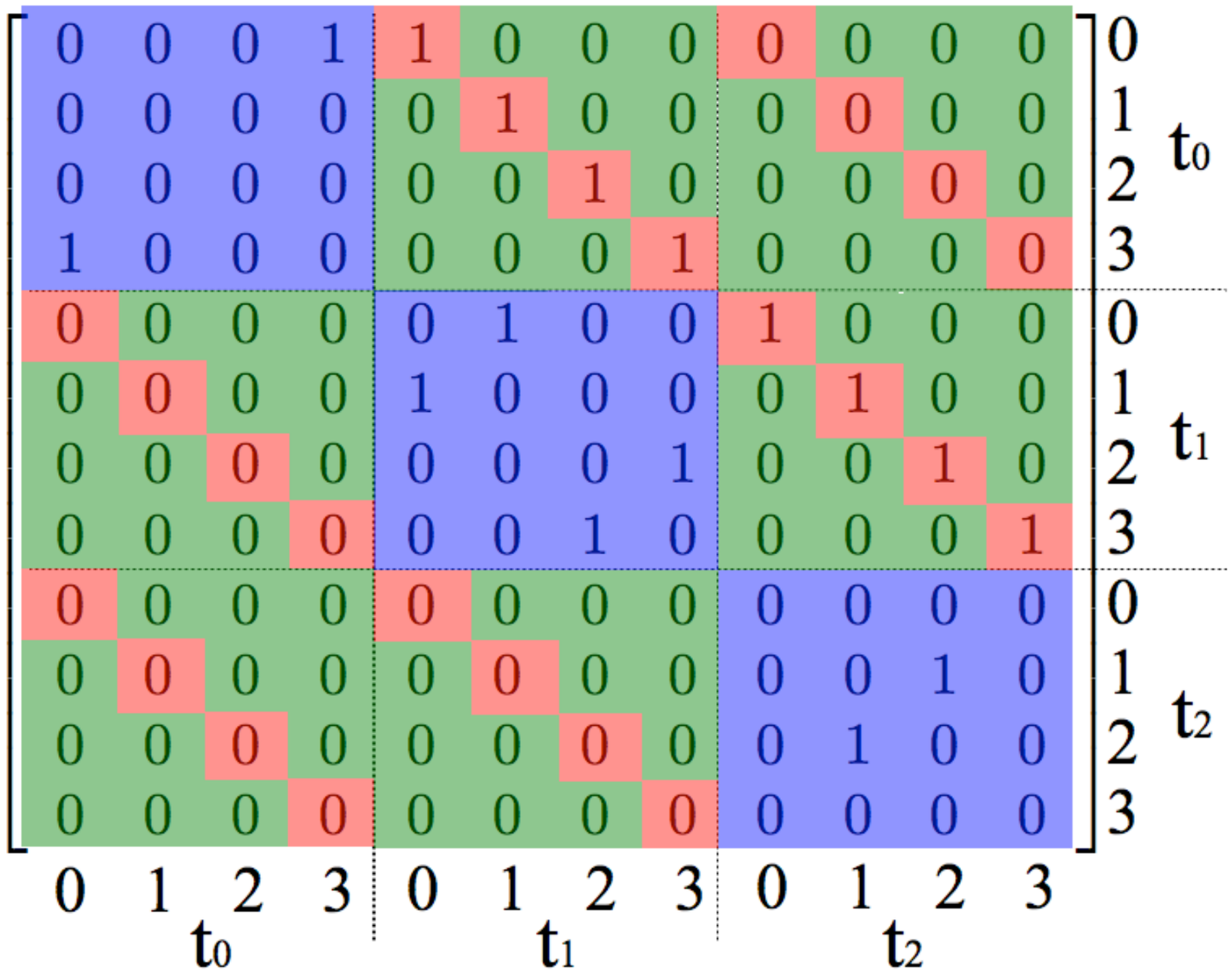
TVG Example



Adjacency Tensor in Matrix Form

$$Mat(\mathbf{A}_{G1}) =$$

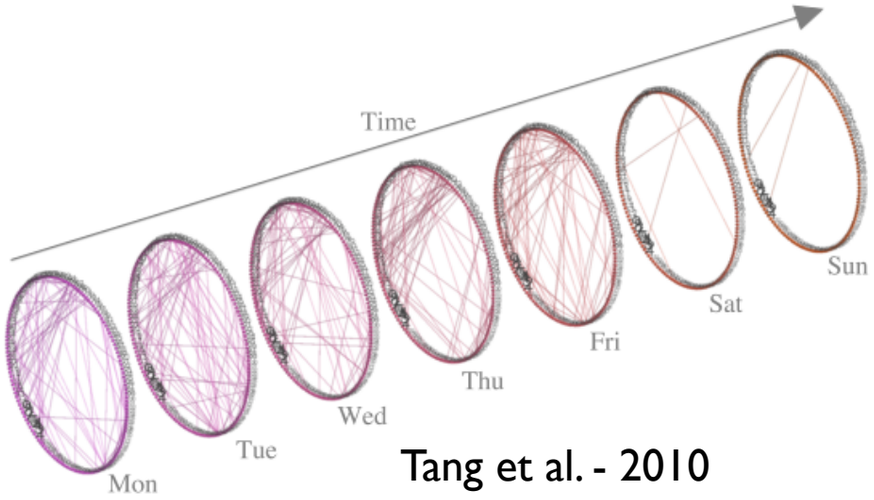




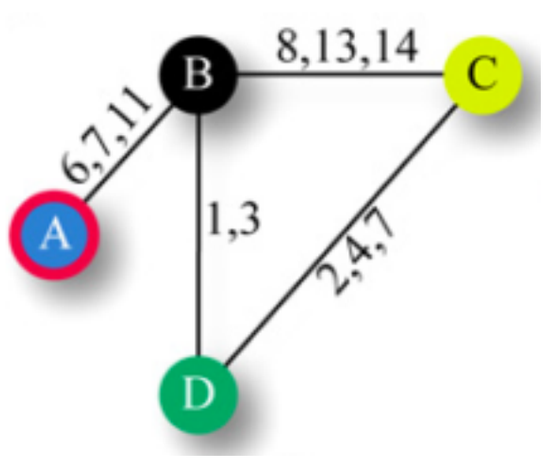
Proposed model as a unifying model

It represents other TVG models
that don’t necessarily represent each other

Snapshot Model Representation

