SIS Epidemics based on Random walks in Networks

Pedro Younes

Daniel R. Figueiredo

Department of Systems Engineering and Computer Science
(PESC/COPPE)

COPPE
Instituto Alberto Luiz Coimbra de Pós-Graduação e Pesquisa de Engenharia UFRJ

Stefanella Boatto

Department of Applied Mathematics
(DMA/IM)

Federal University of Rio de Janeiro (UFRJ) Rio de Janeiro - Brazil

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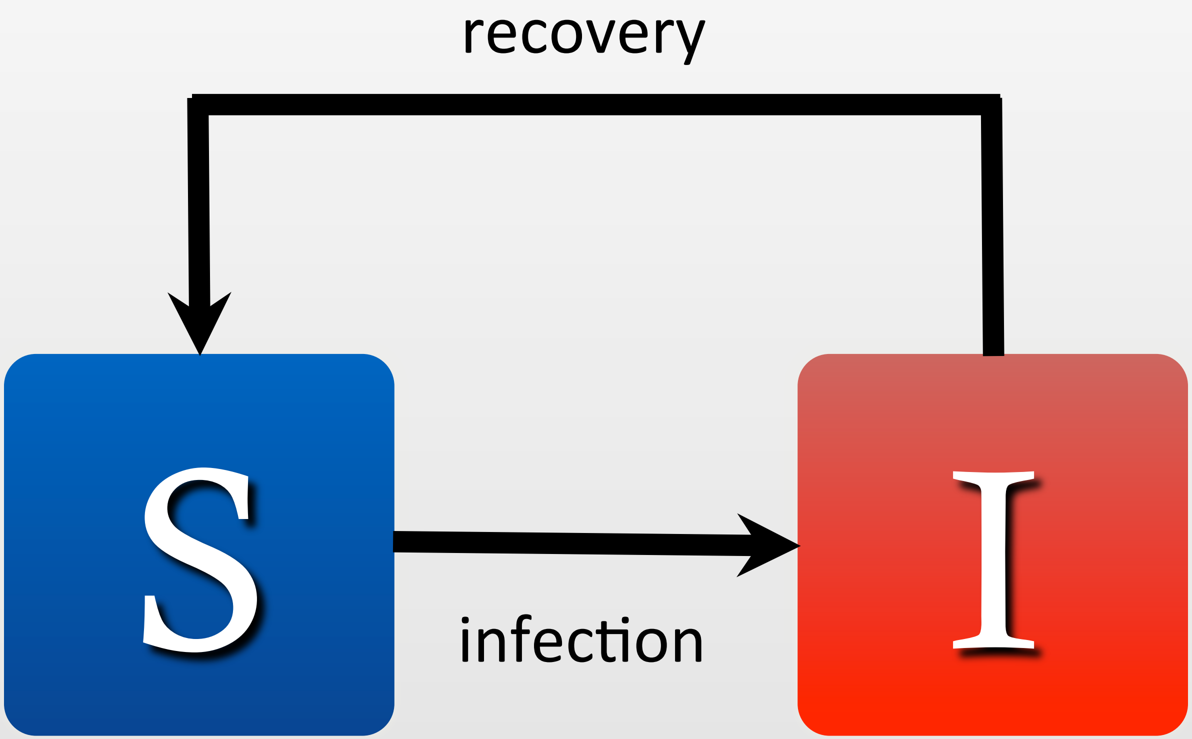
Outline

- SIS model with mobility
- Simulations
- Approximate model
- Conclusion

SIS epidemics

- Epidemics is a fundamental problem concerning spread of information to diseases



