Combinatorial Models of Complex Systems

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Complex Networks and Data Communications Group

Topics

Topics

Complexity Sciences

Complexity

Large number of variables -

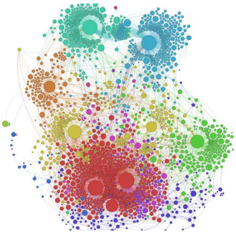
Unformalized relations -

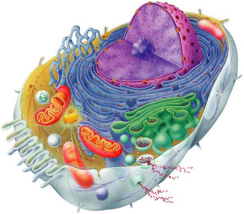
Organic (macroscopic) description -

Interdisciplinarity -

What makes a system *complex*?

- Emergence
  - Collective behavior, global function
  - Not present in the individual components
- Self-Organization
  - Presence of an ordered structure
  - Spontaneous and decentralized





Example: Social Networks

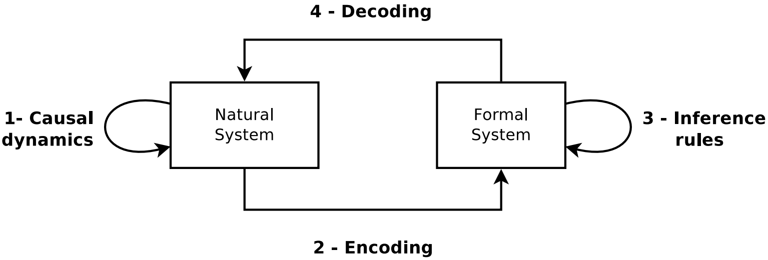
Properties

- Modular structure
  - Communities
- Small-world phenomenon
  - Small average distances
  - High clustering
- Assortative mixing

Relevant problems

- Spreading (information, epidemics, rumors)
- Predict group behavior (segregation, mixing, opinion)
- Understand social relationships

Modeling complex systems



Robert Rosen's modeling relation

Aims of this work

We aimed at...

- Studying combinatorial models
- Proposing efficient models
- Evaluating the performance
- Evaluating scalability
- Visualizing results

Topics

Our contributions

1. Connectivity in the Internet
2. Clustering in networks
3. Social networks: Discovery of community structure

Internet

Two main levels

- The inter-router level (IR)
- The Autonomous Systems level (AS)
  - Internet backbone

    e.g., LEVEL3, HURRICANE, ...
  - Transit ASes (e.g., medium-sized ISPs)
  - Stubs

Internet

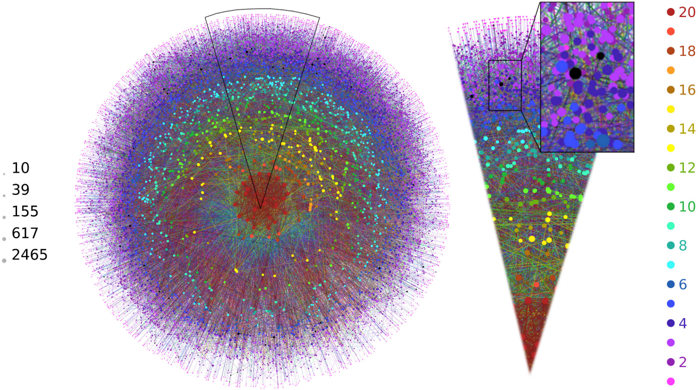
Some results on the Internet topology

- Design of exploration techniques
  - CAIDA, DIMES, RouteViews projects
- Power-laws on the degree distribution
  - Both at the IR-level and the AS-level [FFF99]
- Disassortative mixing
  - Disassortative behavior at the AS-level [PSVV01]
  - Robust-yet-fragile
- Design of hierarchical models
  - Jellyfish model [STF06]
  - MEDUSA model [CHKS07]
  - k-core decomposition [AHDBA08]

Revealed a rich hierarchical structure

Internet

Our contribution: Estimating Internet AS connectivity

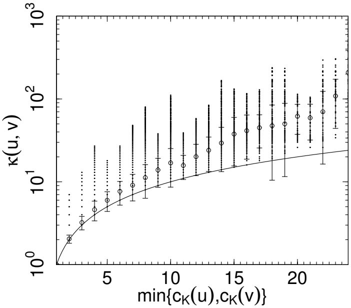


(picture generated with LaNet-vi)