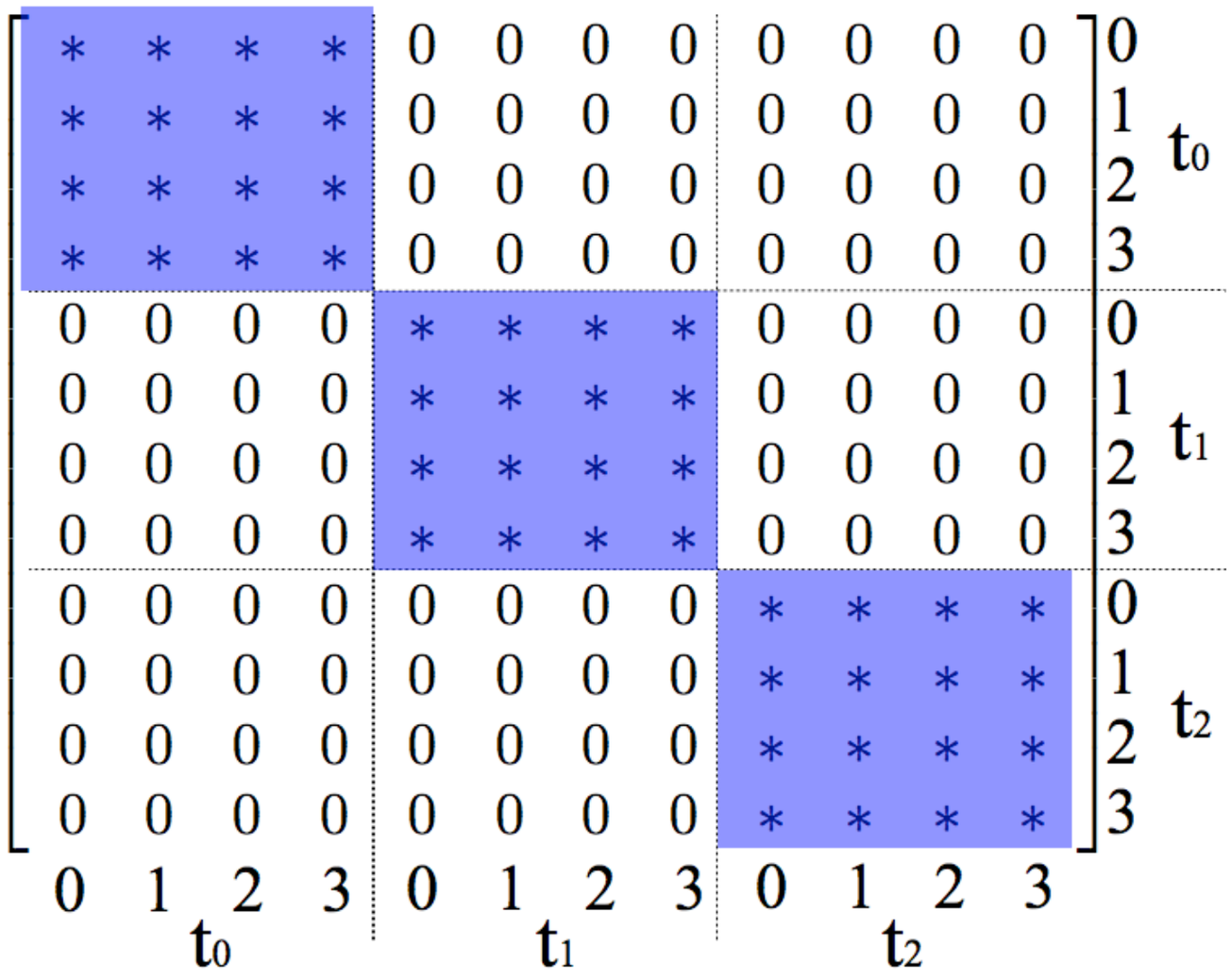


Tang et al. - 2010



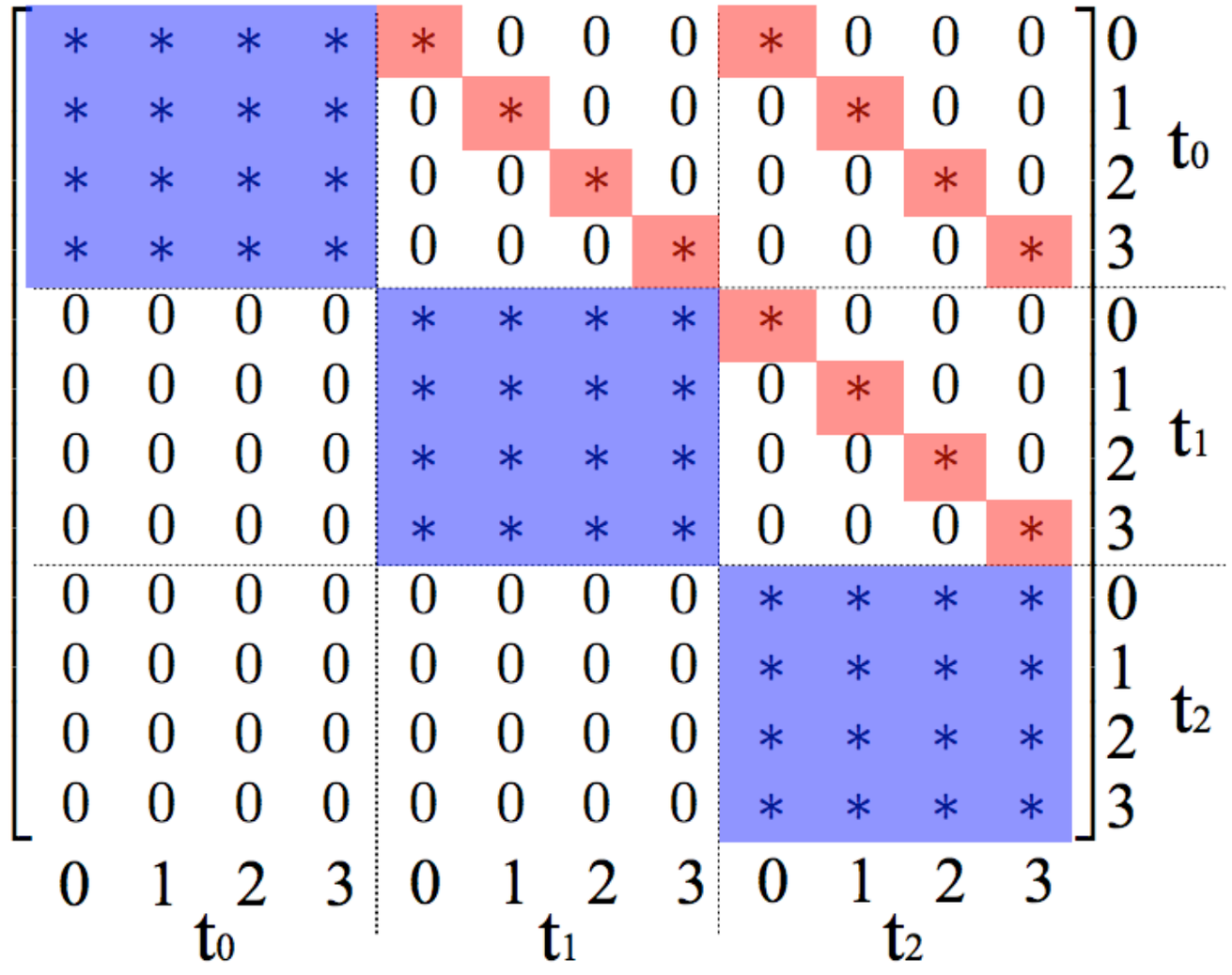
Saramäki & Holme - 2012

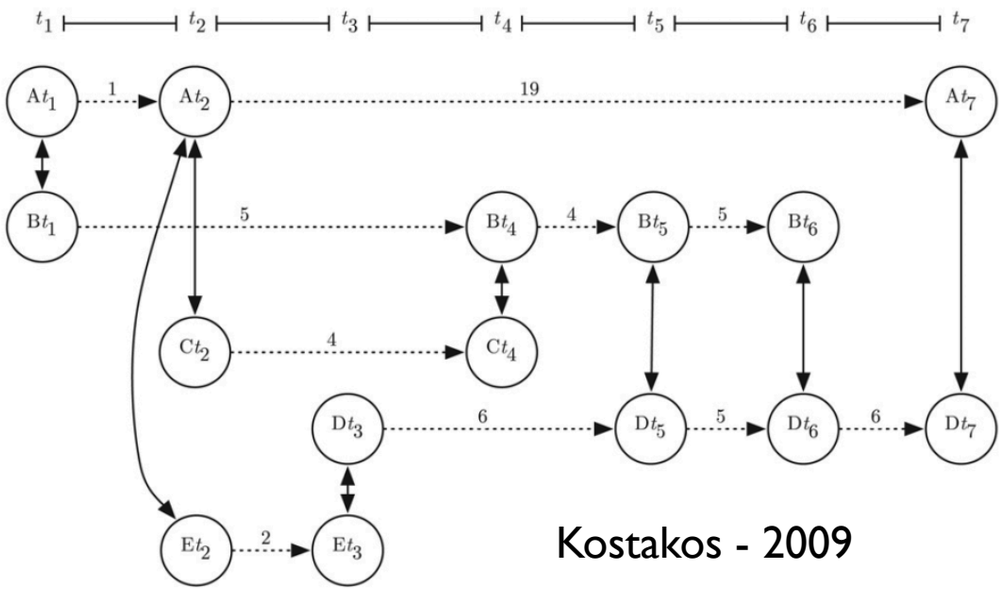
$$Mat(A_{G1}) =$$



Kostakos 2009 Model Representation

$$Mat(A_{G1}) =$$





Kostakos - 2009

Take Away Messages

The new model we propose for representing TVGs

Can represent commonly used previous models

Shows a static graph closely related to a TVG

Does not use external assumptions

Klaus Wehmuth & Artur Ziviani
LNCC
{klaus,ziviani}@lncc.br

Eric Fleury
eric.fleury@inria.fr

Thanks!

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPERJ
Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

Next steps...