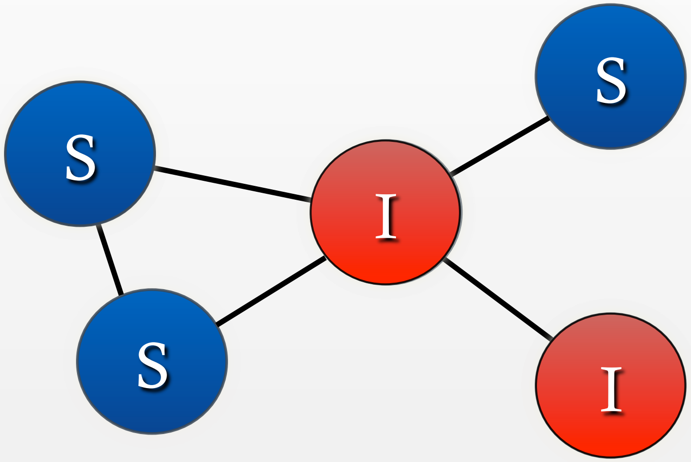
recovery

S

infection

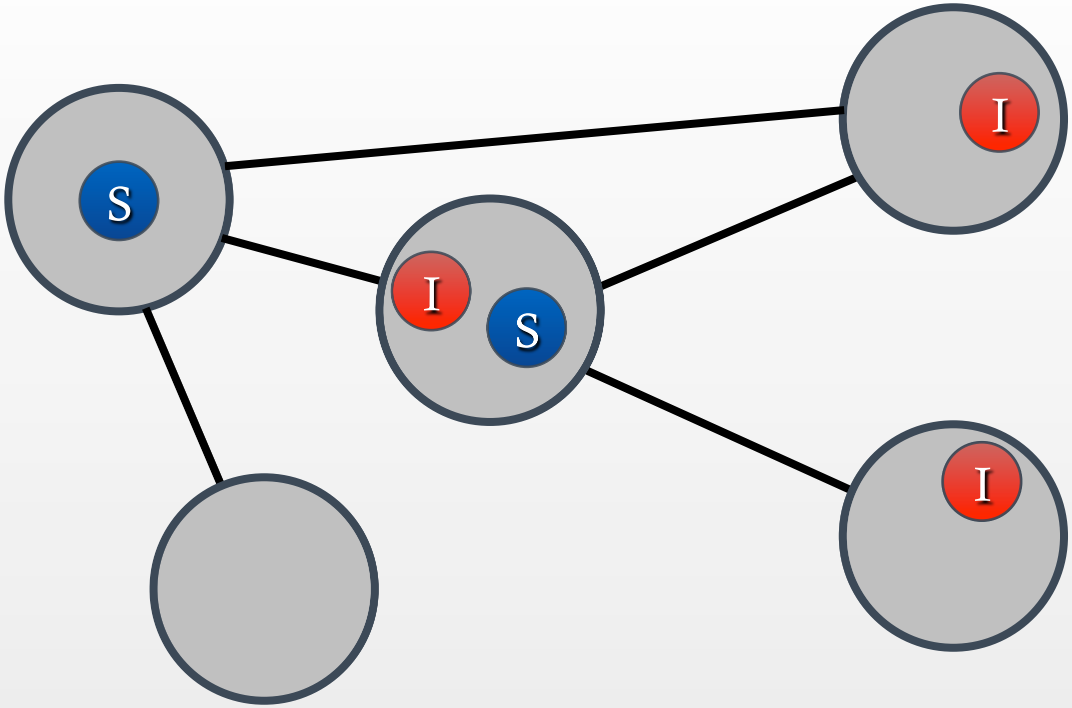
I

Classical Approach



- nodes have state
- contagion occurs through edges
- states of nodes change over time
- epidemic structure is fixed

More Recent Approach



- individuals have state and move around the network
- infection occurs upon encounter
- state and position of individuals change over time
- two coupled dynamics: mobility and epidemic

Model

- network structure $G = (V, E)$ with $n = |V|$ nodes

- k individuals move independently according to identical continuous-time random walks

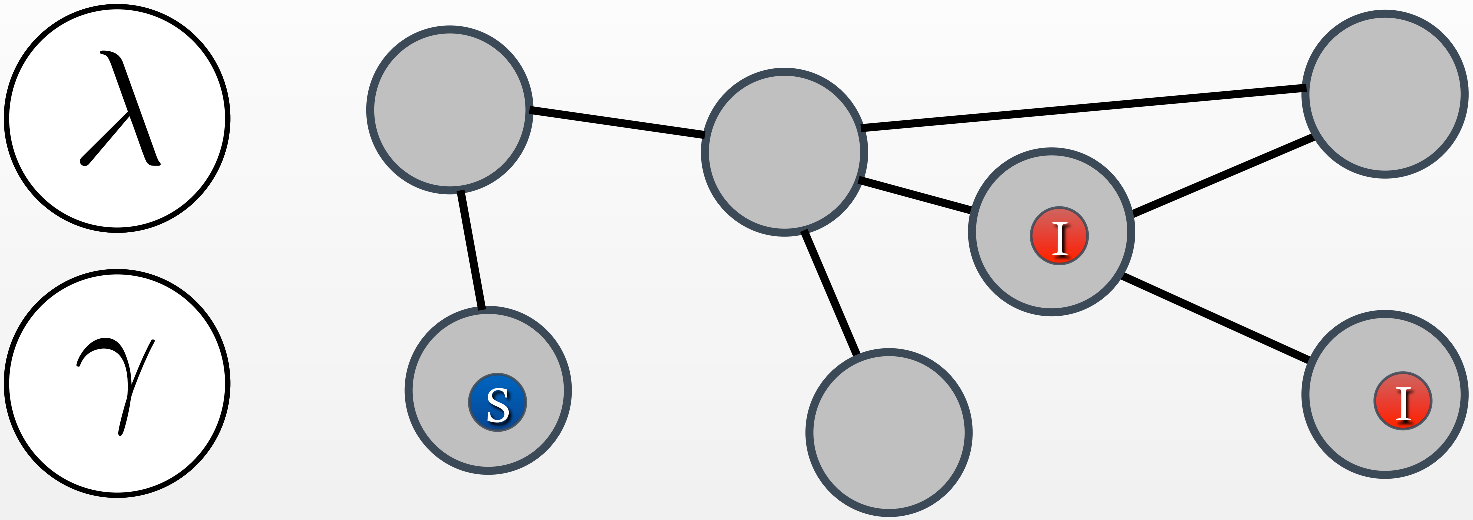
  - holding time in node is exponentially distributed with rate λ

  - transition takes zero time and neighboring node chosen uniformly at random

- infection occurs with probability τ upon encounter

- recovery exponentially distributed with rate γ, independently

Example



Parameters	value
n nodes	7
k individuals	3
i_0 initial infected	2
movement rate λ	1
recovery rate γ	5
infection probability τ	1

low density : $\frac{k}{n} << 1$

inter-step time: $\frac{\lambda}{\gamma}$

Exact Model

- System state : position and state of every individual

$$X(t) = \times_{i=1,\ldots,k}(v_i(t), s_i(t))$$

position: $v_i(t) \in \{1, \ldots, n\}$

state: $s_i(t) \in \{S, I\}$

- Number of infected individuals: $I(t) = \sum_{i=1}^{k} \mathbb{1}\{s_i(t) = I\}$

- $\{X(t)\}$ is characterized by continuous-time Markov chain:

  - state space is large: $(2n)^k$

  - absorbing set (all individuals susceptible): $S_{\mathcal{A}} = \times_{i=1,\ldots,k}(v_i, S)$

- Metric of interest: $E[I(t)]$

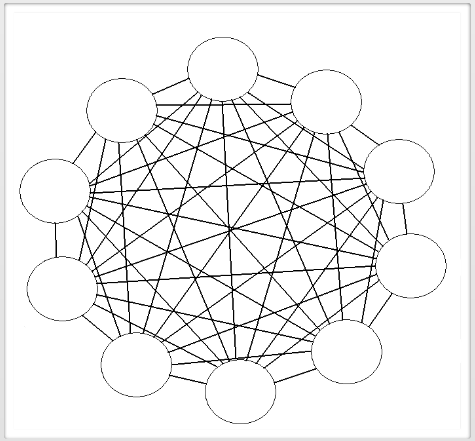
  - exact meta-stationary (transient) solution is hard