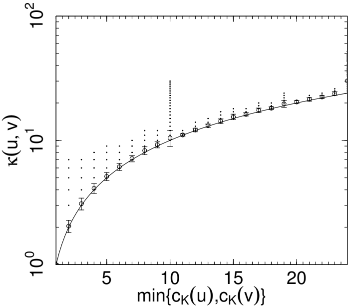
- Sufficient conditions for assuring connectivity k in k-cores [AHBB11]

Internet

Our contribution: Estimating Internet AS connectivity



Total connectivity



Connectivity through core

$O(n(G)e(G))$ vs. $O(e(G))$

Clustering

Clustering:

- Presence of triangles in the network
- Consequence of correlations between nodes
- Effect on the structural and dynamical properties

The Watts-Strogatz model was the first with high clustering coefficients [WS98]

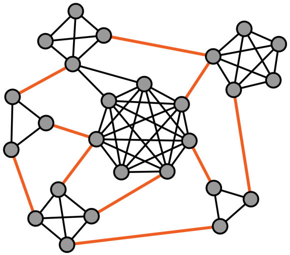
In this work, we compare the hierarchical and modular structure of clustered network models

Clustering

Maximally-ordered (CB) vs. Maximally-random (MR)

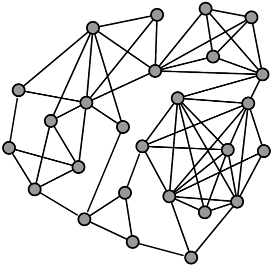
Maximally-ordered network (CB model)

- Fixed degree distribution, $p(k)$
- Fixed clustering distribution, $C(k)$
- Build a set of cliques
- Join cliques with configuration model



Maximally-random network (MR model)

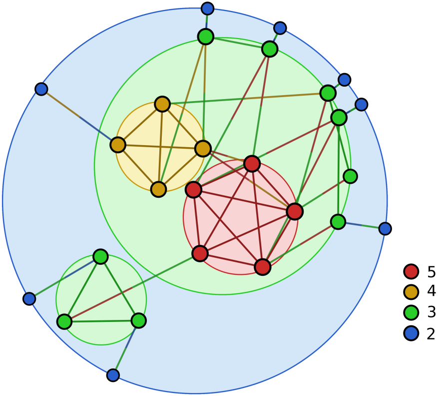
- Fixed degree distribution, $p(k)$
- Fixed clustering distribution, $C(k)$
- Define an exponential random graph
- Minimize Hamiltonian
  - Node rewiring
  - Simulated annealing



Clustering

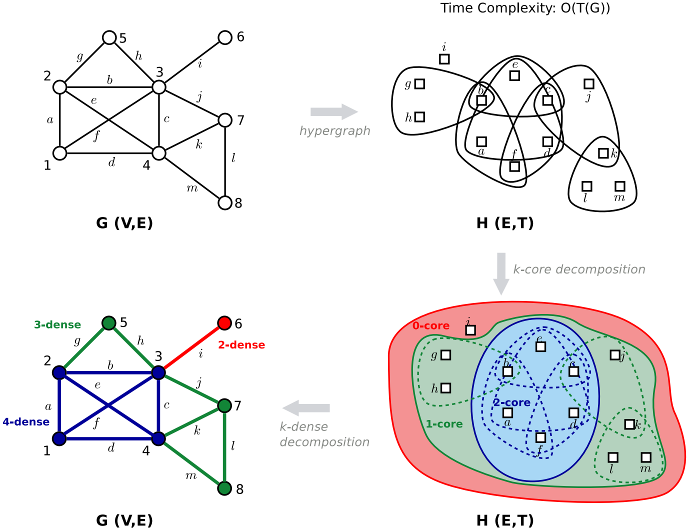
k-dense decomposition

- Based on the notion of edge multiplicity
- Edge multiplicity is the *number of common neighbors between the edge endpoints*
- A k-dense is a *maximal subgraph such that every edge has multiplicity at least* $(k-2)$ *inside the subgraph* [SYK08]



Clustering

k-dense decomposition: Algorithm



Clustering
Positioning algorithm

Combinatorial Models of Complex Systems

M.G. Beiró

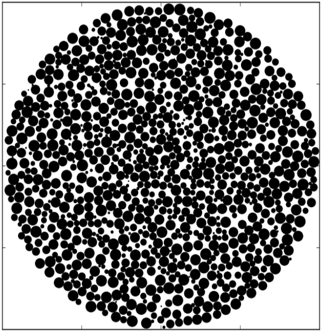
Introduction
Complex Systems
Example
Models of Complex Systems
Aims