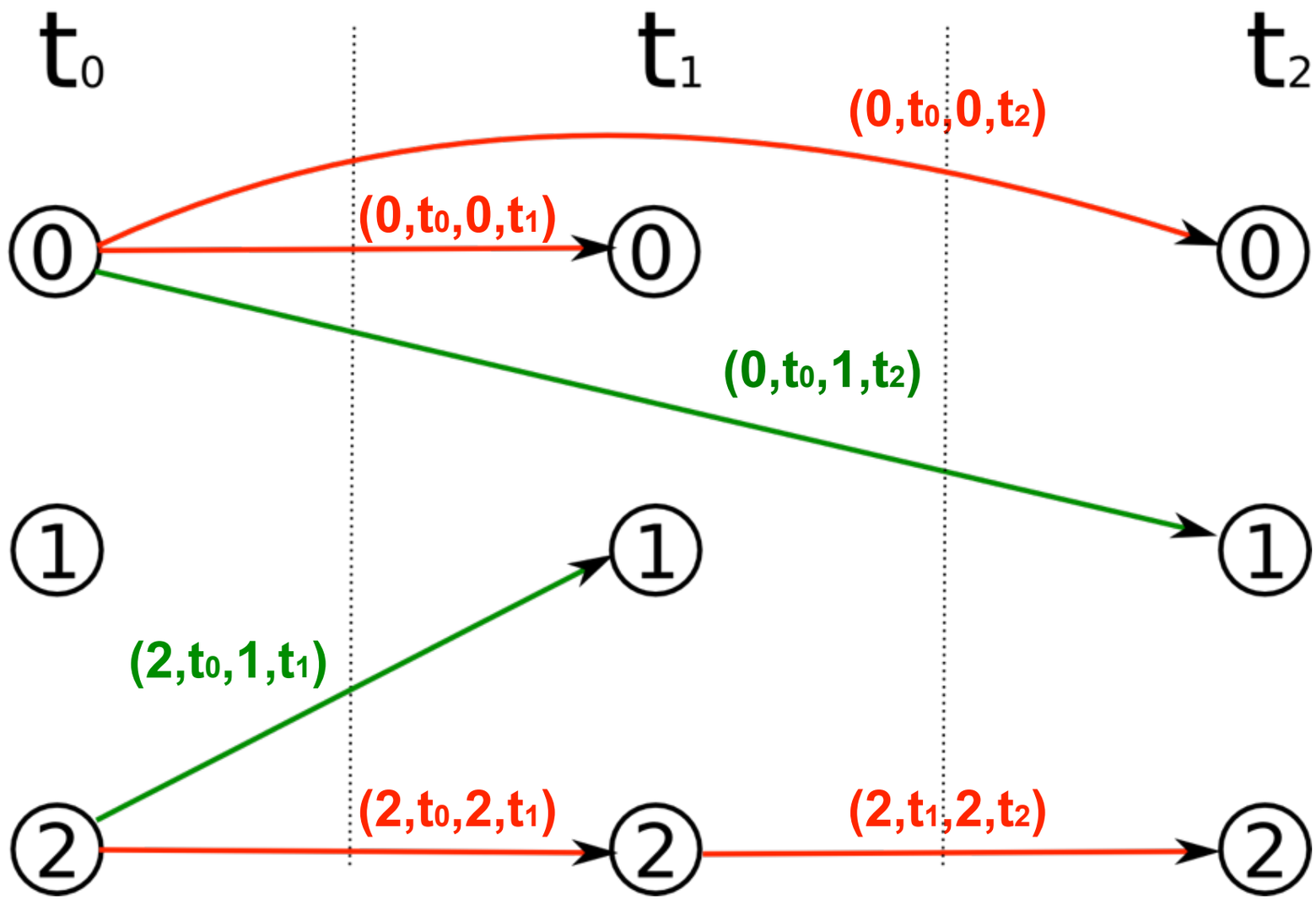
Further analysis of properties

Algebraic analysis

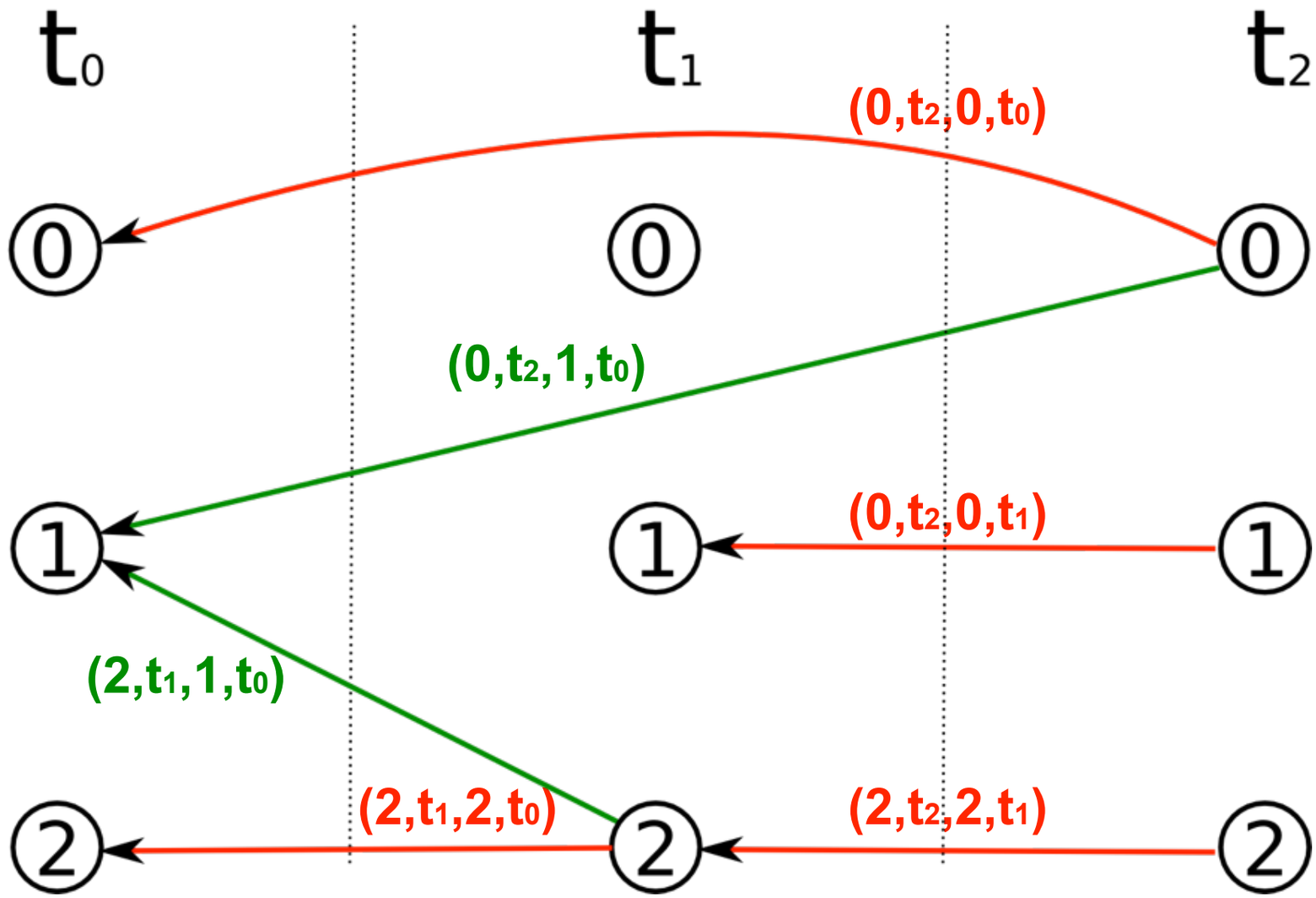
Temporal centralities

...