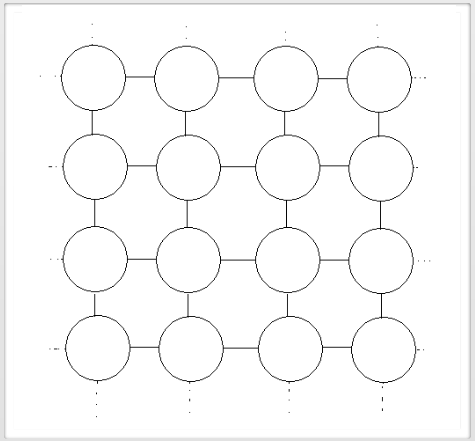
Known Results for RW

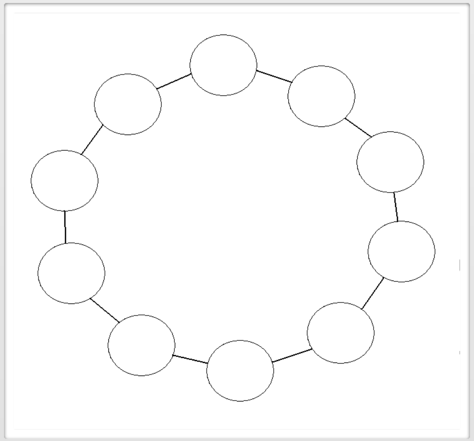
- random walk position in steady state: $\pi_j = \dfrac{d_j}{\sum_{i=1}^{n} d_i}$

- encounter rate of two random walks: $\omega = 2\lambda \sum_{j=1}^{n} (\pi_j)^2$

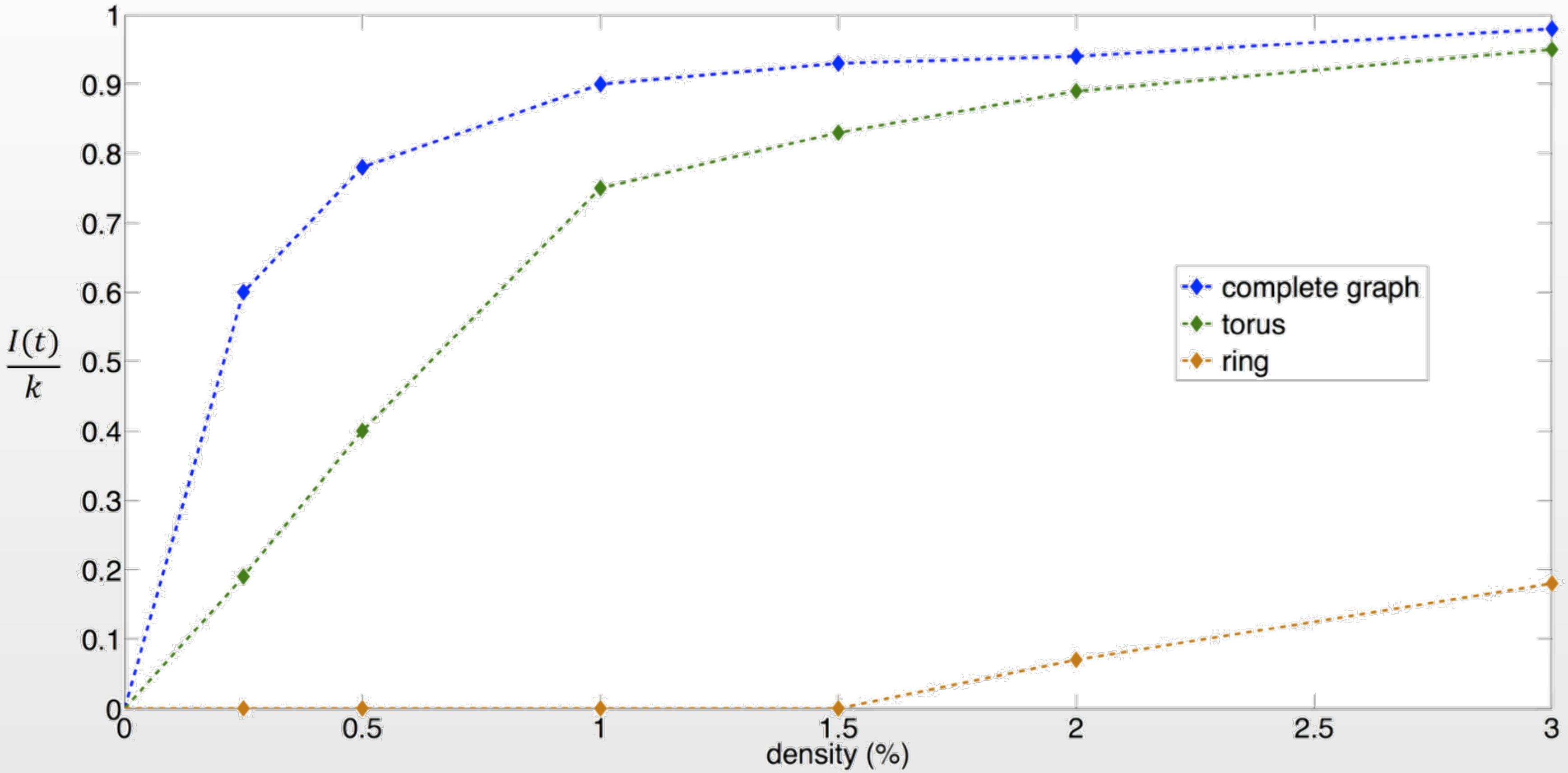
- On regular networks: $\pi_j = \dfrac{1}{n}$ $\omega = 2\lambda \sum_{j=1}^{n} (\dfrac{1}{n})^2 = \dfrac{2\lambda}{n}$