Contributions
Internet
Clustering
Social Networks: Community Structure

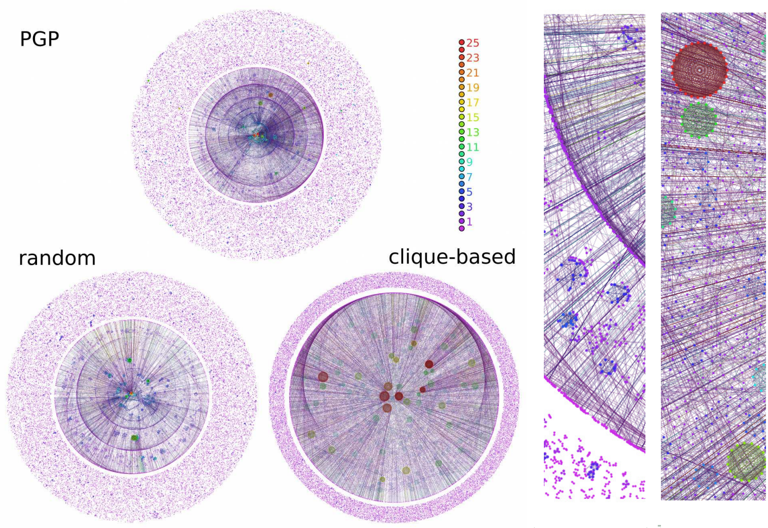
Extension: Community Discovery

Conclusions

- A connected component (CC) may split when we consider the $(k+1)$-dense
- Each CC is drawn into a circle
- Circles are randomly placed
- Random re-positioning until circles do not overlap

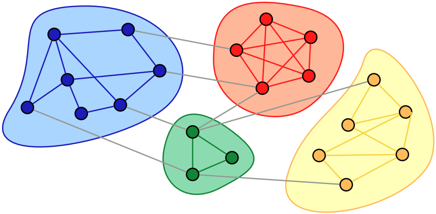


Clustering
PGP



Social Networks: Community Structure

- Community structure is the presence of groups of densily connected vertices
- Also called *modular structure*
- Found in many complex systems
  - Friendship networks, collaboration networks
  - The World Wide Web
  - Metabolic networks, networks of protein interactions
- But the concept lacks of a precise notion
  - Sometimes it depends on the application



- Notion of community
  - Concept of *web community* [FLG00]
  - Communities in the scientific collaboration network [New01]

- Some of the first methods:
  - Edge-betweenness, dendrogram [GN02]
  - Edge-clustering [RCC+04]
  - Modularity maximization
  - Label propagation (diffusion process) [RAK07]
  - INFOMAP (minimum description length) [RB08]
  - Lancichinetti's natural community (overlapping) [LFK09]

Social Networks: Community Structure

A local community detection method

Our method has 3 ingredients:

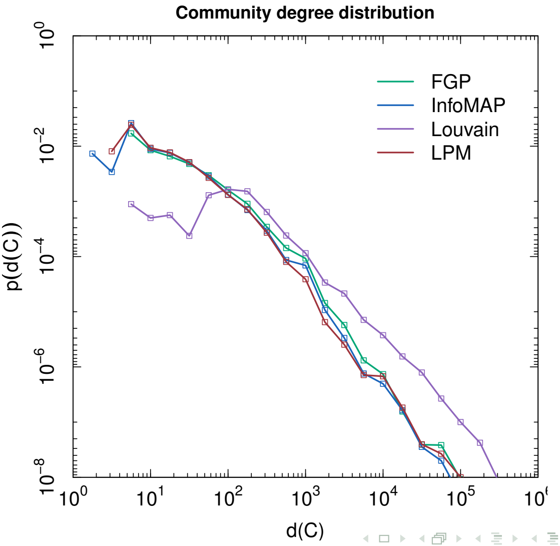
- A *fitness function* with a resolution parameter
- A *growth process*
  - The process "spreads" throughout the graph
  - The fitness function always increases
  - The resolution parameter is dynamically updated
- A *cutting technique*

The result:

- A *partition into communities*

The method was tested in:

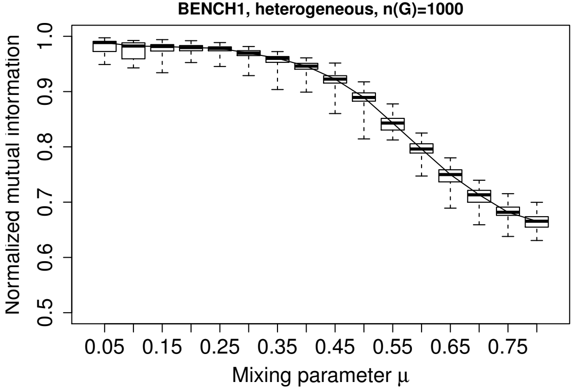
- Simple networks (karate, football, jazz, ...)
- Large real networks (web-Stanford (2M), LiveJournal (42M))
  - Power-law in the community size distribution
- LFR benchmark
  - Good performance in terms of the normalized mutual information


Community degree distribution
FGP
InfoMAP
Louvain
LPM
p(d(C))
d(C)
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^0
10^1
10^2
10^3
10^4
10^5

Social Networks: Community Structure

Results: Benchmark LFR

- Network instances with 1000 vertices
- Variable mixing parameters $[0{,}05 - 0{,}80]$



Topics

Our method has 3 ingredients:

- A *fitness function*
- A *growth process*
- A *cutting technique*

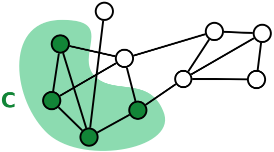
Its complexity is:

- Temporal: $O(n(G) \cdot d_{\max} + e(G) \cdot \log(n(G)))$
- Spatial: $O(n(G) + e(G))$

Given a subset $C \in V(G)$, we define the fitness function $H_t(C)$ as:

$$H_t(C) = m_V(C)\left(1 - \frac{m_V(C)}{2t}\right) - c_E(C) \qquad (1)$$

- What is $m_V(C)$?
  - Relative "mass" of C
- What is $c_E(C)$?
  - Relative external degree of C



- The growth process incorporates vertices into the set C, one after the other

- It always chooses the one which maximizes ΔH

- When no vertex can increase H we increase the resolution t as little as possible

What is the result?

- The growth process always prefers to incorporate vertices in the same local community
- When a community ends, we must incorporate a vertex from another community

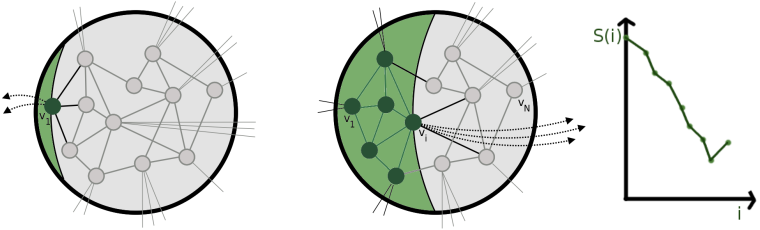
  Tipically, the resolution must be increased at this point
- Thus, the communities are traversed one after the other

Social Networks: Community Structure
Cutting technique

Each community evolves as $C_1, C_2, \ldots\ldots, C_{FINAL}$

We observe the behavior of the function:

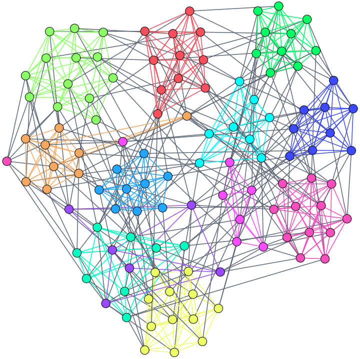
$$S(i) = \frac{c_E(C_i)}{m_V(C_i)} = \frac{\text{external connections}}{\text{community degree}}$$

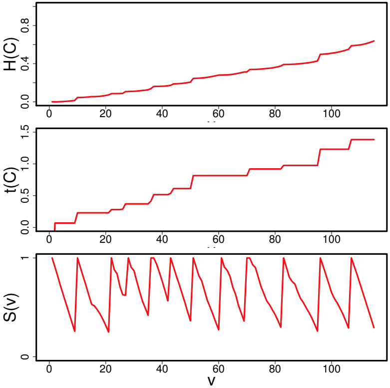


Social Networks: Community Structure

The growth process: An example

The “football” network:

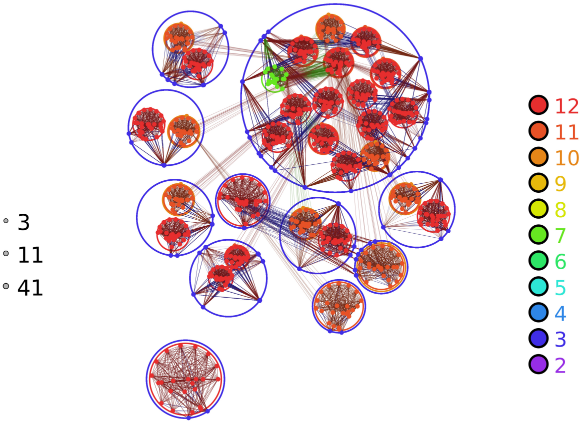




Social Networks: Community Structure

Community visualization

Stanford.edu web graph (250000 vertices) $\rightarrow$
Induced subgraph with **30 consecutive communities**
Node positioning according to the k-dense decomposition with LaNet-vi



(picture generated with SnailVis)

Topics

Conclusions

1. Connectivity in the Internet
   - Proposal estimator for edge-connectivity in the Internet based on the k-core decomposition
   - Edge-connectivity visualization with the LaNet-vi software
2. Clustering in networks
   - Efficient implementation of the k-dense decomposition
   - Development of a visualization tool for comparing clustered network models
3. Social networks: Discovery of community structure
   - Proposal of a local community detection algorithm
   - Visualization of community structure and evolution with the SnailVis software

Conclusions

- We performed a complexity analysis for each proposed algorithm
- We used visualization as a tool for understanding complex systems structure

Possible future lines of work:

- New criteria for evaluating quality of community structure
- Further applications of the k-dense decomposition

Thank you!

J.I. Alvarez-Hamelin, M.G. Beiró, and J.R. Busch.
Understanding edge connectivity in the internet through core decomposition.
Internet Mathematics, 7(1):45–66, 2011.

J.I. Alvarez-Hamelin, L. Dall'Asta, A. Barrat, and Vespignani A.
k-core decomposition of internet graphs: hierarchies, self-similarity and measurement biases.
Networks and Heterogeneous Media, 3(2):371, 2008.

S. Carmi, S. Havlin, S. Kirkpatrick, and E. Shir.
A model of internet topology using k-shell decomposition.
PNAS, 104:11150–11154, 2007.

M. Faloutsos, P. Faloutsos, and C. Faloutsos.
On power-law relationships of the internet topology.
In *Proceedings of the conference on Applications, technologies, architectures, and protocols for computer communication*, SIGCOMM '99, pages 251–262, New York, NY, USA, 1999. ACM.

G.W. Flake, S. Lawrence, and C.L. Giles.
Efficient identification of web communities.
In *Proceedings of the sixth ACM SIGKDD international conference on Knowledge discovery and data mining*, KDD '00, pages 150–160, New York, NY, USA, 2000. ACM.

M. Girvan and M.E.J. Newman.
Community structure in social and biological networks.
Proceedings of the National Academy of Sciences, 99(12):7821–7826, June 2002.

A. Lancichinetti, S. Fortunato, and J. Kertész.
Detecting the overlapping and hierarchical community structure in complex networks.
New Journal of Physics, 11(3):033015, 2009.

M.E.J. Newman.
The structure of scientific collaboration networks.
Proceedings of the National Academy of Sciences, 98(2):404–409, January 2001.

R. Pastor-Satorras, A. Vázquez, and A. Vespignani.
Dynamical and correlation properties of the internet.
Physical Review Letters, 87:258701+, 2001.

U.N. Raghavan, R. Albert, and S. Kumara.
Near linear time algorithm to detect community structures in large-scale networks.
Physical Review E, 76(3):036106+, September 2007.

M. Rosvall and C.T. Bergstrom.
Maps of random walks on complex networks reveal community structure.
Proceedings of the National Academy of Sciences, 105(4):1118–1123, 2008.

F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, and D. Parisi.
Defining and identifying communities in networks.
Proceedings of the National Academy of Sciences, 101(9):2658, 2004.

G. Siganos, S.L. Tauro, and M. Faloutsos.
Jellyfish: A conceptual model for the as internet topology.
Journal of Communications and Networks, 8(3):339–350, 2006.

K. Saito, T. Yamada, and K. Kazama.
Extracting communities from complex networks by the k-dense method.
IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences, E91-A(11):3304–3311, November 2008.

D.J Watts and S.H. Strogatz.
Collective dynamics of small-world networks.
Nature, 393:440–442, 1998.