Progressive Dynamic Edges

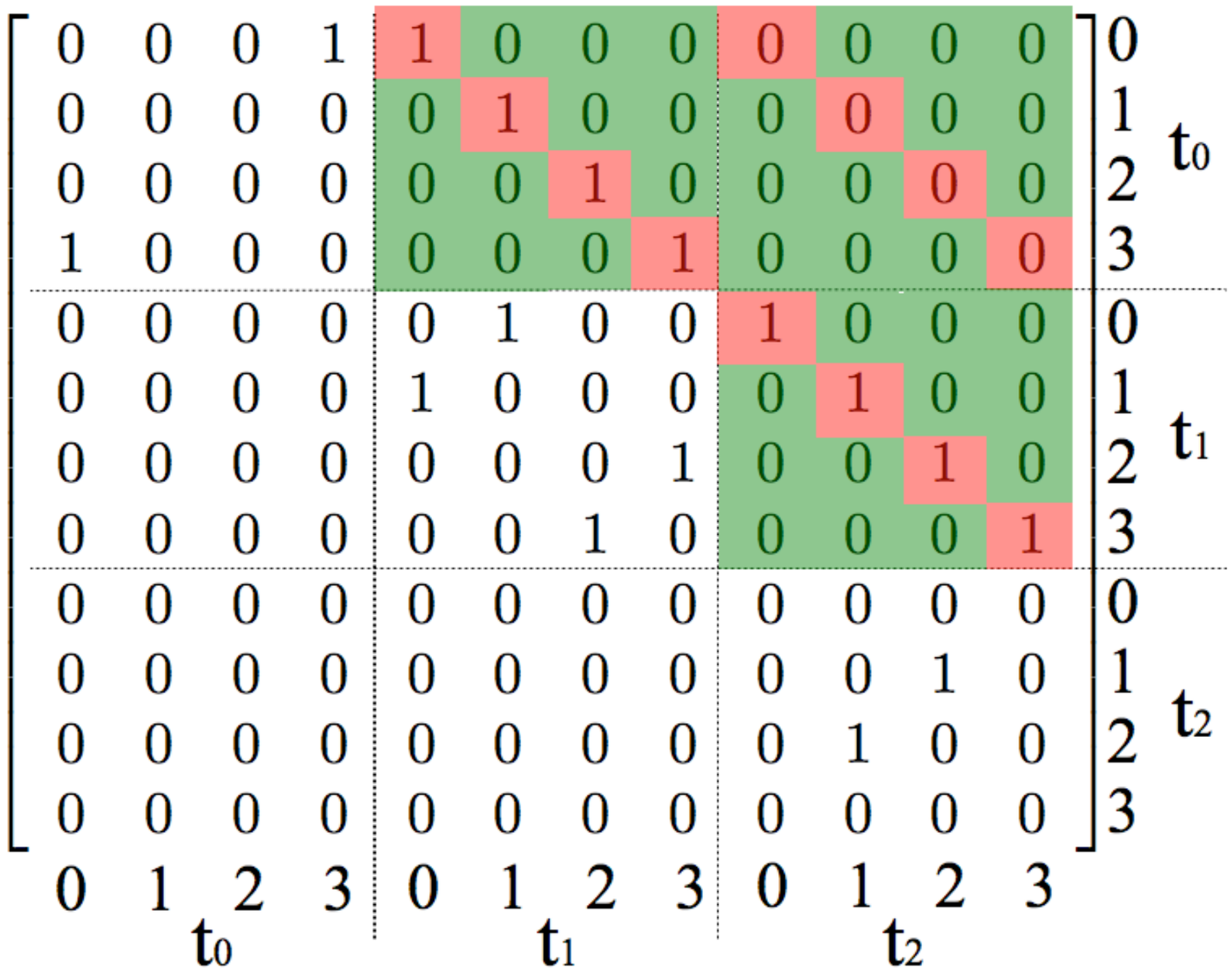


Regressive Dynamic Edges



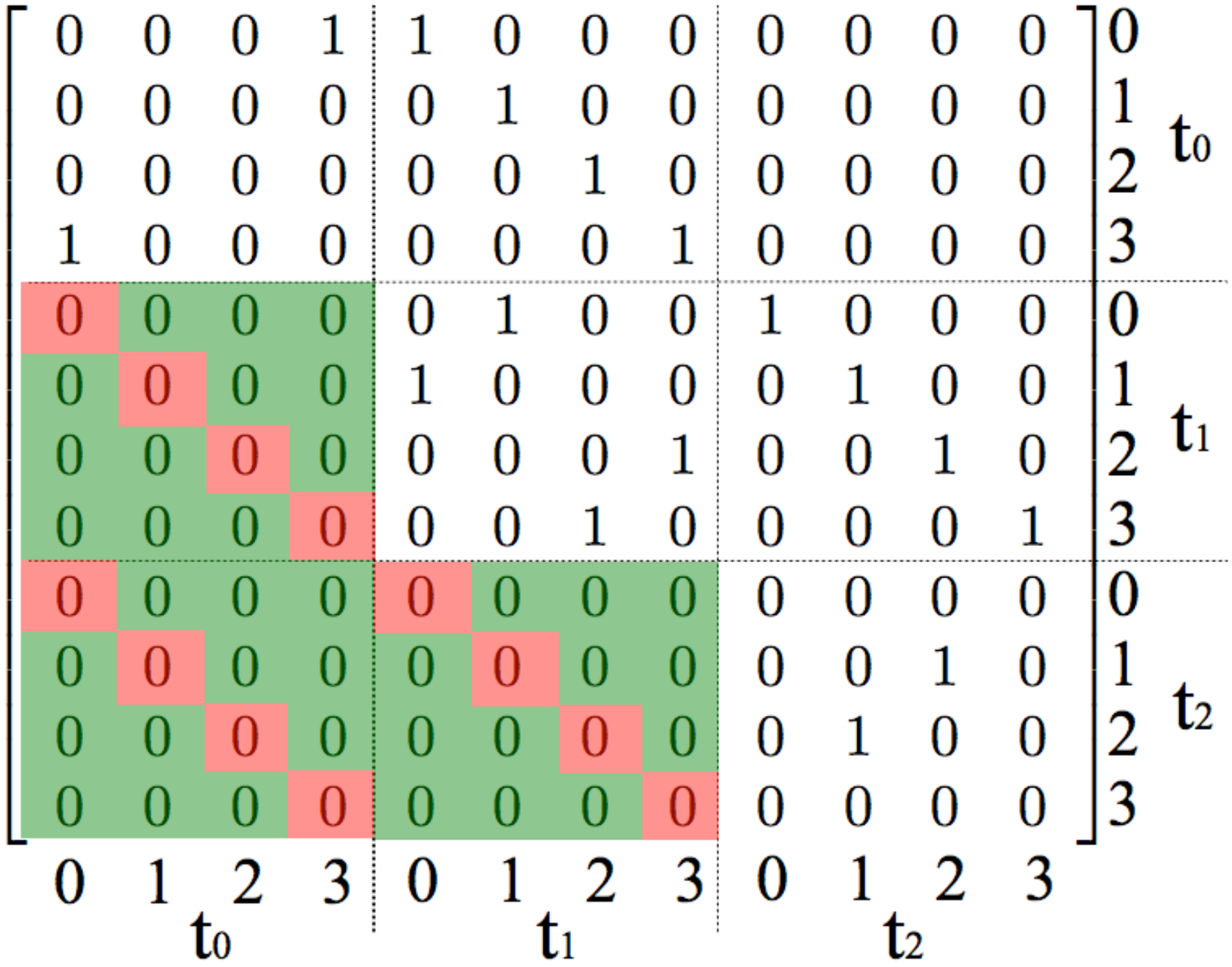
Adjacency Tensor Progressive Edges

$$Mat(\mathbf{A}_{G1}) =$$



Adjacency Tensor
Regressive Edges

$$Mat(\mathbf{A}_{G1}) =$$



Adjacency Tensor
Spatial Edges

$$Mat(\mathbf{A}_{G1}) =$$