complete graph


$2d$ torus

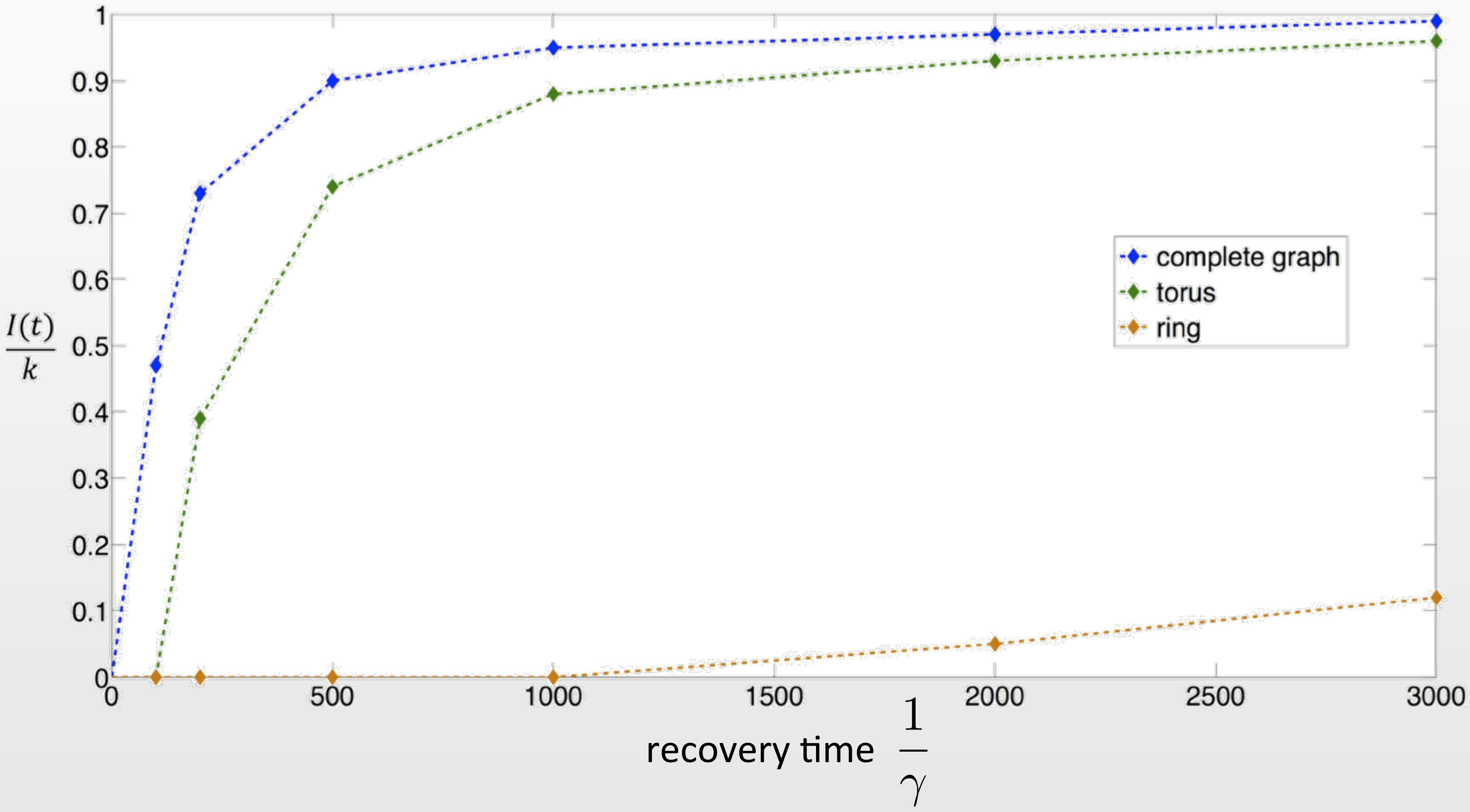

ring

Simulations: Results



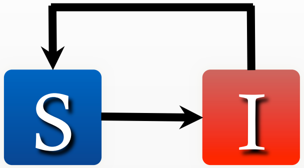
- epidemic intensity increases with density
- behavior depends on network structure

Simulations: Results



- epidemic intensity increases with recovery time
- behavior depends on network structure