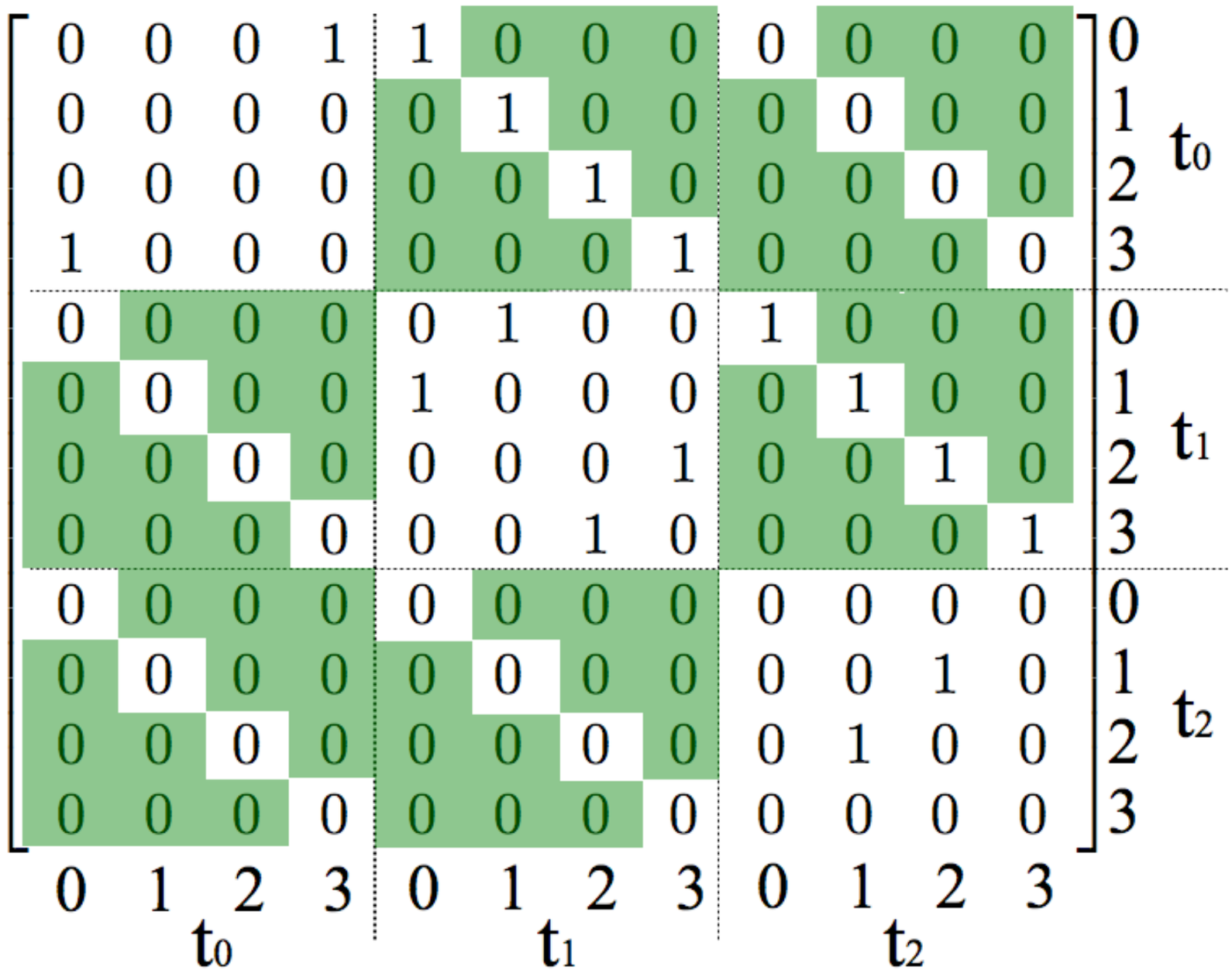
0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	0	0	2
1	0	0	0	0	0	0	1	0	0	0	0	3
0	0	0	0	0	1	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	1	0	0	1	0	2
0	0	0	0	0	0	1	0	0	0	0	1	3
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	0	0	2
0	0	0	0	0	0	0	0	0	0	0	0	3
0	1	2	3	0	1	2	3	0	1	2	3	
t_0				t_1				t_2				

Adjacency Tensor
Temporal Edges

$$Mat(\mathbf{A}_{G1}) =$$

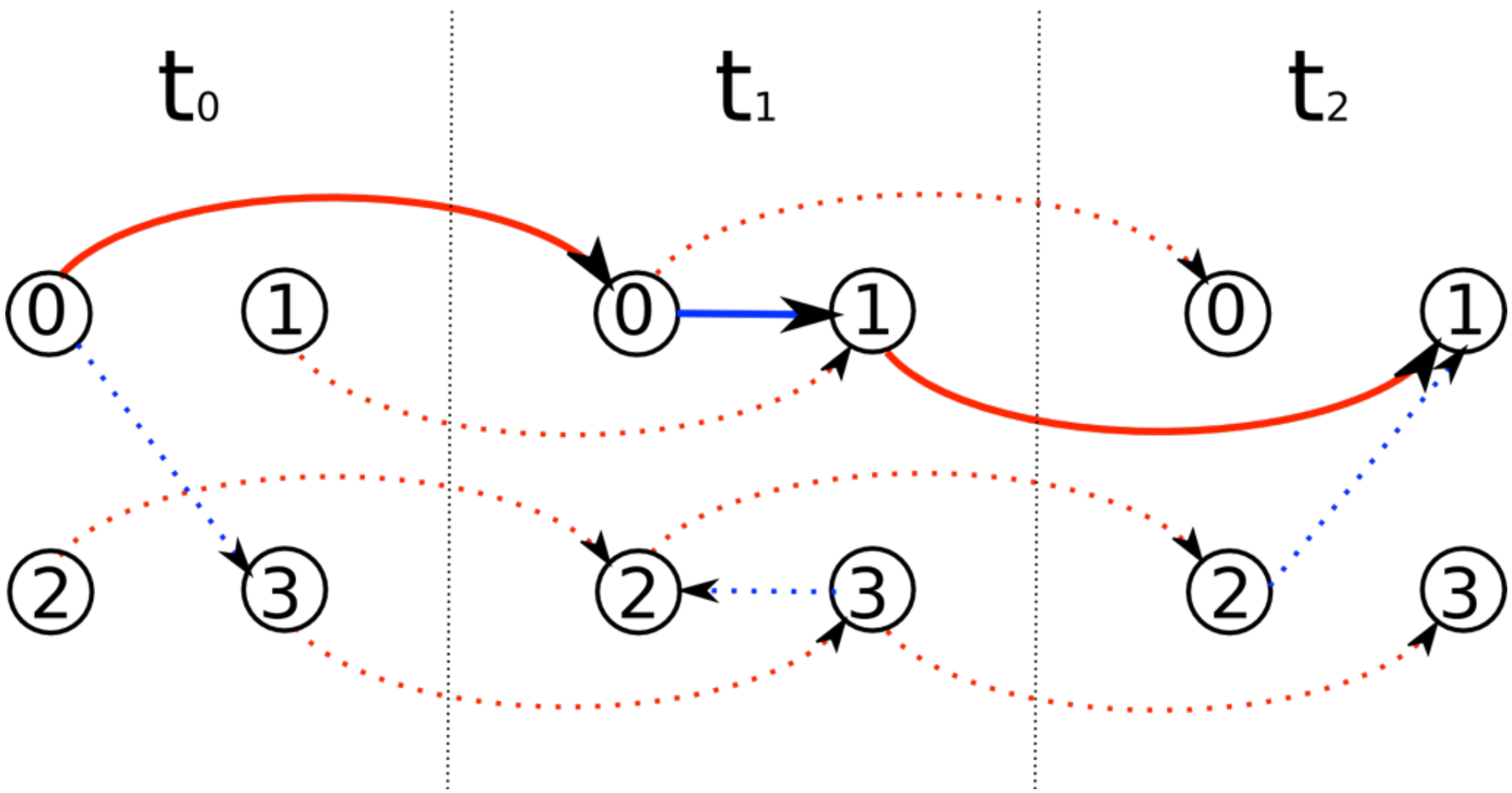
Adjacency Tensor
Mixed Edges

$$Mat(\mathbf{A}_{G1}) =$$



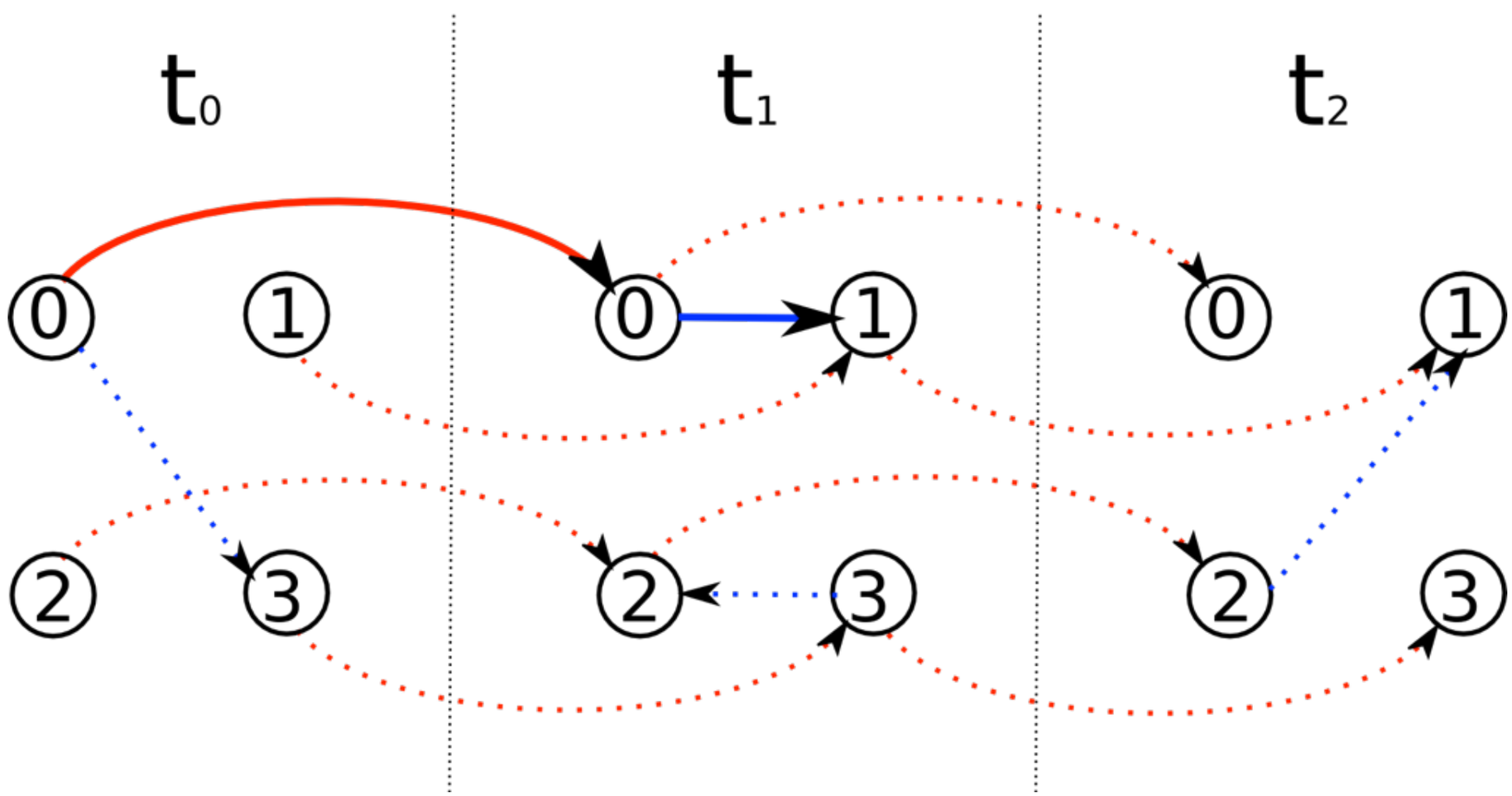