Approximate Model



- Based on classical epidemic ODE-models

  - determine infection (S->I) and recovery (I->S) rates

  - encounter rate of two random walks: $\omega = \frac{2\lambda}{n}$

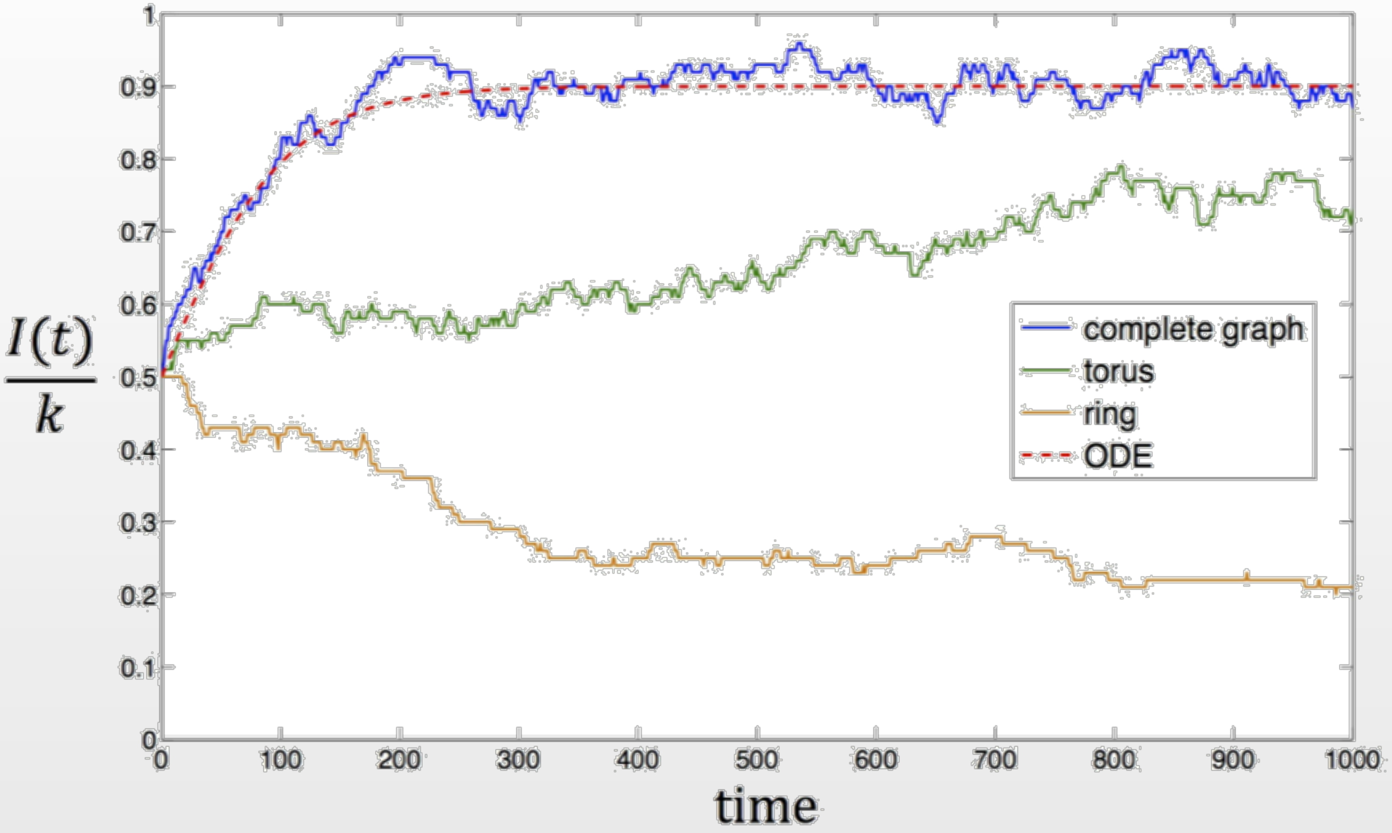
  - three different types of encounter:

  - probability that encounter is between individuals S and I : $P_{SI} = \frac{I(k-I)}{\binom{k}{2}}$

- ODE-based model: $$\frac{dI}{dt} = \binom{k}{2}\omega\tau P_{SI} - \gamma I$$

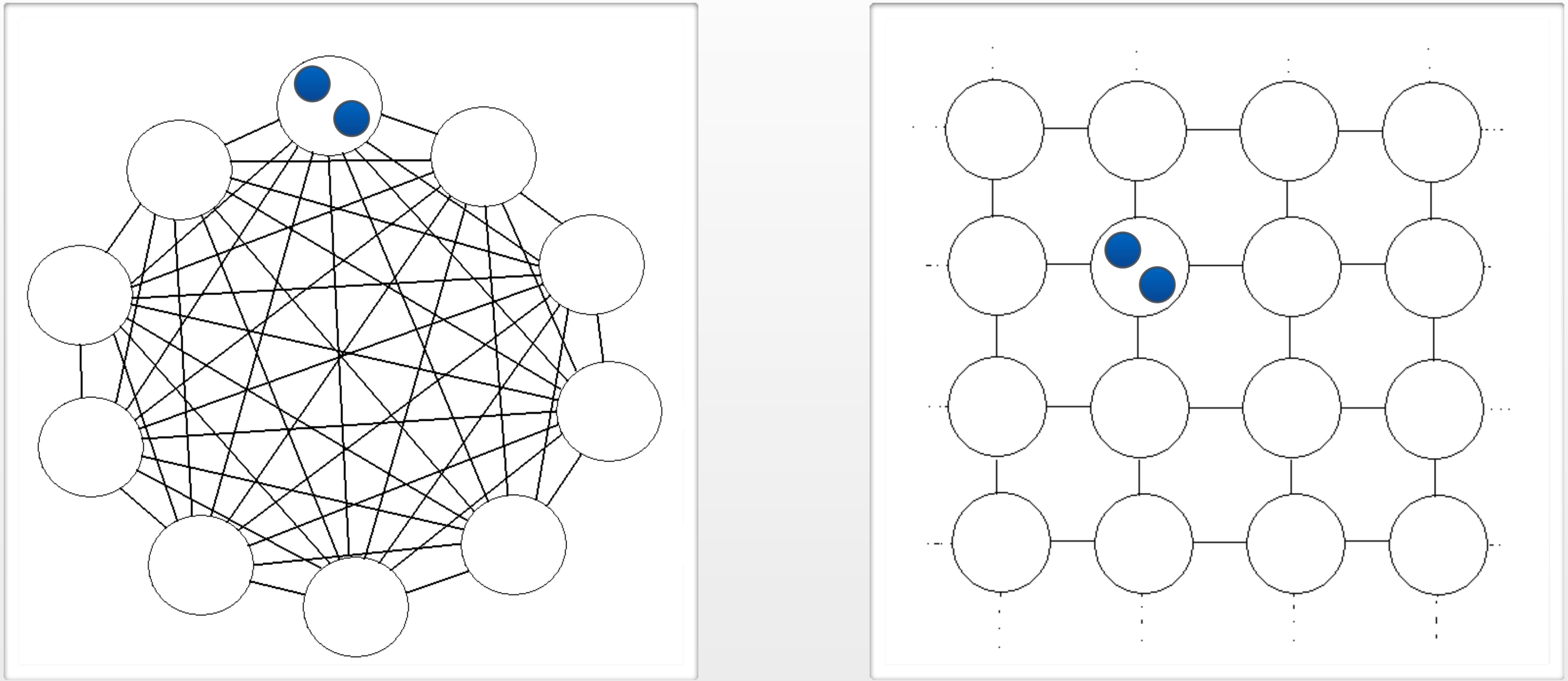
- threshold for onset of epidemic: $$R_0 = 2\frac{k}{n}\frac{\lambda}{\gamma}$$