TVG shortest paths

Between temporal nodes



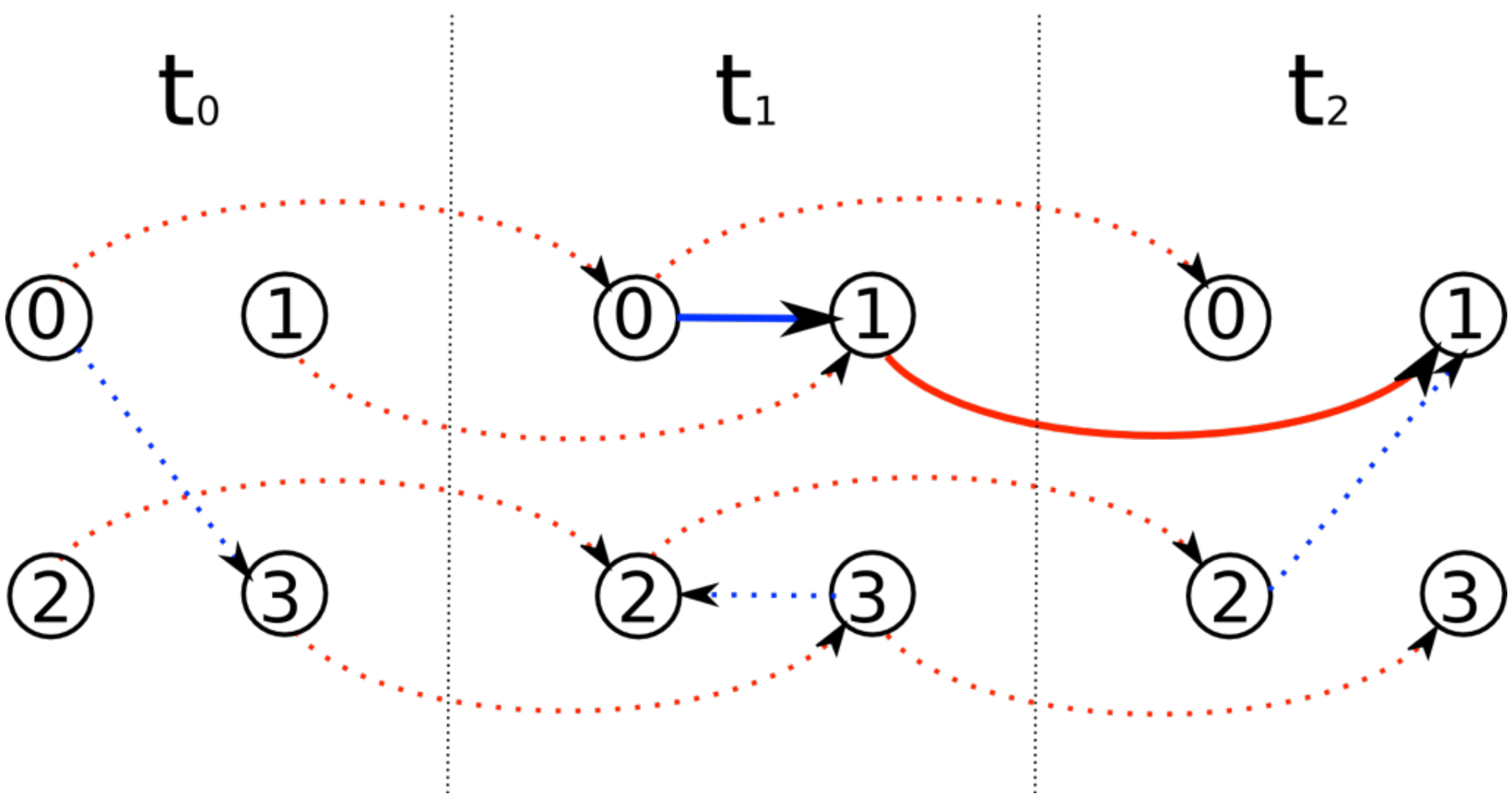
TVG shortest paths

Temporal node to node



TVG shortest paths

Node to temporal node



TVG shortest paths

Node to node