Approximate Model: Results



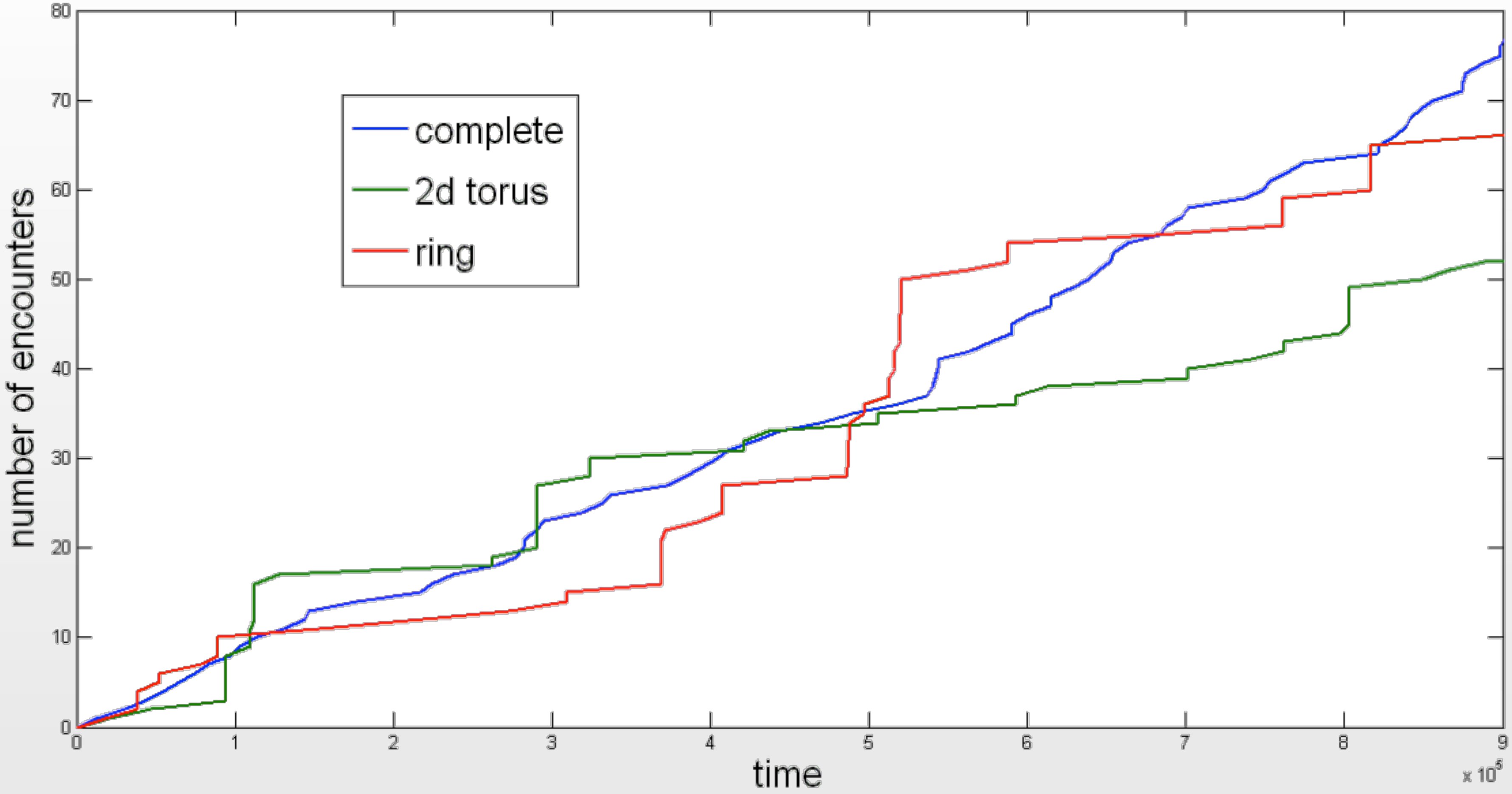
- approximate model very accurate for complete graph and inaccurate for torus and ring

Problem



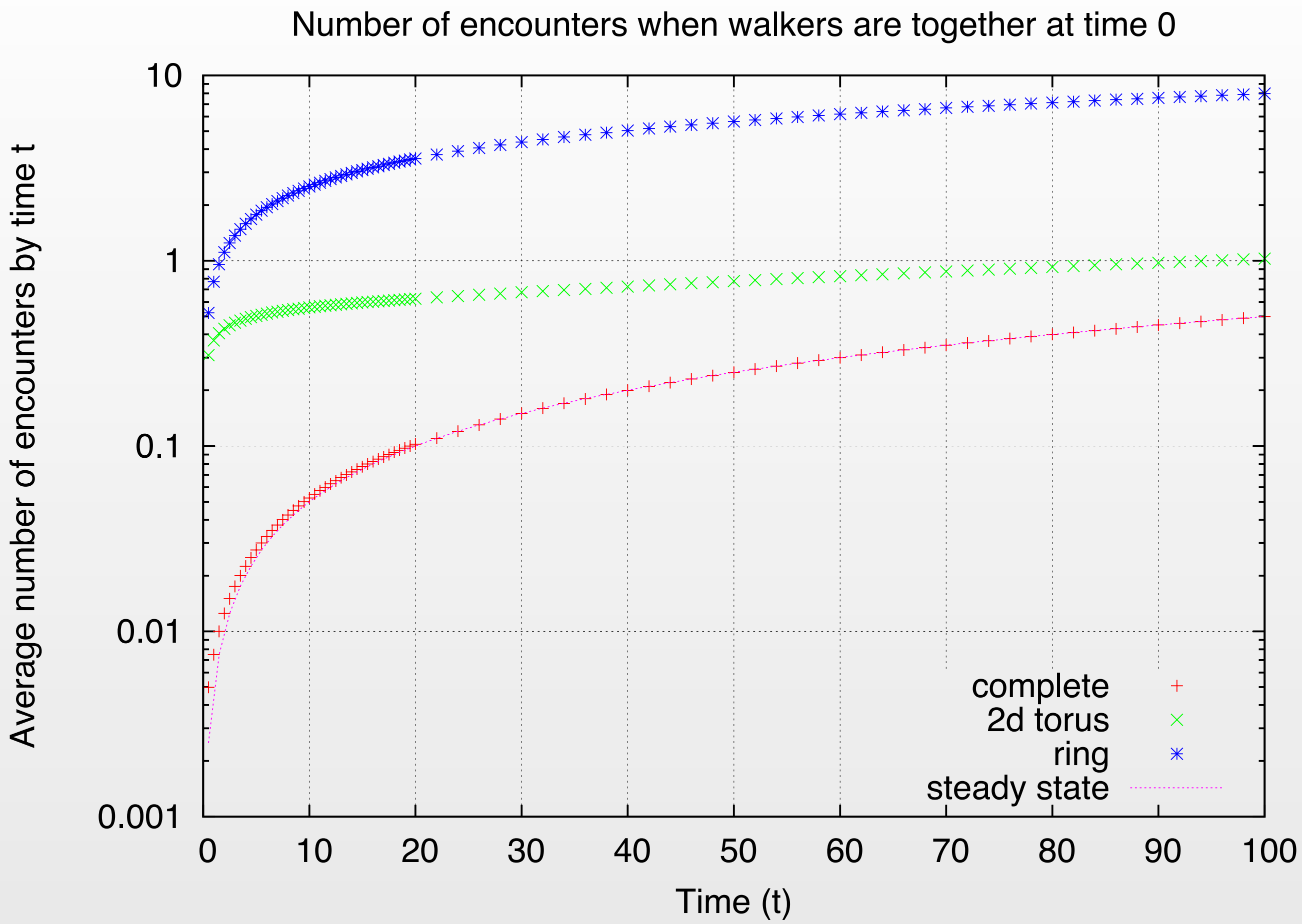
- encounters are bursty in torus and ring despite same average encounter rate

Problem



- encounters are bursty in torus and ring despite same average encounter rate

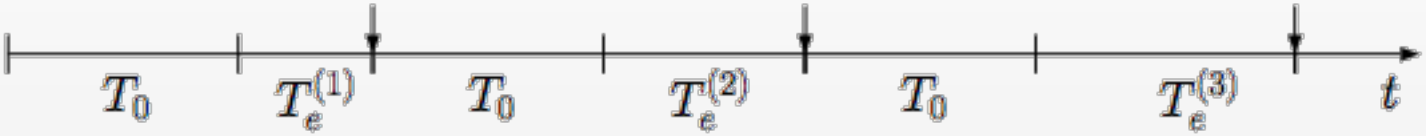
New Mechanism to estimate encounter rate



- $e(t)$: average number of encounters by time *t* given that they are together at time 0.

New Mechanism to estimate encounter rate

- we are interested in when the next encounter will occur after one of them recovers



- T_e : time for next encounter after $T_0 = \frac{1}{2\gamma}$

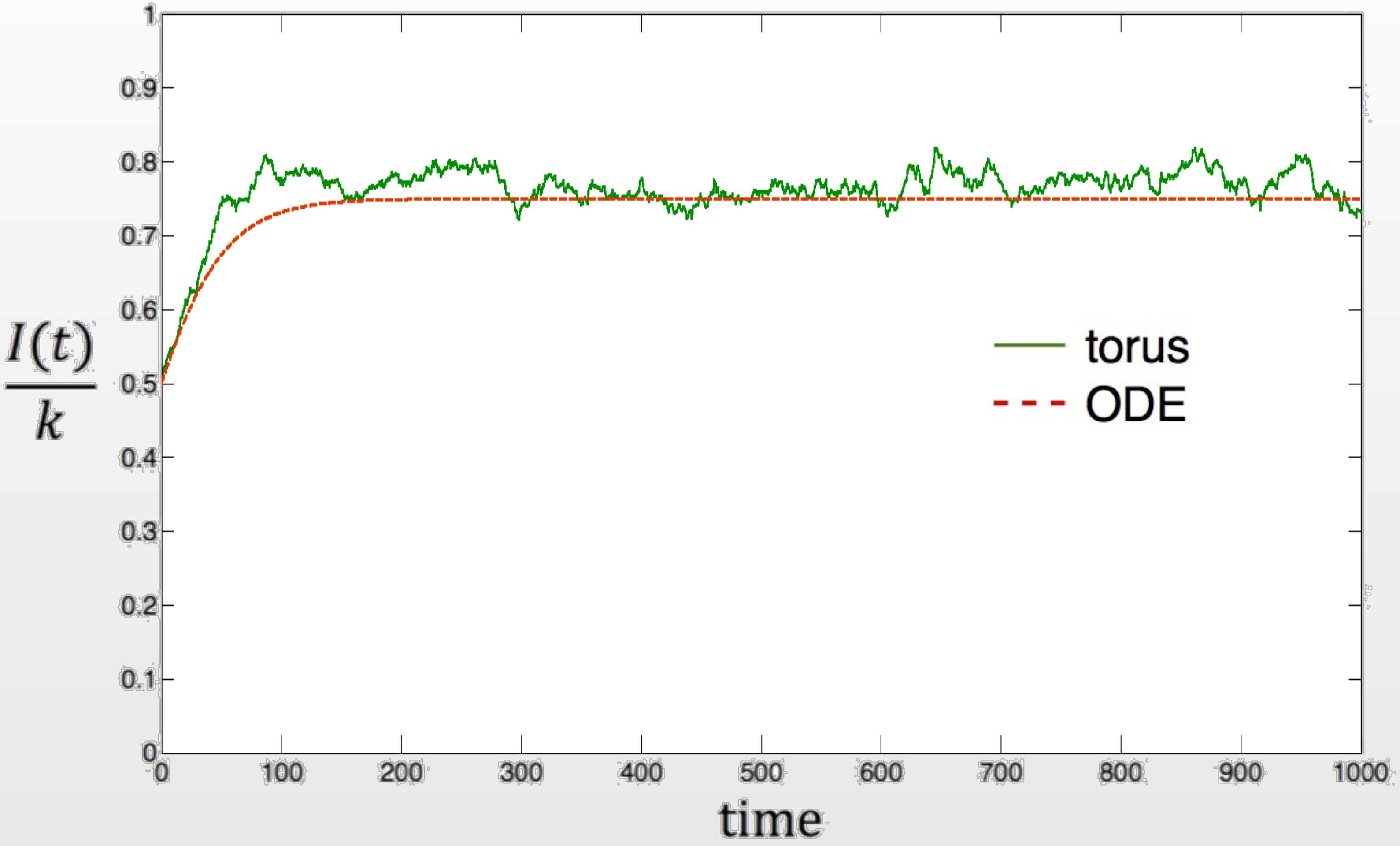
- approximate by the number of encounter in steady state:

$$\omega(T_0 + E[T_e]) = e(T_0) + 1$$

- new encounter rate: $\theta = \frac{1}{T_0 + E[T_e]} = \frac{\omega}{e(T_0)+1}$

- new equation: $\frac{dI}{dt} = \binom{k}{2}\theta\tau P_{SI} - \gamma I$

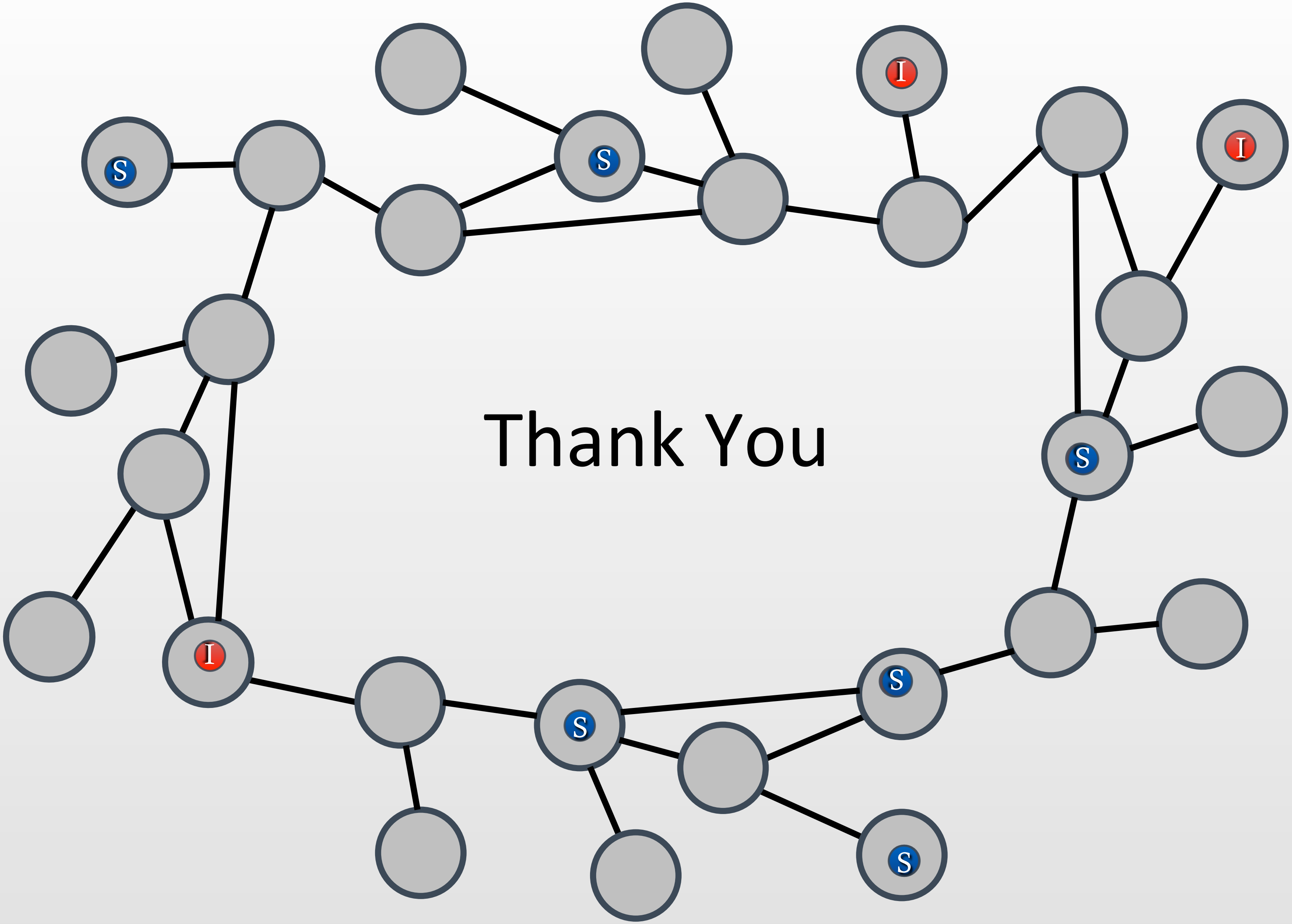
Preliminary Results



- better approximation with the new encounter rate

Conclusion

- Epidemic model on networks with mobility
- Coupled dynamics induces non trivial behavior
  - regular symmetric networks have different epidemic behaviors
- ODE approach seems adequate if correctly parameterized
  - need proper SI encounter rate
- Challenge (ongoing work): parameterize encounter rate
  - for non regular networks as well



Thank You